Classical holography

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Content

What is thermodynamics?

Fundamental or emergent?

- Statistical physics is special, thermodynamics is general.
- Separation of universal from particular.
- Second Law is general, there are statistical demonstrations.
- Second Law can be applied for fields.

Thermodynamics is a stability theory.
(📖T. Matolcsi, 📖W.M. Haddad, 📖, VP (PTRSA 2023))

Are there some original, genuine consequences??

Local, nonlocal and weakly nonlocal

Locality in space(time)

- Local fields and local field equations. Example: $\varphi(t, x)$, Poisson equation.
- Space integrals in the field equations: strong nonlocality.
- Nonlocal fields: Example: $f(t, x_1, x_2)$, Liouville equation, entanglement.
- Weak nonlocality: extension of the field equations with higher order space derivatives. Example: gradient fluids, Horndeski gravity.

Locality in time

- Locality in time. No memory. Markov process.
- Memory functionals in the field equations: strong memory. Example: principle of fading memory.
- Weak 'nonlocality' in time: higher order time derivatives in the field equations. Example: second sound, delay and inertia.

Temporal nonlocality and spatial locality are interdependent.
Action at a distance: vacuum solution of a local theory.

Holography

Holography ← holos+graphe = complete, whole + drawing, writing.

Optical holography

- Dennis Gábor. Reproduction of 3 dimensional information from 2 dimensional projections.
- Interferometric. Amplitude and phase. For any wavelike propagation. E.g. ambisonic sound.

Holography in quantum field theories

- Generalisation of black hole thermodynamics. Hawking, t'Hooft, Susskind. Entropy is area.
- Abstracted in string theory. Expected in quantum gravity.
- AdS-CFT correspondence.

Holographic principle + Unruh effect ⇒ field equation of gravity (Newtonian and GR)

📖 Jacobson (PRL 1995), ..., 📖 Verlinde (JHEP 2011)

Classical holography I

Newtonian gravity ($\Delta\varphi = 4\pi G\rho$):

$$\rho\nabla\varphi = \nabla \cdot \boldsymbol{P}_{grav}(\nabla\varphi) = \nabla \cdot \left(\frac{1}{4\pi G} \left[\nabla\varphi\nabla\varphi - \frac{1}{2}(\nabla\varphi)^2 \boldsymbol{I} \right] \right)$$

Maxwell stress tensor.

Euler fluids are holographic

Ideal Euler fluids: $\boldsymbol{P}_{Euler} = p(u,\rho)\boldsymbol{I}$. p is the thermostatic pressure, e.g. ideal gas.

$$\nabla \cdot \boldsymbol{P}_{Euler} = \nabla p = \rho\nabla\mu + \rho s \nabla T$$

Follows from the Gibbs-Duhem relation: $0 = sdT - vdp + d\mu$. For isothermal processes of ANY fluid the chemical potential is a mechanical potential. Friedmann equation.

Classical holographic property:

$$\nabla \cdot \boldsymbol{P}(\ldots) = \rho\nabla\phi(\ldots)$$

Constitutive (...), material property. Thermodynamics or field equation dependent?

Further remarks

Balance of momentum. Global form:

$$\dot{M} = -F_{surf} + F_{bulk}.$$

Local form and substantial forms:

$$\rho\dot{\boldsymbol{v}} + \nabla \cdot \boldsymbol{P} = -\rho\nabla\varphi, \qquad \rho\dot{v}^i + \partial_k P^{ik} = -\rho\partial^i\varphi. \tag{1}$$

Bulk and surface forces. Substantial or comoving derivative, Convective and conductive current densities, $\boldsymbol{P}_{conv} = \boldsymbol{P}_{cond} + \rho\boldsymbol{v} \circ \boldsymbol{v}$. Hidden Galilean covariance.

Particle or field??

$$\rho\dot{\boldsymbol{v}} + \nabla \cdot \boldsymbol{P}_{grav} = 0 \quad \Longleftrightarrow \quad \dot{\boldsymbol{v}} = -\nabla\varphi$$

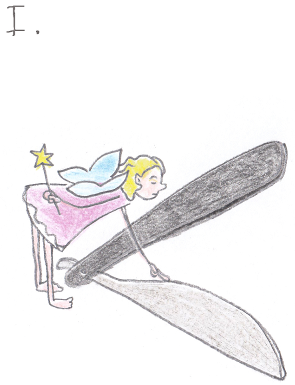
Test particle and integrating screens. Constant background field or field theory?
$(\dot{\rho} + \rho\nabla \cdot \boldsymbol{v} = 0,\ \Delta\varphi = 4\pi G\rho)$
Newtonian form:

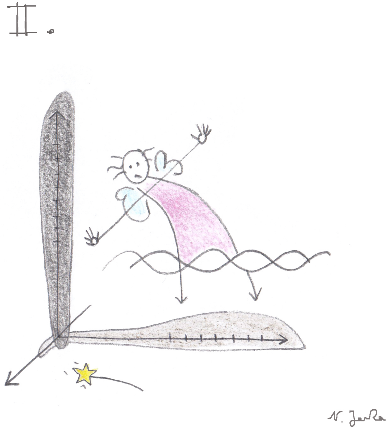
$$\dot{M} = F$$

The universality of point mass modell.

Entropy inequality

Ockham's razor





"Is there a harmony of mathematics and physics??"

Constitutive state space

Coleman-Noll and Liu procedures. Separation of functions and variables.
The entropy inequality is *conditional*:

$$\rho\dot{e} + \nabla \cdot \boldsymbol{q}(e, \nabla e) = 0,$$
$$\rho\dot{s}(e, \nabla e) + \nabla \cdot \boldsymbol{J}(e, \nabla e) - \Lambda(e, \nabla e)(\rho\dot{e} + \nabla \cdot \boldsymbol{q}(e, \nabla e)) =$$
$$\rho \frac{\partial s}{\partial \nabla e}(\nabla e)^{\cdot} + \rho \left(\frac{1}{T} - \Lambda \right) \dot{e} + ... \geq 0$$

Liu-procedure, Lagrange–Farkas-multipliers. It follows that:

$$\frac{\partial s}{\partial \nabla e}(e, \nabla e) = 0, \qquad \Lambda = \frac{1}{T}, \quad \text{and} \quad \boldsymbol{q}(e, \nabla e) \cdot \nabla \left(\frac{1}{T}(e) \right) \geq 0$$

Constitutive state variables: $(e, \nabla e)$
→ thermodynamic state variables: (e)
Process direction variables: $(\dot{e}, (\nabla e)^{\cdot}, \nabla^2 e)$

Weakly nonlocal extensions

Classified by constitutive state spaces and constraints

- Fluid mechanics. Mass, velocity and energy. $(\rho, \nabla\rho, v, \nabla v,\ e, \nabla e)$
  Constraints: balances of mass, momentum and energy ($\rightarrow$ quantum mechanics and more)
  $\rightarrow$ Fourier-Navier-Stokes equations.
- Fluid mechanics +scalar field $(\rho, \nabla\rho, v, \nabla v, e, \nabla e, \varphi, \nabla\varphi,\ \nabla^2\varphi)$
  Constraint: evolution equation, balances of mass momentum and energy.
  $\rightarrow$ Fourier-Navier-Stokes + Newtonian gravity and more
- Fluid mechanics + second order weak nonlocality in density. Mass, velocity and energy. $(\rho, \nabla\rho, \nabla^2\rho, v, \nabla v,\ e, \nabla e)$
  Constraints: balances of mass, momentum and energy
  $\rightarrow$ Korteweg fluids, superfluids, quantum mechanics and more

Newtonian gravity

📖VP-Abe (Physica A, 2022)
📖Abe-VP (Symmetry, 2022)
📖Pszota-VP (arXiv: 2306.01825)

Scalar field and hydrodynamics

$s(e - \varphi - \frac{\nabla\varphi\cdot\nabla\varphi}{8\pi G\rho}, \rho)$. Gibbs relation:

$$du = Tds + \frac{p}{\rho^2}d\rho = de - d\left(\varphi + \frac{\nabla\varphi\cdot\nabla\varphi}{8\pi G\rho}\right).$$

The potential energy, φ, the field energy and internal energy are separated.

Balances of mass, momentum, internal energy + field equation:

$$\dot{\rho} + \rho\nabla\cdot\boldsymbol{v} = 0,$$
$$\rho\dot{\boldsymbol{v}} + \nabla\cdot\boldsymbol{P} = \boldsymbol{0},$$
$$\rho\dot{e} + \nabla\cdot\boldsymbol{q} = -\boldsymbol{P} : \nabla\boldsymbol{v},$$
$$\dot{\varphi} = f.$$

Constraints of the entropy inequality:

$$\boxed{\rho\dot{s} + \nabla\cdot\boldsymbol{J} = \Sigma \geq 0}$$

Constitutive state variables: $(e, \nabla e, \rho, \nabla\rho, (\boldsymbol{v}), \nabla\boldsymbol{v}, \varphi, \nabla\varphi, \nabla^2\varphi)$

$\rightarrow$ thermodynamic state variabless: $(e, \rho, \varphi, \nabla\varphi)$

$$\rho\dot{s} + \nabla\cdot\boldsymbol{J} =$$

$$\left(\boldsymbol{q} + \frac{\dot{\varphi}}{4\pi G}\nabla\varphi\right)\cdot\nabla\left(\frac{1}{T}\right)$$

$$+\boxed{\frac{f}{4\pi G T}\left(\Delta\varphi - 4\pi G\rho\right)}$$

$$-\left[\boldsymbol{P} - p\boldsymbol{I} - \frac{1}{4\pi G}\left(\nabla\varphi\nabla\varphi - \frac{1}{2}\nabla\varphi\cdot\nabla\varphi\boldsymbol{I}\right)\right] : \frac{\nabla\boldsymbol{v}}{T} \geq 0$$

- Perfect self-gravitating (isothermal) fluids are holographic:

$$\nabla\cdot\left(p\boldsymbol{I} + \frac{1}{4\pi G}\left(\nabla\varphi\nabla\varphi - \frac{1}{2}\nabla\varphi\cdot\nabla\varphi\boldsymbol{I}\right)\right) = \rho\nabla(\mu + \varphi)$$

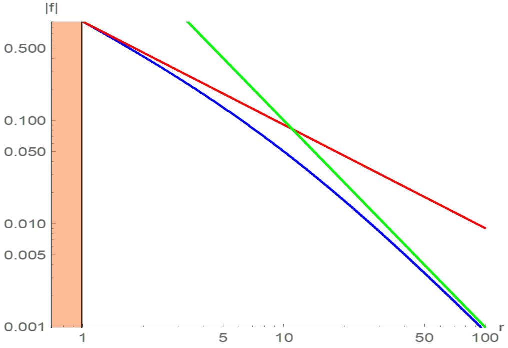
Nonlinear extension, static, nondissipative field

Stationary nondissipative field equation:

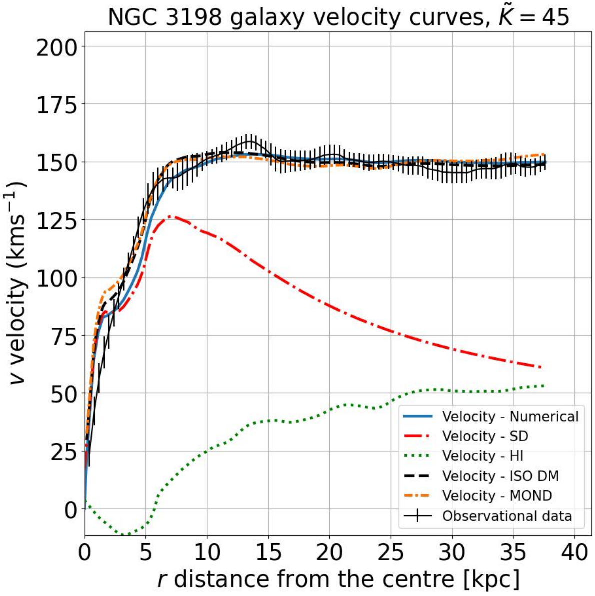
$$\boxed{0 = \Delta\varphi - 4\pi G\rho - K\nabla\varphi\cdot\nabla\varphi.}$$

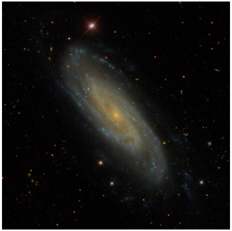
Spherical symmetric force field. Crossover. Apparent and real masses:

$$f(r) = -\frac{r_1}{Kr(r+r_1)} = -\frac{GM_{aa}}{r(r+r_1)}$$



Thermodynamic gravity, MOND and Dark Matter





NGC 3198	
M_{DM+BM}	M_{aa}
190	110

Unit: $10^9 M_{\odot}$

Korteweg fluids

VP-Fülöp (Proc. Roy. Soc., 2004)
VP-Kovács (Phil. Trans. Roy. Soc. A, 2020)
VP (Physics of Fluids, 2023)

Korteweg fluids: history

Capillarity.
Van der Waals: gradient of density is a thermodynamic variable.
Korteweg (1905): second gradient of density, pressure expansion.

Balances of mass, momentum and internal energy:

$$\dot{\rho} + \rho \nabla \cdot \boldsymbol{v} = 0,$$
$$\rho \dot{\boldsymbol{v}} + \nabla \cdot \boldsymbol{P} = \boldsymbol{0},$$
$$(\rho \dot{e} + \nabla \cdot \boldsymbol{q} = -\boldsymbol{P} : \nabla \boldsymbol{v}.)$$

$$\boldsymbol{P} = \left(p - \alpha \Delta \rho - \beta (\nabla \rho)^2\right) \boldsymbol{I} - \delta \nabla \rho \circ \nabla \rho - \gamma \nabla^2 \rho$$

$\alpha, \beta, \gamma, \delta$ are density dependent material parameters.

Instable. Second law? 🕮Eckart fluids 1948, 🕮Dunn and Serrin (ARMA, 1985).

Constitutive state variables: $(e, \nabla e, \rho, \nabla\rho, \nabla^2\rho, (\boldsymbol{v}), \nabla\boldsymbol{v})$

$\rightarrow$ thermodynamic state variables: $(e, \rho, \nabla\rho)$

Process direction: $(\dot{e}, (\nabla e)^{\cdot}, \nabla^2 e, \dot{\rho}, (\nabla\rho)^{\cdot}, (\nabla^2\rho)^{\cdot}, \nabla^3\rho, \dot{\boldsymbol{v}}, (\nabla^2\boldsymbol{v})^{\cdot})$

$$\rho\dot{s} + \nabla\cdot\boldsymbol{J} = \boldsymbol{q}\cdot\nabla\left(\frac{1}{T}\right) -$$

$$-\left[\boldsymbol{P} - p\boldsymbol{I} - \frac{\rho^2}{2}\left(\nabla\cdot\frac{\partial s}{\partial\nabla\rho}\boldsymbol{I} + \nabla\frac{\partial s}{\partial\nabla\rho}\right)\right] : \frac{\nabla\boldsymbol{v}}{T} \geq 0$$

- Rigorous methods are essential.
- The pressure of an ideal, non-dissipative Korteweg fluid is:

$$\boxed{\boldsymbol{P} = p(e,\rho)\boldsymbol{I} + \frac{\rho^2}{2}\left(\nabla\cdot\frac{\partial s}{\partial\nabla\rho}\boldsymbol{I} + \nabla\frac{\partial s}{\partial\nabla\rho}\right)}$$

Perfect Korteweg fluids are holographic

$$\boldsymbol{P}_K = \frac{\rho^2}{2}\left(\nabla \cdot \frac{\partial s}{\partial \nabla\rho}\boldsymbol{I} + \nabla\frac{\partial s}{\partial \nabla\rho}\right)$$

- Classical holographic property, with internal energy:

$$\boxed{\nabla \cdot \boldsymbol{P}_K = \rho(\nabla\phi + T\nabla s)}, \quad \text{where} \quad \phi = \frac{\partial \rho u}{\partial \rho} - \nabla \cdot \frac{\partial(\rho u)}{\partial \nabla\rho} = \delta_\rho(\rho u)\big|_{\rho s}$$

Functional derivative. Isothermal, adiabatic, ...

- Momentum balance: continuum AND point mass

$$\rho\dot{\boldsymbol{v}} + \nabla \cdot \boldsymbol{P}_K = \rho(\dot{\boldsymbol{v}} + \nabla\phi) = 0 \quad \rightarrow \quad \dot{\boldsymbol{v}} = -\nabla\phi$$

- Conserved vorticity follows.
- Bohm potential, superfluids, Schrödinger equation, ...

Probabilistic Korteweg fluids – additivity

Zeroth Law of thermodynamics: separability of independent physical systems.
Multicomponent normal fluids. Notation: $\rho_1 = \rho_1(\boldsymbol{x}_1)$.

$$\mathrm{u}(\rho_1 + \rho_2) = \mathrm{u}(\rho_1) + \mathrm{u}(\rho_2).$$

Multicomponent probabilistic fluids:

$$\mathrm{u}(\rho_1 \rho_2) = \mathrm{u}(\rho_1) + \mathrm{u}(\rho_2).$$

Functional condition, $\rho_{tot} = \rho_1 \rho_1$:

$$\mathrm{u}(\rho_{tot}, (\nabla \rho_{tot})^2) = \mathrm{u}\left(\rho_1 \rho_2, (\rho_2 \nabla_1 \rho_1)^2 + (\rho_1 \nabla_2 \rho_2)^2\right) =$$
$$\mathrm{u}(\rho_1, (\nabla_1 \rho_1)^2) + \mathrm{u}(\rho_2, (\nabla_2 \rho_2)^2).$$

Unique solution:

$$\boxed{\mathrm{u}(\rho, (\nabla \rho)^2) = k \ln \rho + \frac{K}{2} \frac{(\nabla \rho)^2}{\rho^2}}$$

Independent Schrödinger equations for independent particles/components.

QFT, GR can be fluids: 🕮 Jackiw et al. (JP A, 2004), 🕮 Biró-VP(FP, 2015), ...

Summary

Emergent classical holography and emergent evolution.

- The Second Law of Thermodynamics is applicable for fields and informative in the marginal case of zero dissipation.
- Variational principles are not necessary.
- The Second Law of Thermodynamics is (looks like) fundamental.

Case 1: There is a thermodynamic road to gravity.

$$\text{Fluid} + \text{scalar internal variable} \longrightarrow \text{gravity}$$

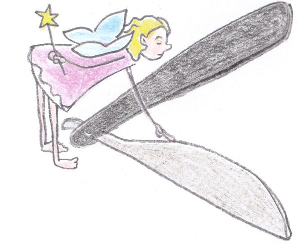
- Second law with zero dissipation $\Longrightarrow$ classical holography
- Energy type, quadratic $\Longrightarrow$ gravity

Case 2: There is a thermodynamic road to quantum physics.

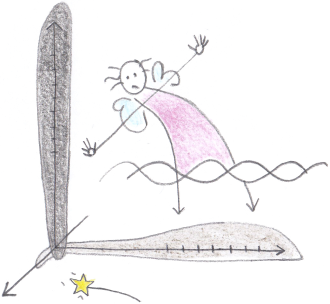
$$\text{Korteweg fluids} \longrightarrow \text{quantum mechanics}$$

- Second law with zero dissipation $\Longrightarrow$ classical holography
- Additivity $\Longrightarrow$ quantum systems

I.



II.



"This may be true, because it is mathematically trivial."
(somebody from Princeton, according to R. Pisarski)

Thank you for the attention!

Interplay: hidden Galilean covariance

Spacetime aspects - separation of material and motion

$$\frac{\partial(\rho e_{total})}{\partial t} + \nabla \cdot (\boldsymbol{q} + \rho \boldsymbol{v} e_{total} + \boldsymbol{P} \cdot \boldsymbol{v}) = 0, \quad \to \quad \rho \dot{e}_{total} + \nabla \cdot (\boldsymbol{q} + \boldsymbol{P} \cdot \boldsymbol{v}) = 0$$

It is a change of frame:

- Comoving(substantial) time derivative: $\dot{e} = \frac{\partial e}{\partial t} + \boldsymbol{v} \cdot \nabla e$,
- Galilean four-vector: $(\rho e, \boldsymbol{q})$, convective and conductive current densities.
- constitutive state space: ∇e is spacelike covector,
- total and internal energies: $e = e_{TOT} - v^2/2$.

Consequences

- What is comoving? Mass? Energy? Observer representations. Flow-frame.
- Total energy, kinetic energy and internal energy. Galilean relativistic energy-momentum-mass four-tensor. Consequence: entropy production is objective.
- Temperature is a Galilean relativistic four-vector: thermal reference frames.