2023

Higher order corrections to beyond-all-order effects in a fifth order Korteweg–de Vries equation. — The Korteweg-de Vries (KdV) equation describes shallow water waves that are weakly and nonlinearly interacting, ion acoustic waves in a plasma, long internal waves in a density, and other phenomena. The KdV equation modified by a fifth order derivative term (fKdV) plays an important role in many applications in plasma physics and in hydrodynamics. A crucial point of interest of the fKdV equation from our point of view is that the familiar solitary wave solutions of the KdV equation are deformed into oscillon-type objects, losing continuously some of their mass by radiating small amplitude waves in the direction of propagation. Perturbative and numerical methods have been developed to calculate approximate and numerical solutions when the coefficient of the fifth order term is small. These studies have clearly indicated that the localized soliton solution becomes delocalized, in that a large core is connected to small amplitude wave trains.
We have computed higher order perturbative corrections to such asymptotic wave tails of the fKdV equation, clarifying a long-standing puzzle concerning the 2^{nd} order perturbative correction term and developed a very performant, arbitrary precision spectral code to compute numerical solutions [1].

Exact mass corrections to Higgs production in association with a jet. — Since the discovery of the Higgs boson, one of the principal objectives of the Large Hadron Collider physics program has been to investigate its couplings and quantum numbers with the highest attainable accuracy. Such measurements are extremely important, as subtle deviations from Standard Model predictions could potentially be footprints of New Physics (NP). One promising observable to probe possible NP effects is the Higgs transverse momentum (pT) distribution, which must be modeled as precisely as possible. In particular, effects associated with the finite masses of top and bottom quarks must be included in theoretical predictions. Therefore, we have computed the Higgs pT distribution at next-to-leading order (NLO) accuracy in the strong coupling with top quark mass dependence and for the first time also including the exact bottom quark mass contribution [2]. Since in our computation the top and bottom quark masses are treated as dynamical parameters and not as fixed numbers, we were able to compute also for the first time the Higgs pT distribution at NLO using a dynamical mass renormalization scheme, the MSbar scheme, see Fig. 1. Besides establishing the impact of the bottom quark mass contribution and the size of the effects associated with renormalization scheme dependence, our work has opened up new avenues of investigation. For example, treating the top quark mass as a dynamical parameter will allow the measurement of the pT spectrum of the Higgs boson to be translated into a measurement of the running mass of the top quark.

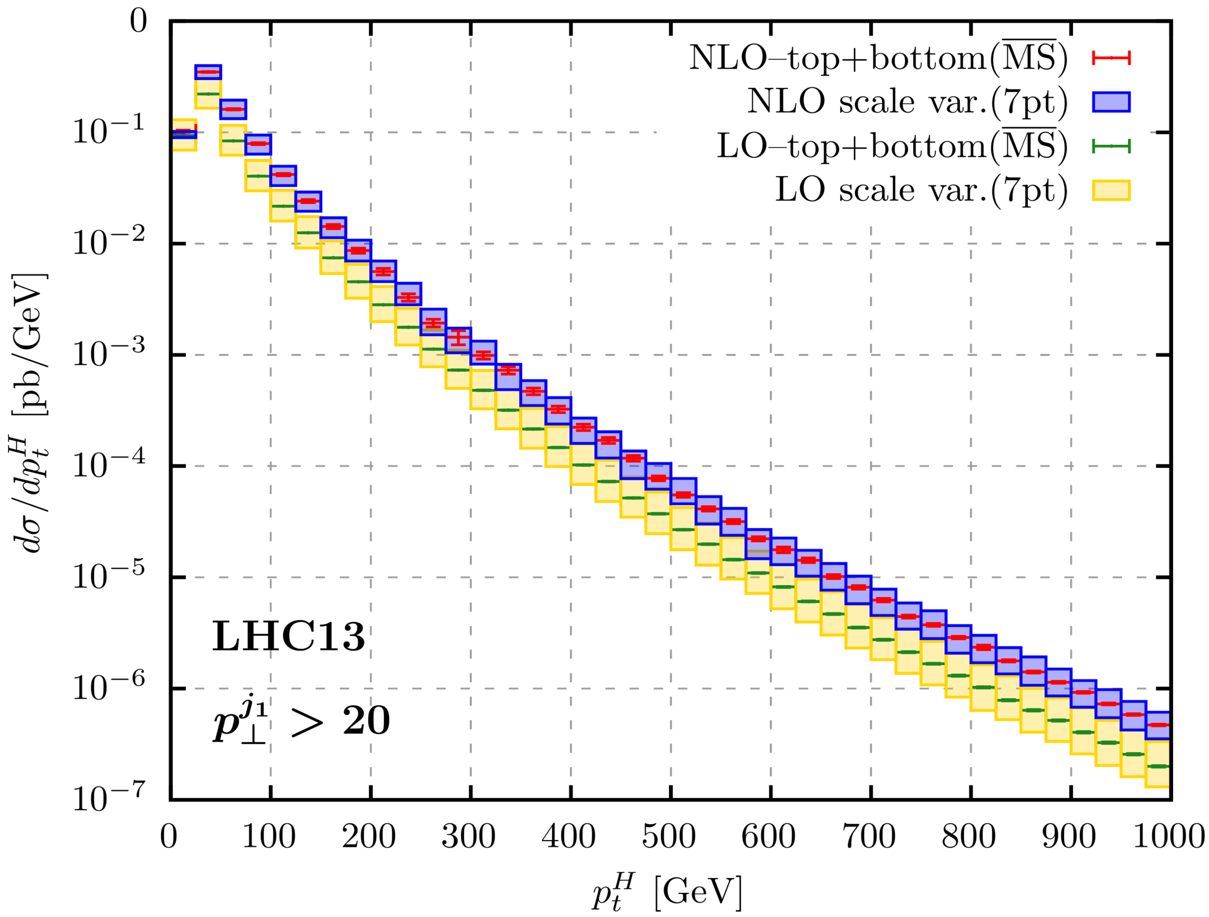

Figure 1. The Higgs boson pT distribution with top and bottom quarks.

Group theoretic approach to integrable Hamiltonian systems. — One of the principal ideas for studying integrable Hamiltonian systems is to represent the systems of interest as low dimensional projections (alias Hamiltonian or Poisson reductions) of higher dimensional manifestly solvable systems with rich symmetries. In the paper [3] we applied this method starting from the natural phase spaces provided by the three `classical doubles' of any simple, connected and simply connected compact Lie group G: the cotangent bundle, the Heisenberg double and the internally fused quasi-Poisson double. On each double we identified a pair of `master integrable systems' and investigated their Poisson reductions. In the simplest cotangent bundle case, the reduction was defined by taking quotient by the cotangent lift of the conjugation action of G on itself, and this was then generalized to the other two doubles. In each case, we derived explicit formulas for the reduced Poisson structure and equations of motion, and found that they are associated with well-known classical dynamical r-matrices. This led to a unified treatment of a large family of reduced systems, which contains new integrable models as well as well familiar spin Sutherland and Ruijsenaars-Schneider models. In the paper [4] similar ideas were applied to a spin extension of the quasi-Poisson double of the unitary group U(n), resulting in new multi-Hamiltonian systems living on the pertinent reduced phase space.

The `most classical' states of SU(2) and Euclidean-invariant quantum mechanical systems. — In [5], all the states of the SU(2) invariant quantum mechanical systems are determined in which the equality holds in the uncertainty relations for the components of the angular momentum vector operator in two given directions. Allowing the angle between the two angular momentum components to be arbitrary, *a new genuine quantum mechanical phenomenon emerges*: it is shown that although the standard deviations change continuously, one of the expectation values changes *discontinuously* on the classical parameter space parameterizing the states. Since physically neither of the angular momentum components is distinguished over the other, this

discontinuity suggests that the genuine parameter space must be a *double cover* of this classical one: it must be diffeomorphic to a *Riemann surface* known in connection with the complex function √z. The consequences in the *simultaneous* measurements of these angular momentum components are also discussed briefly.
In [6], *all* the states in which the equality holds in uncertainty relations for non-commuting basic observables of Euclidean invariant elementary quantum mechanical systems, i.e. the analog of the coherent states of these systems, were determined. Contrary to expectations, these solutions show that the existence of such states depends not only on the Lie algebra, but on the choice of its generators as well. In particular, none of the characteristic properties of the well-known coherent states of Heisenberg systems holds.

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2022

The geometry of 3-space from quantum mechanics. — According to the so-called Spin Geometry Theorem of Penrose, the angles between directions of the Euclidean 3-space, i.e. its conformal structure, can be recovered in the classical limit from his SU(2)-spin network. In the publication [1 (https://link.springer.com/article/10.1007/s10701-022-00616-3)], we gave a simple, elementary proof of this result in the algebraic formulation of quantum mechanics: We introduced the notion of `empirical quantum angle' between the angular momentum vectors of elementary SU(2)-invariant quantum mechanical systems, and showed that, in the classical limit, these angles tend with asymptotically vanishing uncertainty to angles between directions of the Euclidean 3-space.
In the publication [2 (https://link.springer.com/article/10.1007/s10701-022-00617-2)], the previous investigations and results were extended from SU(2) to E(3) (i.e. Euclidean) invariant quantum mechanical systems. We introduced the `empirical distance' between elementary E(3)-invariant

quantum mechanical systems exclusively in terms of quantum observables, and we show that, in the classical limit, these distances tend with asymptotically vanishing uncertainty to distances between straight lines of the Euclidean 3-space. Thus, the metric structure of the Euclidean 3-space can also be recovered from a purely algebraic formulation of quantum mechanics, i.e. the geometry of the 3-space emerges from abstract quantum mechanics.

Bi-Hamiltonian structures of integrable many-body models. — Exactly solvable models of interacting particles moving along one-dimensional space govern special solutions of soliton equations and appear ubiquitously in mathematical physics. Integrable dynamical systems often admit alternative Hamiltonian formulations, where two different Hamiltonians generate the same dynamics via compatible Poisson bracket structures; and this can be used to explain the solvability of such systems. We discovered the bi-Hamiltonian structure behind a recently found integrable model of an arbitrary number of mass points moving on the circle, in which the coupling parameters depend on two spin variables interacting with each other and with the particles. The model has been related previously to Higgs bundles on an algebraic curve, but we provided another derivation viewing it as a symmetry reduction of a free particle on the complex general linear group of n by n matrices, which led to the bi-Hamiltonian structure [3 (https://iopscience.iop.org/article/10.1088/1361-6544/ac6c71)]. Our reduction method also led to a new group theoretic interpretation of the bi-Hamiltonian structure of the Toda molecule, a prime example of integrable Hamiltonian systems [4 (https://link.springer.com/article/10.1007/s11005-022-01537-y)].

Particle Physics Applications of Mellin-Barnes Integrals. — The successful operation of the Large Hadron Collider (LHC) has opened up a new era of discovery in particle physics. However, in order to fully exploit the physics potential of the LHC, precise measurements must be confronted with accurate theoretical predictions. As the latter are computed using perturbative quantum field theory (pQFT), the development of methods for performing higher-order calculations in pQFT is of paramount importance. In particular, the evaluation of complicated multi-loop Feynman integrals must be addressed. One particular approach to this problem is the method of Mellin-Barnes integrals. This approach has been successfully applied to the analytic and numeric analysis of virtual and real higher-order perturbative corrections in state-of-the-art problems and its further development is an area of active research. In the book [5 (https://link.springer.com/book/10.1007/978-3-031-14272-7)], we have endeavored to provide an easy-to-follow, yet thorough introduction to this important technique at the level of advanced students and young researchers. The book is supplemented with extensive online material demonstrating the practical use of the described methods and software packages.

Energy-momentum tensor of linearized gravity. The problem of finding a satisfactory definition of the energy and momentum of the gravitational field has received very much attention. It is known, that a local energy-momentum tensor of the linearized gravitational field, that has all the desirable properties one would expect, does not exist. Nevertheless, a less ideal energy-momentum with sufficiently good properties might still be found – this is what we aimed to achieve in [6 (https://iopscience.iop.org/article/10.1088/1361-6382/ac50eb)]. We found an energy-momentum tensor (T_{lg}) for the linearized gravitational field that exhibits remarkable similarity to the standard energy-momentum tensor of the electromagnetic field, and thus has several desirable properties. Notably, T_{lg} is traceless, and satisfies the dominant energy condition in the transverse traceless gauge. This means that under appropriate gauge fixing conditions the energy of the linearized gravitational field has physically sensible behaviour, i.e. its density is positive and it does not flow faster than light. A further property of T_{lg} is that it does not depend on higher than first derivatives of the gravitational field and it is a quadratic expression in a field strength tensor that can be regarded as a gravitational counterpart of the electromagnetic field strength. T_{lg} also has a partial ("scalar") gauge symmetry, it is invariant under the duality symmetry of linearized gravity, and it can be obtained via Noether's theorem. Considering its remarkable properties, T_{lg} appears to be a satisfactory local energy-momentum tensor for linearized gravity.

2021

Gravitational multipole moments — The multipole moments of stationary axially symmetric vacuum or electrovacuum spacetimes can be expressed in terms of the power series expansion coefficients of the Ernst potential along the symmetry axis. We have worked out a new, more efficient method for the calculation of the multipole moments, using a method which is based on the introduction of a complex null vector field [1] (https://journals.aps.org/prd/abstract/10.1103/PhysRevD.104.064012). For the case when electromagnetic fields are also allowed, our results for the octupole and higher moments differ from the results already published in the literature. The reason for this difference is that we correct a mistake in the power series solution of the Ernst equations that remained unnoticed in the literature for 30 years. We have considered it important to publish the correct expressions for the power series solution and for the moments, since these results have been used in several subsequent papers. We have also applied our method for the calculation of the multipole moments of a 5-parameter charged magnetized generalization of the Kerr and Tomimatsu-Sato exact solutions.

Multipole moments describe the exterior of black holes and neutron stars in a coordinate system independent way. Our results are expected to have concrete physical applications, since the gravitational radiation emitted by a compact object orbiting around a much larger central object can be used to determine the multipole moments of the central body. In case of extreme-mass-ratio inspirals, one is expected to be able to determine the first few gravitational multipole moments of the larger object by the proposed space-based gravitational wave detector LISA. Measurement of multipole moments should provide a practical way of testing the no-hair theorem of black holes, and also would give information about the internal structure of neutron stars.

Screening in Q-balls — The fundamental observation of the Higgs mechanism, spontaneously breaking a gauge symmetry and giving a mass to the originally massless gauge fields, has its origins in the physics of superconductivity. In a spontaneously broken Abelian Higgs model the long-range interaction becomes a short-range one, as the scalar field generates a mass to the massless gauge field. This shows a close analogy to how long-range electromagnetic interactions are shielded by mobile charges in superconductors. Since the Abelian Higgs model is nonlinear, with an exact analytical approach lacking, investigations are necessarily based on approximations.

Motivated by recent works of charge screening in the Abelian Higgs model, we have worked out a simple but systematic perturbative framework to study this phenomenon analytically, complementing previous, mostly numerical investigations [2] (https://link.springer.com/article/10.1140/epjc/s10052-021-09022-x). In our work the physics of screening in the Abelian Higgs model is studied both numerically and analytically for both point-like and for extended charge distributions. Our perturbative framework is based on introducing the external charge strength as a ``small'' perturbative parameter. Computing corrections in the charge strength to the massive Green's function turns out to be well suited to study the physics of charge screening in this context. In our approach it is easy to investigate local charge screening, while global screening has been shown to follow from Gauss' theorem in our previous work reported one year earlier. Our perturbative results are shown to agree remarkably well with the numerical ones. To illustrate the precision of the agreement between the full numerical and the perturbative result we note that the two are *within line width of each other,* which is remarkable for a leading order perturbative result.

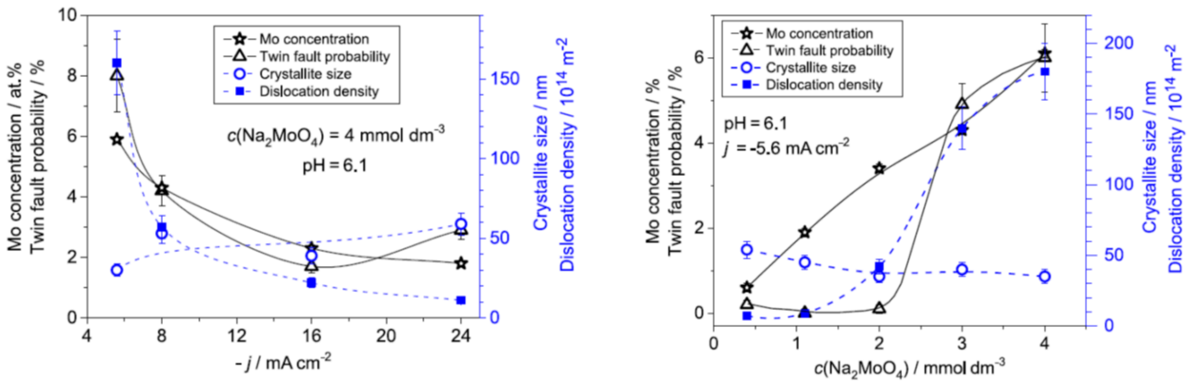

Figure 1. Left : Profile functions of the electric field, A_0 of the scalar field for point charge; Right : The charge distributions of the same solution. The dashed vertical line shows the charge radius of the solution.

2020

Q-balls in U(1)×U(1) Abelian gauge theories. — Q-balls are finite energy, non-radiating solutions in theories containing scalar fields with time-periodic phases and associated conserved charges. Prototype Q-balls appear in (complex) scalar field theories. In many cases they are stable, their stability being related to their conserved charge. It turned out that similar lumps appear in gauge theories too, stimulating a number of investigations as to their nature, stability, etc. Q-balls gained large attention, as they could form in the early universe, and play a possible role in baryogenesis. Moreover, stable Q-balls are good candidates for dark matter.

In a series of recent papers Ishihara and Ogawa, observed numerically that for spherically symmetric Q-balls in a class of Abelian Higgs model coupled to another charged, massive scalar field, the Higgs field provides for charge screening, cancelling out all long-range fields due to the charged scalars. In our work we have generalized the results of Ishihara and Ogawa to the case of the most general U(1)×U(1) symmetric scalar sector with quartic self-interaction potentials [1] (https://journals.aps.org/prd/abstract/10.1103/PhysRevD.102.076017). We have shown that the remarkably precise numerically observed cancellation of the charge contribution between the two charged scalar fields pointed out in their works holds for this general class of theories, and we have proven that the global charge cancellation is exact. We have also found a new interesting subfamily of charged Q-balls with vanishing Higgs potential.

Evolution of spinning bodies moving in rotating black hole spacetimes. - We have investigated the evolution of spinning bodies moving on bound/unbound orbits in different rotating (singular/regular) black hole spacetimes using the *Mathisson-Papapetrou-Dixon* equations [2] (http://www.astro.ro/~roaj/30_1/07-spinning_body_1925.pdf)[3 (http://www.minkowskiinstitute.org/mip/books/2019conf.html) or 3 (http://real.mtak.hu/114765/1/Proceedings_Second_Hermann_Minkowski_Meeting.pdf)]. At the closest approach distance of the central rotating black hole, the body crossed into the ergosphere. In the considered numerical simulations, the initial values were chosen such that the relatively small mass body without spin would have moved in the equatorial plane. However, since the initial spin was not aligned or anti-aligned with the rotation axis of the central black hole, the body moved out of the equatorial plane. We have presented that the spin precession was highly increased in the near region of the central black hole, especially within the ergosphere. In the case of unbound orbits, the evolutions describe such scattering processes during which the bodies enter the ergosphere of the rotating black hole but remain outside of the outer event horizon (Fig. 1).

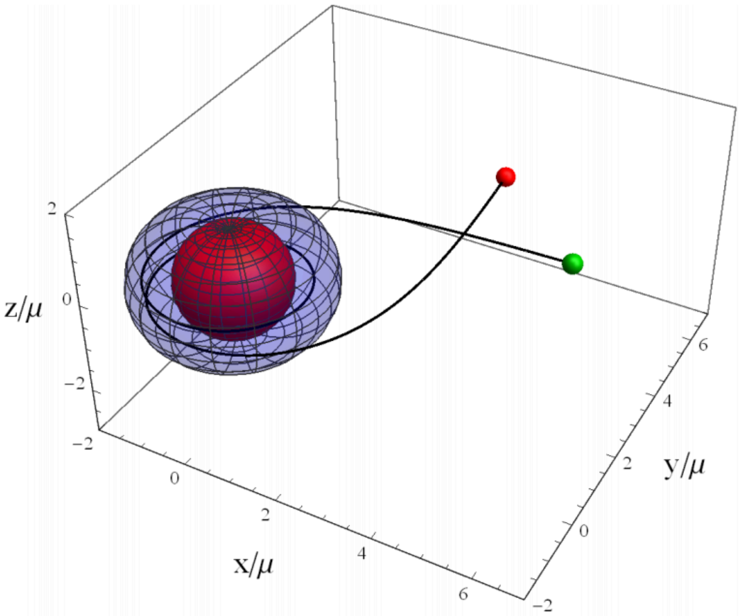

Figure 1. The evolution along an unbound orbit. The red and the blue surfaces depict the outer event horizon and outer stationary limit surface, respectitvely. The ergosphere is the region between these two surfaces.

The scattering processes are characterized by the final values of the spin and orbital plane orientation angles and the azimuthal *Boyer-Lindquist* coordinate. We have demonstrated their dependencies on the initial spin angles and a characteristic black hole parameter (Fig. 2).

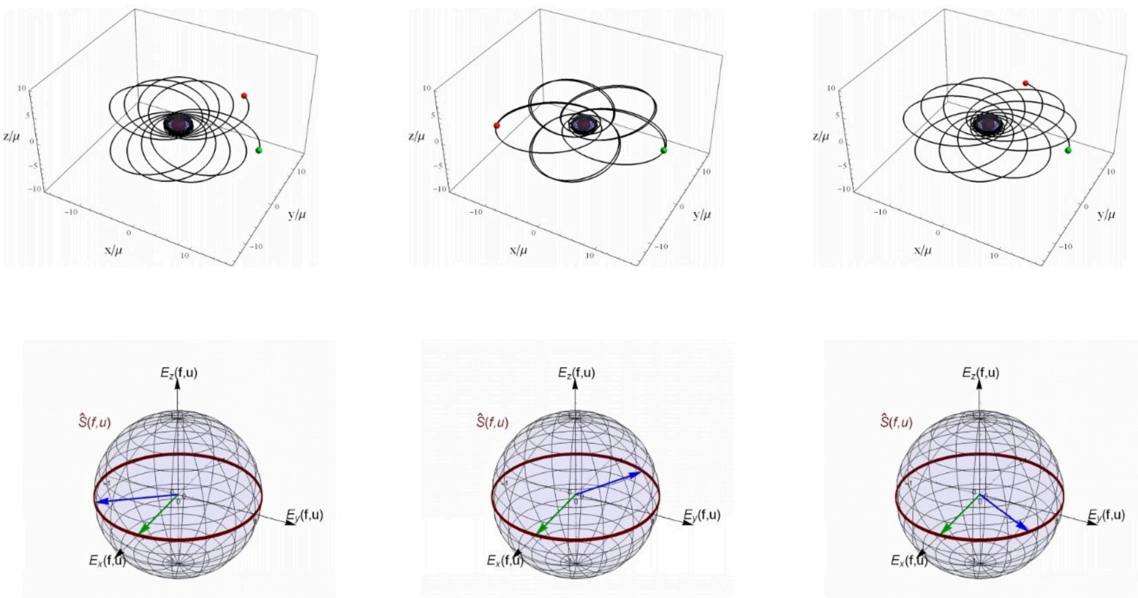

Figure 2. Evolutions along zoom-whirl orbits with initial rotation parameter a = 0.99μ (μ is the mass parameter of the black hole). The dimensionless spin magnitude and the initial spin polar angles are s=0.01 and $\theta^{(S)}(0) = \pi/2$, $\varphi^{(S)}(0) = 0$, respectively. The columns represent the evolution for three cases: when the background spacetime is the Kerr, or the rotating Hayward with q=0.216, or rotating Bardeen with q=0.081 (here q is the dimensionless parameter). The bottom row represents the evolutions of the unit spin vector in the boosted ZAMO co-moving Cartesian-like frame.

2019

Poisson-Lie analogues of spin Sutherland models. — Exactly solvable models of point particles moving along one dimension have been studied intensively for nearly 50 years, due to their diverse physical applications ranging from effective descriptions of solitons to supersymmetric Yang-Mills theories. These models involve rational, trigonometric or elliptic interaction potentials, and the `Sutherland models' represent the trigonometric case. They have interesting extensions in which the interacting particles are coupled to internal, `spin' degrees of freedom. In Ref. [1] (https://doi.org/10.1016/j.nuclphysb.2019.114807), we constructed new generalizations of the spin Sutherland models by applying symplectic symmetry reduction to certain free systems that live on the Heisenberg double of an arbitrary compact simple Poisson-Lie group. The Hamiltonian structure of the reduced systems was fully described, and it was shown that the leading terms of the pertinent Poisson brackets and Hamiltonians reproduce the previously known spin Sutherland models. It turned out that an analytic continuation of the models associated with the unitary group U(n) reproduces models introduced earlier by Braden and Hone in a study of interacting solitons of affine Toda field theories. In another paper [2] (https://doi.org/10.1088/1361-6544/ab2d5e), the Braden-Hone system of evolution equations was found to admit two distinct Hamiltonian descriptions, which are compatible in the sense that an arbitrary linear combination of the corresponding Poisson brackets is again a Poisson bracket, i.e., the system has a bi-Hamiltonian structure. The generalization of the bi-Hamiltonian structure to other compact Lie groups, as well as the quantization and applications of the new classical integrable systems will be explored in the future.

Exotic entanglement entropy scaling. — Entanglement entropy has become a ubiquitous tools in the study many-body systems. In 1D systems it usually diverges logarithmically for critical systems, while it saturates to a finite value in non-critical systems. In Ref. [3] (https://doi.org/10.1088/1742-5468/ab38b6), we studied the scaling of the entanglement and Rényi entropies in the ground state of long-range Kitaev chains with slowly decaying coupling strengths. We obtained that, under some circumstances, the entropy grows sublogarithmically with the length of the subsystem; this is the first translation-invariant state presented in the literature that has this type of entanglement scaling. Our result is based on the asymptotic behavior of a new class of Toeplitz determinants whose symbol does not lie within the application domain of the Strong Szegő Theorem or the Fisher-Hartwig conjecture.

Short time asymptotics of quantum dynamics. — Dynamical evolution of systems with sparse Hamiltonians can always be recognized as continuous time quantum walks (CTQWs) on graphs. In Ref. [4] (https://doi.org/10.1103/PhysRevA.100.062320), we analyzed the short time asymptotics of CTQWs. In previous studies it was shown that for the classical diffusion process the short time asymptotics of the transition probabilities follow power laws whose exponents are given by the usual combinatorial distances of the nodes. Inspired by this result, we performed a similar analysis for CTQWs both in closed and open systems, including time-dependent couplings. For time-reversal symmetric coherent quantum evolutions, the short time asymptotics of the transition probabilities is completely determined by the topology of the underlying graph analogously to the classical case, but with a doubled power-law exponent. Moreover, this result is robust against the introduction of on-site potential terms. However, we showed that time-reversal symmetry breaking terms and non-coherent effects can significantly alter the short time asymptotics (see Figure 1.). Furthermore, we discussed in detail the relevance of our results for quantum evolutions on particular network topologies.

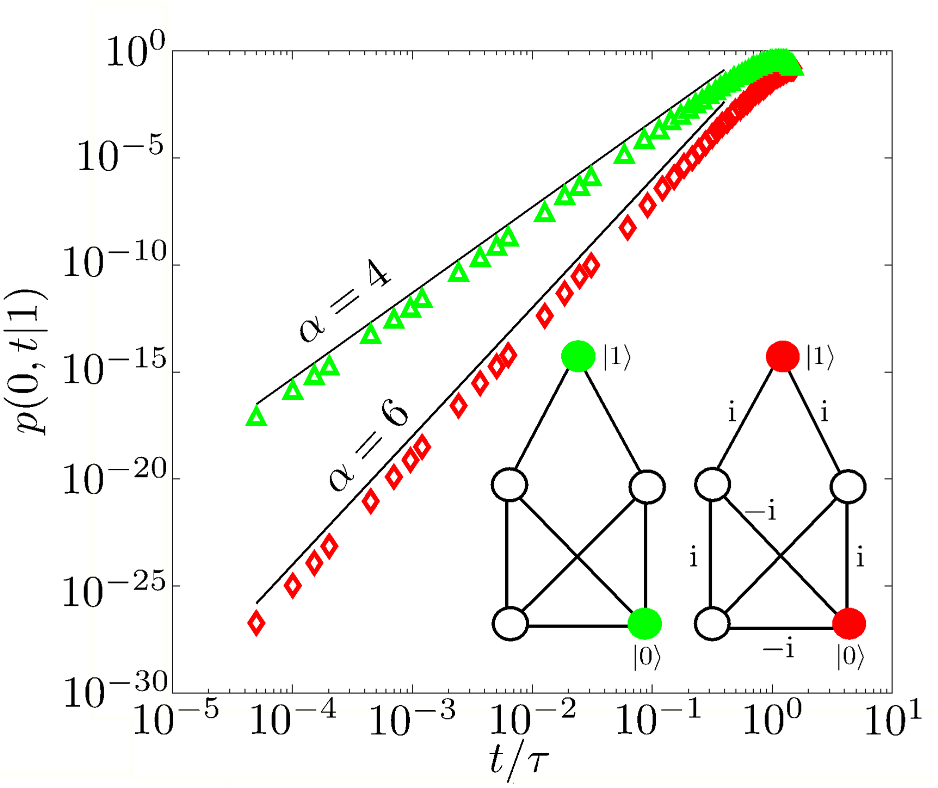

Figure 1. Comparison of short-time asymptotics of arrival probabilities in time-reversal symmetric and chiral quantum walks on the graph depicted in the left lower corner of the figure.

2018

Einstein-conformally coupled Standard Model. — We introduced and studied a classical field theoretical model, the so-called Einstein-conformally coupled Standard Model (EccSM), which is general relativistic and in which (according to the key idea above) the matter sector is coupled to gravity in a conformally invariant manner. We showed that, in this theory, in addition to the usual initial Big Bang singularity there might be a so-called Small Bang singularity, too (in which it is only the spacetime geometry is singular but all the matter field variables remain bounded), and that in the generic case Newton's gravitational constant yields an absolute upper bound for the magnitude of the Higgs field. Furthermore, the resulting rest masses of the fields depend on time, and although their time dependence can be neglected soon after their genesis, but about 10-27 seconds after the initial singularity (which is the characteristic time of the weak interactions) this time dependence could still in principle be shown up in the starting up particle physics processes.

Noether currents for the Teukolsky master equation. — The Teukolsky master equation is an important wave equation that governs the evolution of the extreme spin weight components of the electromagnetic, linearized gravitational, neutrino and spin-3/2 fields in Kerr (i.e., rotating black hole) spacetime. For various purposes, e.g. for testing numerical simulations and for studying the decay properties of the mentioned fields, it is desirable to know conserved currents for this equation. However, the Teukolsky master equation does not follow from a Lagrangian, therefore the usual procedure, which is to apply Noether's theorem, is not suitable for finding conserved currents for it. By applying a less well-known variant of Noether's theorem, we showed that a pair of Teukolsky master equations with opposite spin weights does follow from a Lagrangian, and constructed conserved currents that correspond to the time translation and axial symmetries of the Kerr spacetime and to the scaling symmetry of the Teukolsky master equation. These currents involve two independent solutions of the Teukolsky master equation with opposite spin weights. We also introduced general definitions for the symmetries and conserved currents of boundary conditions of partial differential equations, extended Noether's theorem and its variant to them, and used this extension of the latter variant to construct conserved currents associated with the Sommerfeld boundary condition in the case of the Teukolsky master equation. Such boundary conserved currents are again useful for testing purposes in numerical simulations.

Quantum Correlations in Many-Body Systems. — We studied various types of quantum correlations in many-body systems and field theories. One of these was entanglement negativity, which is a versatile measure of entanglement that has numerous applications in quantum information and in condensed matter theory. It can not only efficiently be computed in the Hilbert space dimension, but for Gaussian bosonic systems, one can compute the negativity efficiently in the number of modes. However, such an efficient computation does not carry over to the fermionic realm, the ultimate reason for this being that the partial transpose of a fermionic Gaussian state is no longer Gaussian. To provide a remedy for this state of affairs, we introduced efficiently computable and rigorous upper and lower bounds to the negativity, making use of techniques of semi-definite programming, building upon the Lagrangian formulation of fermionic linear optics, and exploiting suitable products of Gaussian operators. We also discussed examples in quantum many-body theory with applications in the study of topological properties at finite temperature.

Another investigated measure was the quantum Fisher information. We calculated the Fisher information quantity for different states of atomic ensembles in a magnetic field, see Fig. 1. The value of the Fisher information can signal nonclassicality, but it is also important from a metrological point of view. In particular we calculated precision bounds for estimating the gradient of the magnetic field based on the quantum Fisher information. We also considered a single atomic ensemble with an arbitrary density profile, where the atoms cannot be addressed individually, and which is a very relevant case for experiments.

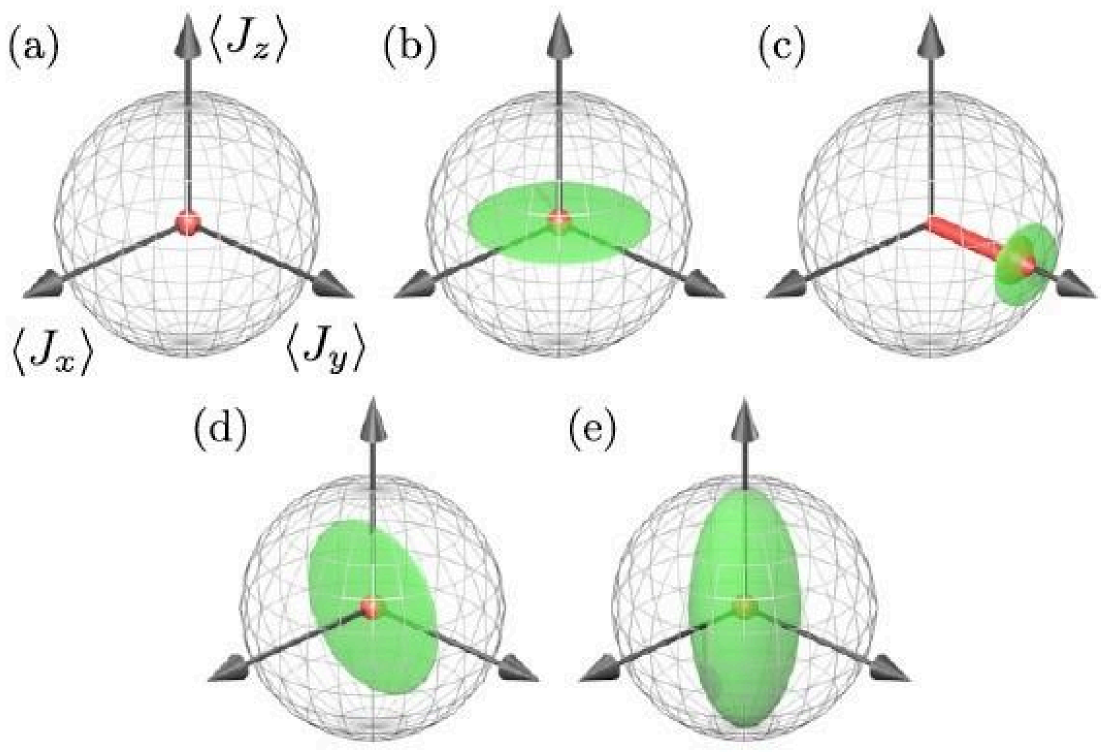

Figure 1. Angular momentum components and their variances for various spin states for few particles are shown: (a) singlet state, (b) z-Dicke state, (c) state totally polarized in the y-direction, (d) x-Dicke state, (e) GHZ state.

2017

Quantum physics: universal gate sets and measurements. — For numerous applications of quantum theory it is desirable to be able to apply arbitrary unitary operations on a given quantum system. However, in particular situations only a subset of unitary operations is easily accessible. This raises the question of what additional unitary gates should be added to a given gate-set in order to attain physical universality, i.e., to be able to perform arbitrary unitary transformation on the relevant system. We studied this problem for three paradigmatic cases of naturally occurring restricted gate-sets: particle-number preserving bosonic linear optics, particle-number preserving fermionic linear optics, and general (not necessarily particle-number preserving) fermionic linear optics. Using tools from group theory, we were able to classify, in each of these scenarios, what sets of gates are generated, if an additional gate is added to the set of allowed transformations. This allowed us to solve the universality problem completely for arbitrary number of particles and for arbitrary dimensions of the single-particle Hilbert space. We also attacked the problem of describing quantum measurements, we constructed a two-step dynamical model for selective measurements in quantum mechanics. The first step is the non-selective measurement or decoherence described by a semigroup of completely positive maps, which is given by the linear, deterministic first order Lindblad differential equation. The second step is a process from the resulted decohered state to a pure state, which is described by an effective non-linear `randomly chosen' toy model dynamics: the pure states arise as asymptotic fixed points, and their emergent probabilities are the relative volumes of their attractor regions.

Integrable systems and quantum groups. — We have derived new integrable many-body models of Ruijsenaars--Schneider--van Diejen type by applying Hamiltonian reduction to the Heisenberg double of the Poisson--Lie group SU(2n), and clarified the global structure of the phase space for another model in the same family. We have also continued our study of describing algebraic structures that go beyond groups. In an earlier work, we have already studied a distinguished class of Hopf monads in monoidal bicategories. Hopf algebras and most of their known generalizations were shown to fit this class. Since then, however, some newer generalizations of Hopf algebras — so-called Hopf categories and Hopf polyads — appeared in the literature. In this year, we constructed a monoidal bicategory in which also these structures, as well as Turaev’s Hopf group algebras, can be regarded as Hopf monads. In order to describe quantum we constructed a two-step dynamical model for selective measurements in quantum mechanics.

Gravitational theory: spinning binary black-hole systems. — The Lagrangian of the spinning binary system contains acceleration-dependent terms in some cases of spin supplementary condition (SSC). We constructed the nontrivial generalized Hamiltonian formalism with the high-order canonical moments proposed by Ostrogradsky and calculated the conserved quantities such as the energy and the magnitude of the orbital angular momentum for each SSC. Thus we computed the first integrals of the spinning dynamics and gave the perturbative radial and angle motion with the help of the generalized radial parameterization. Moreover, we defined the generalized Poisson brackets following the work of Yang and Hirschfelder and we have given the canonical structure of the spinning binary. Finally, we generalized the result of Kidder for the spinning waveform and for the dissipative part of relative motion during the gravitational radiation of each SSC (Fig. 1).

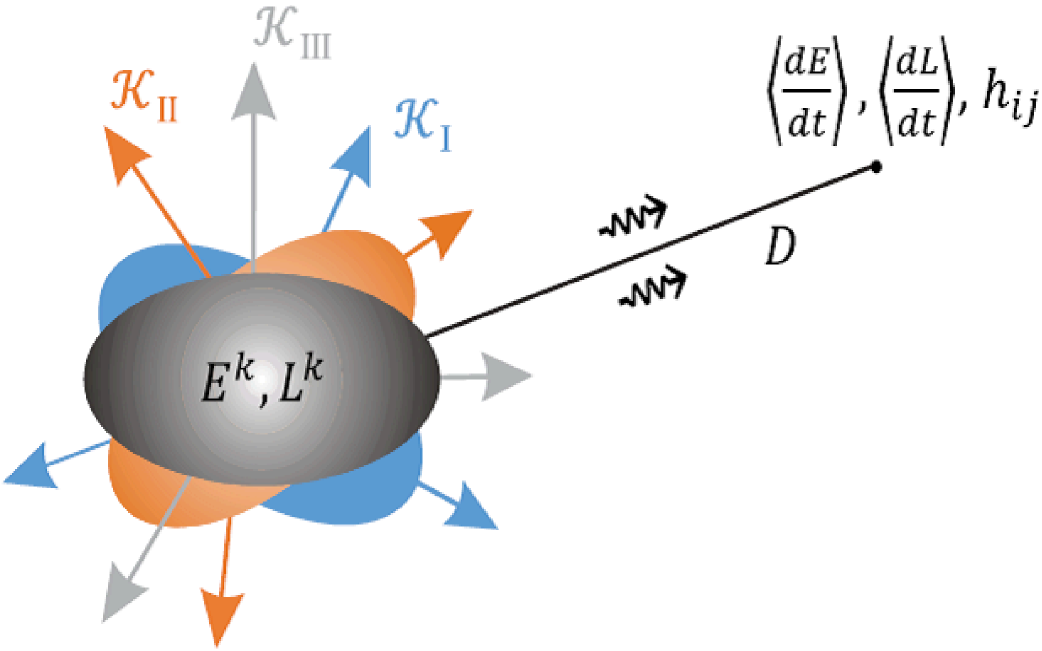

Figure 1. Dissipative quantities of the binary in the different spin supplementary conditions