Some questions of the radiation reaction

Tamás Matolcsi
literature review by Péter Ván

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We have shown

1. the world tube breaks down for accelerations higher than $\frac{1}{\epsilon}$
2. the basic assumption is not valid, proved by Distribution Theory.

Scheme of the presentation

Usual notions of electric energy and force in statics are reappraised

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The role of the self-force and its relation to the LAD equation is examined

Distributions

distribution and Distribution

- measure $\lambda \to$ Distribution: $(\lambda \mid \psi) := \int \psi(x)\ d\lambda(x)$
- locally integrable function $f \to$ Distribution: $(f \mid \psi) := \int f(x)\psi(x)\ dx$
- locally non-integrable $\to$ pole taming $\to$ Distribution

Pole taming

- $|\boldsymbol{q}|$ is the length of the vector $\boldsymbol{q}$ in a three dimensional Euclidean space (the function $|\cdot|$ is the distance from the origin)
- $\frac{1}{|\cdot|^{2+m}}$ is not locally integrable if m is a positive integer
- $T_{\psi}^{(m-1)}$ is the Taylor polynomial of order $m-1$ of ψ at zero
- $\left(\mathrm{tm}\frac{1}{|\cdot|^{2+m}} \,\middle|\, \psi\right) := \int \frac{\psi(\boldsymbol{q})-T_{\psi}^{(m-1)}(\boldsymbol{q})}{|\boldsymbol{q}|^{2+m}} \, d\boldsymbol{q}$!!

Electrostatics

An important notion

- Charge distributions **extraneous** to each other: disjoint supports
- Electric field **extraneous** to a charge distribution: its producing charge distribution is extraneous to the one in question

Electrostatic energies (by Jackson)

- Point charge e at space point q_e, extraneous potential V, extraneous electric energy: $eV(q_e)$; charge density ρ, extraneous potential, extraneous electric energy density: ρV

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- 1. step: no work for the first charge, the self-energy is zero

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- 2. step:

$$\frac{e_1 e_2}{4\pi|q_2 - q_1|} = \frac{1}{2}\left(\frac{e_1 e_2}{4\pi|q_2 - q_1|} + \frac{e_1 e_2}{4\pi|q_1 - q_2|}\right) = \\ = \frac{1}{2}(e_1 V_2(q_1) + e_2 V_1(q_2))$$

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- n. step:

$$\frac{1}{2}\sum_{k \neq i = 1}^{n} \frac{e_k e_i}{4\pi|q_k - q_i|} = \frac{1}{2}\sum_{k=1}^{n}(e_k(V - V_k)(q_k))$$

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- More and more particles with smaller and smaller charges together $\rightarrow$ continuous charge distribution

Without a serious objection, $\frac{1}{2}\rho V$ can be accepted as the self-energy density of a charge density ρ in its own potential V

Electrostatic energies (by Jackson)

- $\rho V = (\mathrm{div}\boldsymbol{E})V = \mathrm{div}(\boldsymbol{E}V) - \boldsymbol{E} \cdot \mathrm{grad}\, V =$
  $= \mathrm{div}(\boldsymbol{E}V) + |\boldsymbol{E}|^2 \qquad (\mathrm{div}\boldsymbol{E} = \rho)$

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- The total self-energy (Gauss theorem)

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- For a point charge: the electric self-energy is infinite

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- 'Electric self-energy density' $= \frac{1}{2}|\boldsymbol{E}|^2$
- For a point charge: the electric self-energy is infinite

Comment

ρV is zero where ρ is zero but $|\boldsymbol{E}|^2$ is not zero there

> *It is unjustified* to consider $\frac{1}{2}|\boldsymbol{E}|^2$ the self-energy density of a static electric field $\boldsymbol{E}$. It has the only physical meaning: its integral – if it is integrable – over all the space is the total self-energy

Amazing and shocking:
first step "the electric self-energy of a point charge is zero"
conclusion "the electric self-energy of a point charge is infinite"

Electric self-energy of a point charge

- The infinite electric self-energy: three times incorrect reasoning:
  1. $\frac{1}{2}\rho V$ does not make sense for a point charge
  2. $\frac{1}{2}|\boldsymbol{E}|^2$ is not the self-energy density even in the continuous case
  3. Gauss theorem is not applicable for a point charge

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- $\frac{1}{2}\mathrm{tm}|\boldsymbol{E}|^2$, mathematical construction; it could have the only physical meaning: its 'integral' is the self-energy of the point charge (a Distribution is 'integrable' if its value on the constant function 1 makes sense)

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- $\frac{1}{2}\mathrm{tm}|\boldsymbol{E}|^2$ does have this physical meaning: the self-energy is zero, in accordance with the 'first step':

$$\left(\frac{1}{2}\mathrm{tm}|\boldsymbol{E}|^2 \;\middle|\; 1\right) = 0$$

Electrostatic forces

- Point charge e at space point q_e, extraneous electric field $\boldsymbol{E}$, extraneous force: $e\boldsymbol{E}(q_e)$; charge density ρ, extraneous electric field $\boldsymbol{E}$, extraneous force density: $\rho\boldsymbol{E}$

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  $-\mathrm{div}\boldsymbol{P} = \rho\boldsymbol{E}$ $\qquad$ ($\mathrm{div}\boldsymbol{E} = \rho$, $\boldsymbol{E}$ is produced by ρ)

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Comment

$\rho\boldsymbol{E}$ is zero where ρ is zero but $\boldsymbol{P}$ is not zero there:

It is unjustified to consider $\boldsymbol{P}$ a real stress tensor of a static electric field. It has the only physical meaning that its negative divergence is the self-force density

Electrostatic self-force of a point charge

- The fictitious stress tensor $\boldsymbol{P} := -\boldsymbol{E} \otimes \boldsymbol{E} + \frac{1}{2}|\boldsymbol{E}|^2 \mathbf{1}_{\mathsf{S}}$ is not locally integrable, it has a pole at the space point where the charge is

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- $\mathrm{tm}\boldsymbol{P}$, mathematical construction; it could have the only physical meaning: its negative divergence is the self-force of the point charge
- $\mathrm{tm}\boldsymbol{P}$ does have this physical meaning: the self-force is zero:

$$-\mathrm{div}(\mathrm{tm}\boldsymbol{P}) = 0$$

Going round

- The path we have taken in statics for self-energy and self-force:
  - point charge (known quantities)
  - system of point charges
  - continuous charge distribution
  - point charge by pole taming (resulting quantities)
- The resulting quantities are **equal** to the starting ones

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- The resulting quantities are **equal** to the starting ones
- Leaving statics, we shall take a similar path: the **unknown** starting quantity, the self-force, will be **obtained** by the resulting one

Beyond statics

- Point charge e, velocity $\boldsymbol{v}$, extraneous electric and magnetic field $\boldsymbol{E}$ and $\boldsymbol{B}$, extraneous force: $e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$

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- More and more particles with smaller and smaller charges together $\rightarrow$ continuous charge distribution:

$$\rho(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) + ?$$

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- More and more particles with smaller and smaller charges together $\to$ continuous charge distribution:

$$\rho(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})+?$$

$? \neq 0$ probably

Contrary to electrostatics, *it is questionable* that $\rho(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})=\rho\boldsymbol{E}+\boldsymbol{i}\times\boldsymbol{B}$ is the self-force density of ρ in its own fields $\boldsymbol{E}$ and $\boldsymbol{B}$

Spacetime formulae

T. Matolcsi: *Spacetime without Reference Frames*, Minkowski Institute Press, 2020

- Spacetime points $x, y \ldots$ spacetime vectors $\boldsymbol{x}, \boldsymbol{y}, \ldots$
- If x, y are spacetime points then $x - y$ is a spacetime vector
- $\boldsymbol{x} \cdot \boldsymbol{y} \sim x_k y^k, \quad \boldsymbol{L} \cdot \boldsymbol{x} \sim L^{ik} x_k, \qquad \boldsymbol{x} \otimes \boldsymbol{y} \sim x^i y^k$
- absolute velocity $\boldsymbol{u}$, $\boldsymbol{u} \cdot \boldsymbol{u} = -1 \quad ((-1, 1, 1, 1))$
- Spacetime differentiation $\mathrm{D} \sim \partial_k, \qquad \mathrm{D} \cdot \boldsymbol{T} \sim \partial_k T^{ik}$

Spacetime formulation; continuous case

- Absolute current density $\boldsymbol{j} \sim (\rho, \boldsymbol{i})$
- Electromagnetic field $\boldsymbol{F} \sim (\boldsymbol{E}, \boldsymbol{B})$
- $\boldsymbol{F} \cdot \boldsymbol{j} \sim (\boldsymbol{E} \cdot \boldsymbol{i}, \rho \boldsymbol{E} + \boldsymbol{i} \times \boldsymbol{B})$; extraneous absolute force density acting on $\boldsymbol{j}$ in an extraneous field $\boldsymbol{F}$
- $\boldsymbol{F} \cdot \boldsymbol{j} + ?$ absolute self-force density of $\boldsymbol{j}$ in its own field $\boldsymbol{F}$

It is questionable that $\boldsymbol{F} \cdot \boldsymbol{j}$ is the absolute self-force density of $\boldsymbol{j}$ in its own field $\boldsymbol{F}$

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- 'Energy-momentum tensor' $\boldsymbol{T} := -\boldsymbol{F} \cdot \boldsymbol{F} - \frac{1}{4}(\mathrm{Tr}\boldsymbol{F} \cdot \boldsymbol{F})\mathbf{1}$
- The time-time component of $\boldsymbol{T}$ is $\frac{1}{2}(|\boldsymbol{E}|^2 + |\boldsymbol{B}|^2)$
- The space-space component of $\boldsymbol{T}$ is $-\boldsymbol{E} \otimes \boldsymbol{E} + \frac{1}{2}|\boldsymbol{E}|^2 \mathbf{1}_{\mathsf{S}} + \dots$

It is unjustified to consider $\boldsymbol{T}$ a real energy-momentum tensor

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It is unjustified to consider $\boldsymbol{T}$ a real energy-momentum tensor

- $-\mathrm{D} \cdot \boldsymbol{T} = \boldsymbol{F} \cdot \boldsymbol{j} \qquad (\mathrm{D} \cdot \boldsymbol{F} = \boldsymbol{j},$ $\boldsymbol{F}$ is produced by $\boldsymbol{j})$

It is questionable that $\boldsymbol{T}$ has the physical meaning that its negative spacetime divergence is the absolute self-force density

Spacetime formulation; point charge

- Existence of a point charge in spacetime:
  world line function r (proper time$\rightarrow$spacetime)
- Absolute velocity of the point charge:
  $\dot{r}$ (proper time$\rightarrow$spacetime vector)

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- World line: $\mathrm{Ran}\, r$ (one dimensional submanifold)
- Lebesgue measure $\lambda_{\mathrm{Ran}\, r}$

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- Lebesgue measure $\lambda_{\mathrm{Ran}\,r}$
- A point charge e, world line function r,
  spacetime current: $e\dot{r}\lambda_{\mathrm{Ran}\,r}$

Spacetime formulation; point charge

- **Given** r, retarded proper time $s_r(x)$:
  $x - r(s_r(x))$ is future-lightlike
- A point charge e, **given** r, produced electromagnetic potential (Liénard–Wiechert)

$$x \mapsto \frac{e}{4\pi} \frac{\dot{r}(s_r(x))}{-\dot{r}(s_r(x)) \cdot (x - r(s_r(x)))}$$

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- **Given** r, retarded proper time $s_r(x)$:
  $x - r(s_r(x))$ is future-lightlike
- A point charge e, **given** r, produced electromagnetic potential (Liénard–Wiechert)

$$x \mapsto \frac{e}{4\pi} \frac{\dot{r}(s_r(x))}{-\dot{r}(s_r(x)) \cdot (x - r(s_r(x)))}$$

- Electromagnetic field $\boldsymbol{F}[r]$ is some functional of the **given** r

Spacetime formulation; point charge

- Fictitious energy-momentum tensor $\boldsymbol{T}[r] := -\boldsymbol{F}[r] \cdot \boldsymbol{F}[r] - \frac{1}{4}(\mathrm{Tr}\boldsymbol{F}[r] \cdot \boldsymbol{F}[r])\mathbf{1}$ is some functional of the **given** r

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- $\mathrm{tm}\,\boldsymbol{T}[r]$, mathematical construction; it could have the only physical meaning: its negative spacetime divergence is the self-force of the point charge but even this meaning is questionable
- It is proved that

$$-\mathrm{D} \cdot \mathrm{tm}\,\boldsymbol{T}[r] = \frac{1}{4\pi}\frac{2e^2}{3}(\dot{r} \wedge \dddot{r}) \cdot \dot{r}\lambda_{\mathrm{Ran}\,r}$$

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- We obtained the usual self-force for a **given** world line function, thus it can be possibly accepted only for a **given** spacetime existence of a point charge
- Enlightment:
  - if the force $\boldsymbol{f}$ acts on the point charge
  - $m\ddot{r} = \boldsymbol{f}(r, \dot{r})$ would valid if there were no radiation
  - then the self-force $\frac{1}{4\pi}\frac{2e^2}{3}(\dot{r} \wedge \dddot{r}) \cdot \dot{r}$ must be compensated

  by an opposite force
  - in order that $m\ddot{r} = \boldsymbol{f}(r, \dot{r})$ remain valid in case of radiation
- An actual example is an elementary particle revolved in a cyclotron

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- Enlightment:
  - if the force $\boldsymbol{f}$ acts on the point charge
  - then the formula for the self-force cannot be added to $\boldsymbol{f}$
  - to obtain the Newtonian-like LAD equation

  $m\ddot{r} = \boldsymbol{f}(r, \dot{r}) + \frac{1}{4\pi}\frac{2e^2}{3}(\dot{r} \wedge \dddot{r}) \cdot \dot{r}$

  for the world line function r

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- We obtained the usual self-force for a **given** world line function
- It is unjustified to put this self-force in a Newtonian-like equation to obtain the LAD equation which would serve **to determine** the world line function
- The LAD equation is a misconception; its pathological properties are not surprising

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- At present we have not a well working theory of **interaction** which would define both $\boldsymbol{F}$ and r together

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- Usual formulation: a point charge under the action of a given force $\boldsymbol{f}$, the balance equation

$$-\mathrm{D} \cdot (\mathcal{T}_m[r] + \mathcal{T}_e[\boldsymbol{F}]) + \boldsymbol{f}(r, \dot{r})\lambda_{\mathrm{Ran}\,r} = 0$$

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- It seems evident that $\boldsymbol{\mathcal{T}}_m[r]=m\dot{r}\otimes\dot{r}\lambda_{\mathrm{Ran}\,r}$ and then

$$m\ddot{r}\lambda_{\mathrm{Ran}\,r}=\boldsymbol{f}(r,\dot{r})\lambda_{\mathrm{Ran}\,r}-\mathrm{D}\cdot\boldsymbol{\mathcal{T}}_e[F]$$

$$\mathrm{D}\cdot\boldsymbol{F}=\dot{r}\lambda_{\mathrm{Ran}\,r},\qquad \mathrm{D}\wedge\boldsymbol{F}=0$$

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- It is not sure that this is a good system of equations with a convenient $\mathcal{T}_e[\boldsymbol{F}]$
- It is sure that $\mathcal{T}_e[F]$ cannot be replaced with $\mathrm{tm}\,\boldsymbol{T}[r]$ obtained for a **given** r

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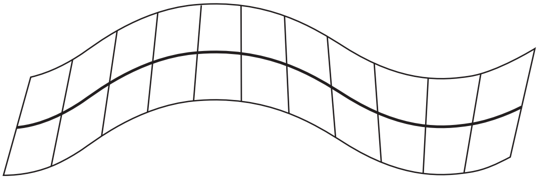

Radiation reaction: recent literature

Peter Ván

Wigner RCP

Budapest, 30.11.2020.

Major approaches

- Nonlocal modifications of the electromagnetic part, the Maxwell equations, [1, 2, 3].
- Dissipative modifications of the mechanical part, the Newton equation [4, 5, 6, 7].
- A suitable interpretation of the LAD equation, e.g. excluding particular solutions, [8, 9, 10].
- Application of continuum charge distributions instead of point charges. This is the method of the classical papers of Lorentz, Abraham and Dirac, too, [11, 12, 13, 8]. There are two main aspects of this strategy:
  - One may improve the classical theory, with the identification and elimination of the mathematical problems, [14, 15],
  - One may modify the point charge model with the help of quantum mechanics, or with various renormalization procedures, [16, 17, 18, 19, 20, 21].

Related issues

- The recent experiments of radiation reaction related phenomena, see, e.g. [22, 23, 24], open the way toward the verification of the mentioned theories.
- There are less rigorous, but more applicable (?) approaches, [25, 26, 27].

M. Born and L. Infeld.
Foundations of the new field theory.
Proceedings of the Royal Society of London. Series A, 144(852):425–451, 1934.

F. Bopp.
Eine lineare Theorie des Elektrons.
Annalen der Physik, 430(5):345–384, 1940.

M. K.-H. Kiessling.
Force on a point charge source of the classical electromagnetic field.
Physical Review D, 100(6):065012, 2019.
arXiv:1907.11239v4, with erratum.

G.W. Ford and O'Connell.
Radiation reaction in electrodynamics and the elimination of runaway solutions.
Physics Letters A, 157(4-5):217–220, 1991.

J. Verhás.

White areas on the map of applying non-equilibrium thermodynamics: on the self accelerating electron.
Atti dell'Accademia Peloritana dei Pericolanti, Classe di Scienze Fisiche, Matematiche e Naturali, Suppl. I., 86:1–7, 2008.

J. Polonyi.
Effective dynamics of a classical point charge.
Annals of Physics, 342:239–263, 2014.

M. Formanek, A. Steinmetz, and J. Rafelski.
Radiation reaction friction: Resistive material medium.
Physical Review D, 102:056015, 2020.
arXiv:2004.09634.

P. A. M. Dirac.
Classical theory of radiating electrons.
Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences, 167(929):148–169, 1938.

H. Spohn.
The critical manifold of the Lorentz-Dirac equation.

EPL (Europhysics Letters), 50(3):287, 2000.

Matolcsi T., Fülöp T., and Weiner M.
Second order equation of motion for electromagnetic radiation back-reaction.
Journal of Modern Physics, 32(27):1750147(18), 2017.
arXiv:1207.0428.

H.A. Lorentz.
La théorie électromagnétique de Maxwell et son application aux corps mouvants.
Archives Neerlandaises des Sciences Exactes et Naturelles, 25:363–552, 1892.

Max Abraham.
Theorie der Elektrizität, Vol. II: Elektromagnetische Theorie der Strahlung.
Teubner, Leipzig, 1905.

H.A. Lorentz.
The Theory of Electrons.
Teubner, Leipzig, 1909.

S. Parrott.

Relativistic electrodynamics and differential geometry.
Springer, 2012.

C. Bild, D.-A. Deckert, and H. Ruhl.
Radiation reaction in classical electrodynamics.
Physical Review D, 99(9):096001, 2019.

E. J. Moniz and D.H. Sharp.
Absence of runaways and divergent self-mass in nonrelativistic quantum electrodynamics.
Physical Review D, 10(4):1133, 1974.

E. J. Moniz and D.H. Sharp.
Radiation reaction in nonrelativistic quantum electrodynamics.
Physical Review D, 15(10):2850, 1977.

C. R. Galley, A. K. Leibovich, and I. Z. Rothstein.
Finite size corrections to the radiation reaction force in classical electrodynamics.
Physical review letters, 105(9):094802, 2010.

P. Forgács, T. Herpay, and P. Kovács.
Comment on "Finite Size Corrections to the Radiation Reaction Force in Classical Electrodynamics".
Physical review letters, 109(2):029501, 2012.

C. R. Galley, A.K. Leibovich, and I.Z. Rothstein.
Galley, Leibovich, and Rothstein reply.
Physical Review Letters, 109(2):029502, 2012.

J. Polonyi.
The Abraham–Lorentz force and electrodynamics at the classical electron radius.
International Journal of Modern Physics A, 34(15):1950077, 2019.

T.N. Wistisen, A. Di Piazza, H. V. Knudsen, and U. I. Uggerhoj.
Experimental evidence of quantum radiation reaction in aligned crystals.
Nature communications, 9(1):1–6, 2018.

K. Poder, M. Tamburini, G. Sarri, A. Di Piazza, S. Kuschel, C.D. Baird, K. Behm, S. Bohlen, J.M. Cole, D.J. Corvan, and +other 14 authors.

Experimental signatures of the quantum nature of radiation reaction in the field of an ultraintense laser.
Physical Review X, 8(3):031004, 2018.

T.G. Blackburn.
Radiation reaction in electron–beam interactions with high-intensity lasers.
Reviews of Modern Plasma Physics, 4(1):1–37, 2020.

L. D. Landau and E. M. Lifshitz.
The Classical Theory of Fields (Course of Theoretical Physics, vol. 2).
Pergamon Press, Oxford, 2th edition, 1959.

I.V. Sokolov.
Renormalization of the Lorentz-Abraham-Dirac equation for radiation reaction force in classical electrodynamics.
Journal of Experimental and Theoretical Physics, 109(2):207–212, 2009.

D.B. Zot'ev.
Critical remarks on Sokolov's equation of the dynamics of a radiating electron.
Physics of Plasmas, 23(9):093302, 2016.