2024:

Late-time power-law behaviour of the electromagnetic field in rotating black hole background. — It is known that small perturbations of black holes exhibit power-law decay in time in the last phase of their evolution. We studied this behaviour by numerical simulations in the case of a sub-extremal Kerr black hole perturbed by an electromagnetic field [1]. We focused on the spin zero component of the perturbing field, which is governed by the Fackerell–Ipser equation, to complement previous investigations focused mainly on the spin ±1 components, governed by the Teukolsky master equation. We used horizon-penetrating compactified hyperboloidal coordinates, which allowed us to observe the behaviour of the perturbing field at the event horizon and at future null infinity as well (see Fig. 1). We determined the exponents characterizing the power-law decay of the spherical harmonic components of the field for various values of the parameters of the initial data, and based on the results we made a proposal for a Price's law relevant to the Fackerell–Ipser equation. In comparison with the spin ±1 components, a remarkable feature of the decay exponents we found is that they show much less dependence on the parameters pertaining to the initial data and the late-time field.

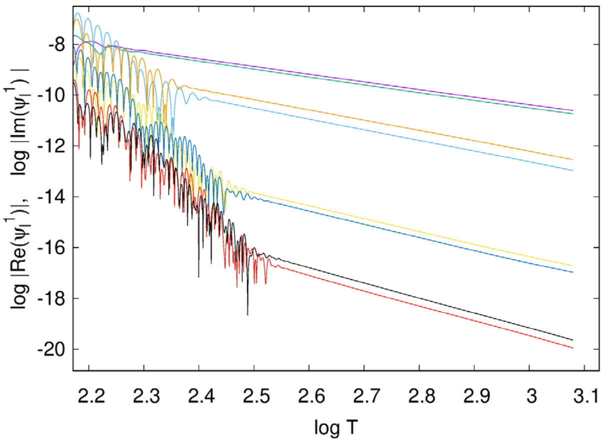

Figure 1. Behaviour of the first few spherical harmonic components of the perturbation as T → ∞ in the case m = 1, a/M = 0.5, at future null infinity.

Feynman rules for gauge-invariant operators in QCD. — The description of hadrons is an important topic in contemporary particle physics. In particular, the full connection between hadronic properties and properties of the constituent quarks and gluons is still not completely clear. For example, there is the long standing proton-spin puzzle which poses the question of how the spin-1/2 of the proton can be explained in terms of the spins and orbital angular momenta of the partons inside the proton. Valuable insight into such questions can be gained by performing high-energy scattering experiments that resolve the inner structure of the proton, which in QCD is described by non-perturbative (generalized) parton distributions. The study of these distributions is a major objective of current and future colliders, such as the high-luminosity LHC and the upcoming electron-ion colliders.

An important aspect of phenomenological studies is the energy scale dependence of the distributions, which is determined by the anomalous dimensions of the operators that define them. The latter can in turn be computed from the corresponding operator matrix elements if their Feynman rules known. In ref. [2], we derived these Feynman rules for the relevant leading-twist gauge-invariant operators with an arbitrary number of derivatives and an arbitrary number of gluons. As such, our results are applicable to all orders in perturbation theory and hence provide an important extension of previously known fixed-order expressions. In order to facilitate the use of our formulae in practical calculations, we also provide implementations in Mathematica and FORM, available at https://github.com/vtsam/NKLO.

Oscillatory tails of weakly delocalized solitons. — The Korteweg-de Vries (KdV) equation describes weakly and nonlinearly interacting shallow water waves. If the KdV equation is modified by a fifth order derivative term the familiar solitary wave solutions are deformed into oscillon-type objects, losing energy by radiating small amplitude waves in the direction of propagation. We show by asymptotic matching techniques that the radiation amplitude can be obtained from the central asymmetry of the unique solution exponentially decaying in one direction [3]. This observation, complemented by some fundamental results of Hammersley and Mazzarino, not only sheds new light on the computation of the radiation amplitude but also greatly facilitates its numerical determination to a remarkable precision, which is beyond the capabilities of standard numerical methods.

References

[1] I. Rácz and G. Zs. Tóth, *Numerical investigation of the late-time tails of the solutions of the Fackerell-Ipser equation*, General Relativity and Gravitation **56,** 131 (2024) https://doi.org/10.1007/s10714-024-03316-7

[2] G. Somogyi and S. Van Thurenhout, *All-order Feynman rules for leading-twist gauge-invariant operators in QCD*, Eur. Phys. J. C **84,** no.7, 740 (2024) https://doi.org/10.1140/epjc/s10052-024-13071-3

[3] G. Fodor, P. Forgács and M. Mushtaq, New derivation of the amplitude of asymptotic oscillatory tails of weakly delocalized solitons, Phys. Rev. D **109,** 125011 (2024) https://doi.org/10.1103/PhysRevD.109.125011