2024

Bayesian inference of causal relations between dynamical systems [1].

From ancient philosophers to modern economists, biologists, and other researchers, there has been a continuous effort to unveil causal relations. The most formidable challenge lies in deducing the nature of the causal relationship: whether it is unidirectional, bidirectional, or merely apparent — implied by an unobserved common cause [2]. While modern technology equips us with tools to collect data from intricate systems such as the planet's ecosystem or the human brain, comprehending their functioning requires the identification and differentiation of causal relationships among the components, often without external interventions. In this context, we introduce a novel method capable of distinguishing and assigning probabilities to the presence of all potential basic causal relations between two or more time series within dynamical systems. The efficacy of this method is verified using synthetic datasets (Fig. 1.) and applied to EEG (electroencephalographic) data recorded from epileptic patients. Given the universal applicability of our method, it holds promise for diverse scientific fields.

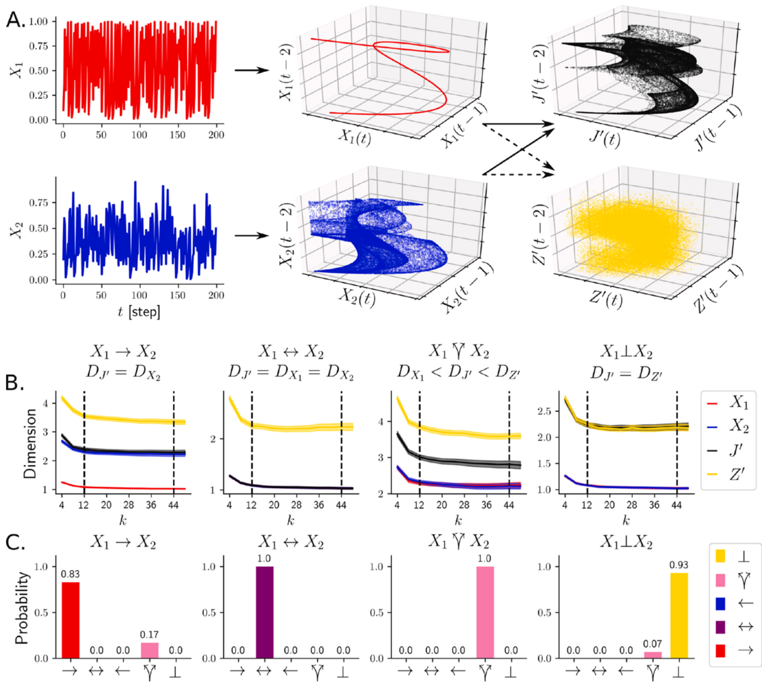

Figure 1. The workflow and testing of our method on coupled logistic map systems. (A) Time series (left) of the driver and the driven time series. Next to them, the state spaces of the systems are reconstructed by time delay embedding of the two time series (red) and (blue), resulting in the red and blue manifolds. Then, the joint of the two datasets, and their time-shuffled version are also embedded, resulting in a reconstruction of the joint state space of the two subsystems (black manifold) and their independent joint (yellow). On (B, C) the test of our method on the four simulated examples of five possible causal interactions (one of the unidirectional, bidirectional:, unidirectional backward, common cause and independence) are demonstrated. (B) The intrinsic dimensionality of each manifold is estimated for different neighborhood sizes. The plateau of dimension-estimates identifies where the estimates can be considered reliable (between dashed lines). Note the match between the actual causal and dimensional relationships: the dimension of the joint manifold relative to the others. (C) Posterior probabilities of the possible causal relationships. The method correctly assigned the highest probability to the actual causal relation in each case.

Validation of dimensional causality on EEG data (Fig 2.). — we aim to assess our approach under real-world conditions, where the true dimensionality and the properties of the noise are unknown. While the precise causal relationships between time series in these systems are not known, external factors can induce changes in the internal causal relationships that our analysis method can detect. Notably, the standard epilepsy-diagnostic photo-stimulation procedure, where patients are exposed to flashing light at different frequencies in a standardized test, serves as an ideal model for an external common cause affecting the two brain hemispheres.

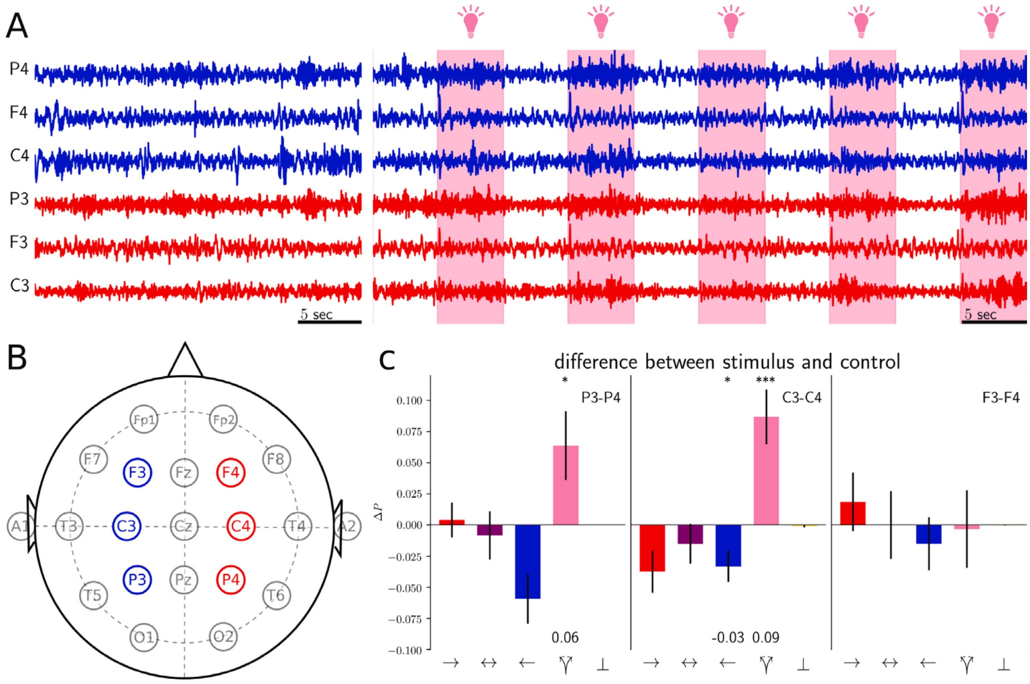

Figure 2. Inter-hemispherical interactions during photo-stimulation. (A) CSD signal in control condition and photo-stimulation periods (light bulbs) at the six analyzed recording-channels. (B) Electrode positions on the scalp. Causal relations were computed between P3–P4, C3–C4 and F3–F4 channel pairs. (C) Difference in probabilities of causal relations between stimulation and control (mean and SE). The probability of the existence of common cause is significantly higher during stimulation periods for P3–P4 and C3–C4 channel-pairs but not for F3–F4.

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