"Integrability in Gauge and String Theory, 22 – 26 August 2016,

Humboldt-Universität zu Berlin"

3pt functions and form factors

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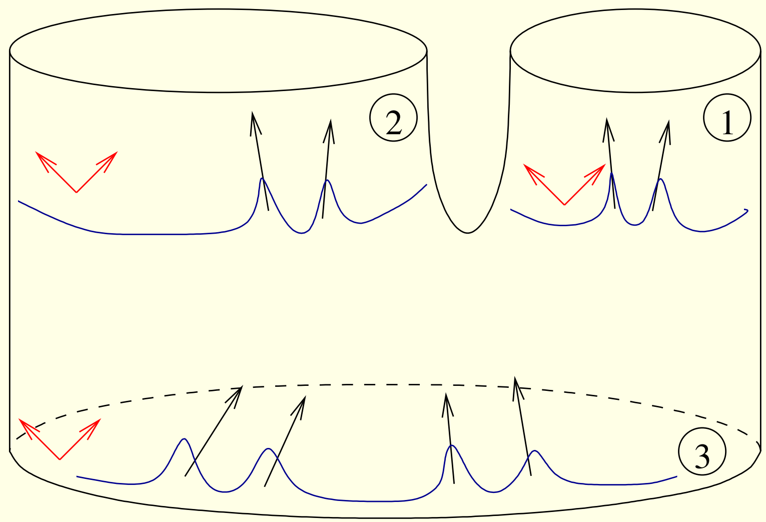
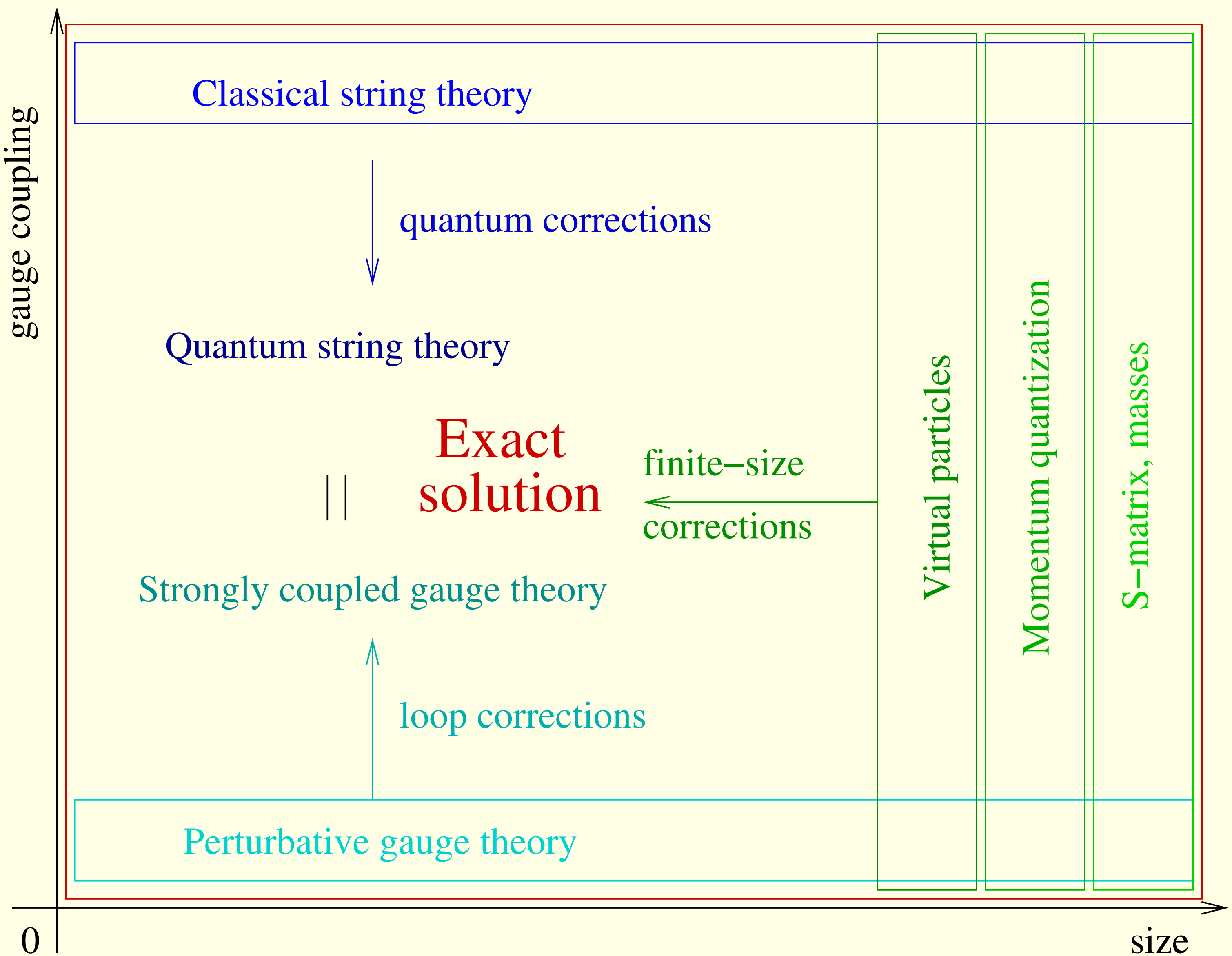
IIB strings on $AdS_5 \times S^5$	Integrability $\longleftrightarrow$	$N = 4$ SYM
SFT vertex	Form factors	3pt functions



Same light-cone gauge fixing: integrable 2D QFT with particle like excitation. Amplitude $\equiv$ string vertex $\longleftrightarrow$ 3pt functions and $1/N$ corrections in dual gauge theory.

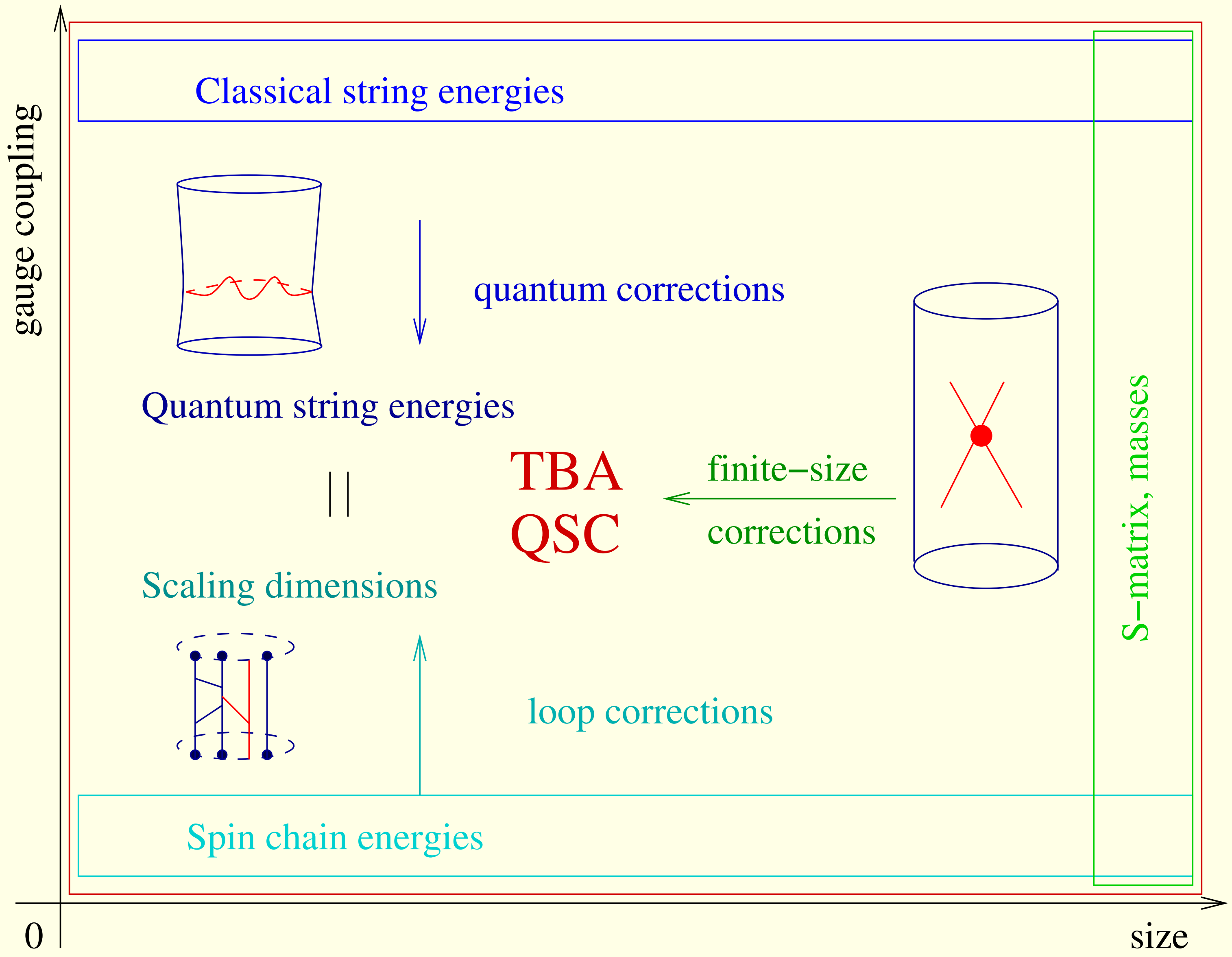
arXiv:1404.4556,1501.04533,1512.01471,1607.02830: work done in collaboration with Romuald Janik and Andrzej Wereszczynski

Motivation:



gauge coupling
Classical string theory
quantum corrections
Quantum string theory
||
Exact solution
finite–size
corrections
Strongly coupled gauge theory
loop corrections
Perturbative gauge theory
Virtual particles
Momentum quantization
S–matrix, masses
0
size

Spectral problem: 2pt functions



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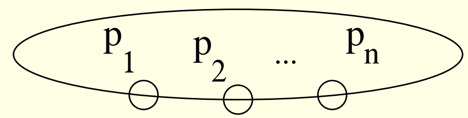
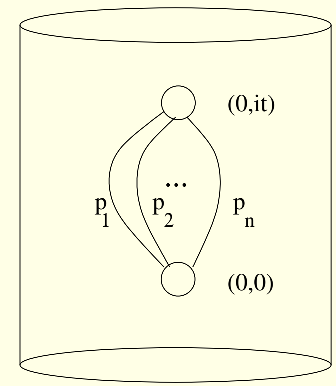
The simplest interacting QFT in 1+1 D: $\mathcal{L} = \frac{1}{2}(\partial_t \varphi)^2 - \frac{1}{2}(\partial_x \varphi)^2 - \frac{m^2}{b^2}(\cosh b\varphi - 1)$

interesting observables: finite size spectrum,





finite size correlators ${}_L\langle 0|\mathcal{O}(it)\mathcal{O}(0)|0\rangle_L = \sum_n |{}_L\langle 0|\mathcal{O}(0)|n\rangle_L|^2 e^{-E_n t}$

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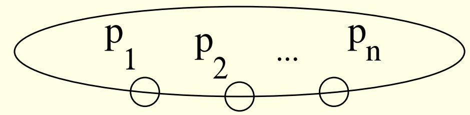

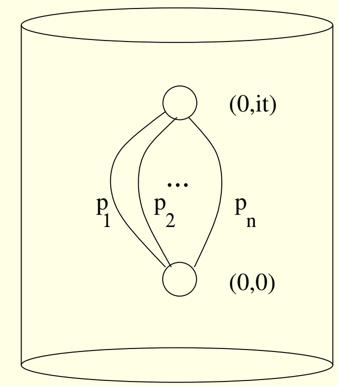

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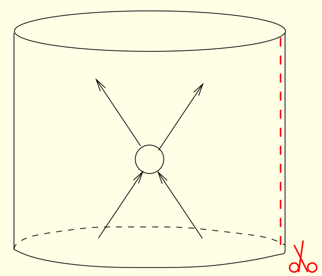
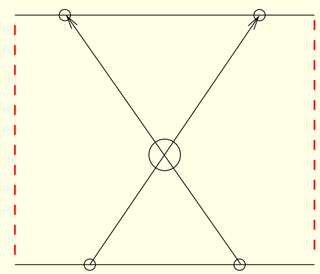
Too difficult, instead

Infinite volume $\rightarrow$ LSZ reduction formula





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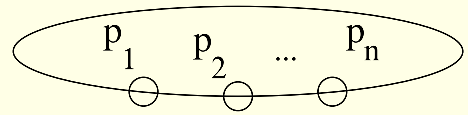

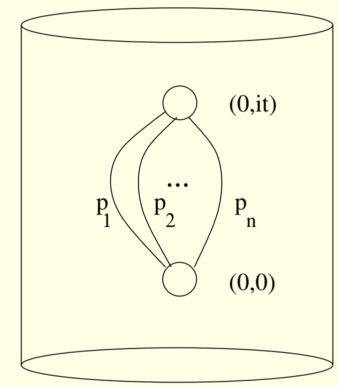

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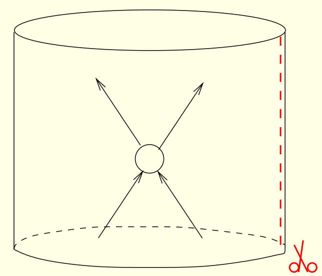

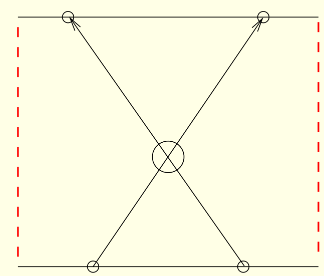

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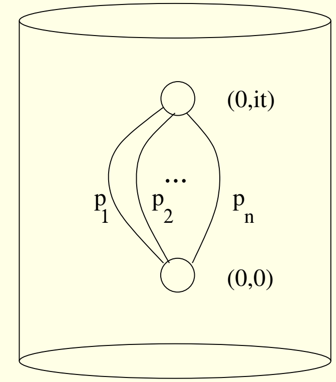

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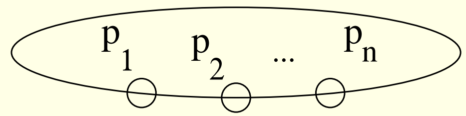

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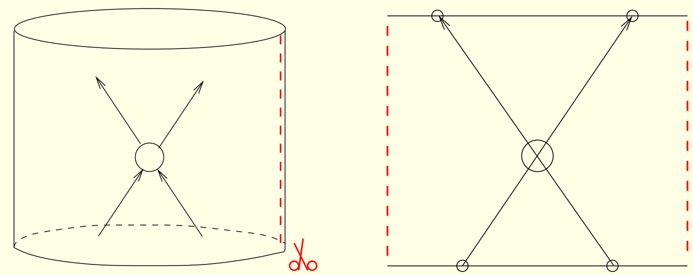

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Consequence: perturbative definition, calculational tool:

$$S(\theta) = 1 - \frac{1}{4} i b^2 \mathrm{csch}\theta - \frac{b^4(\mathrm{csch}\theta(\pi\mathrm{csch}\theta - i))}{32\pi} + \frac{ib^6 \mathrm{csch}\theta(\pi\mathrm{csch}\theta - i)^2}{256\pi^2} + O\left(b^8\right)$$

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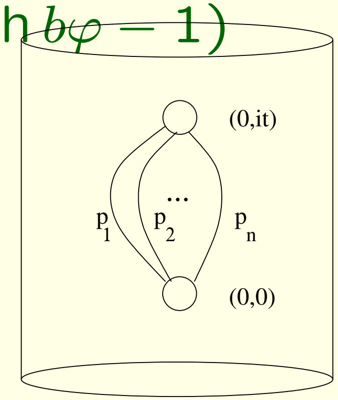

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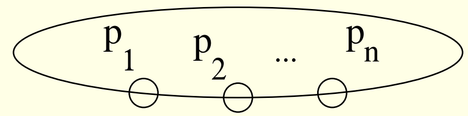

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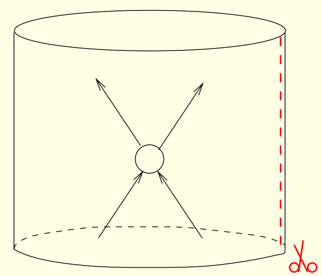

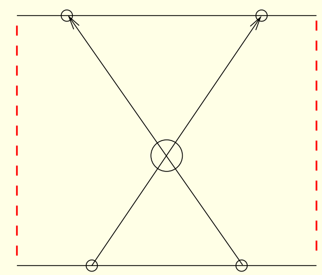

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control over analytical properties: unitarity, crossing $S(\theta) = S(-\theta)^{-1} = S(i\pi - \theta)$

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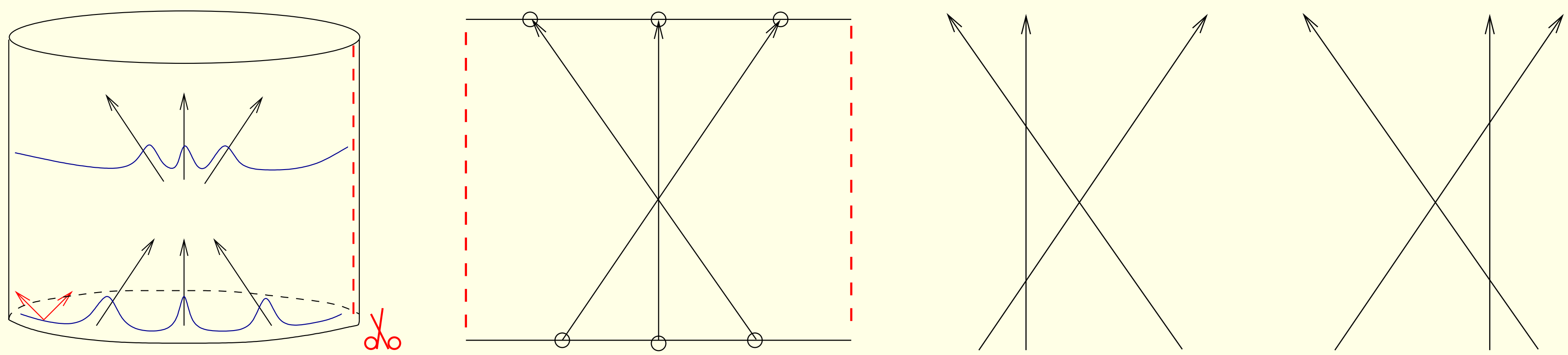
S-matrix bootstrap: fundamental object is the two particle S-matrix [Zamolodchikov[2] '79,Dorey]



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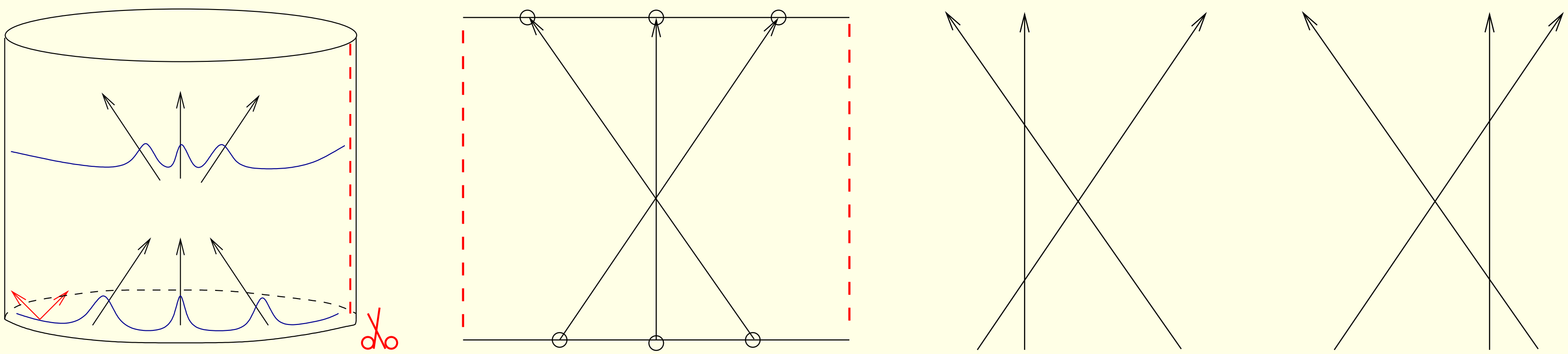
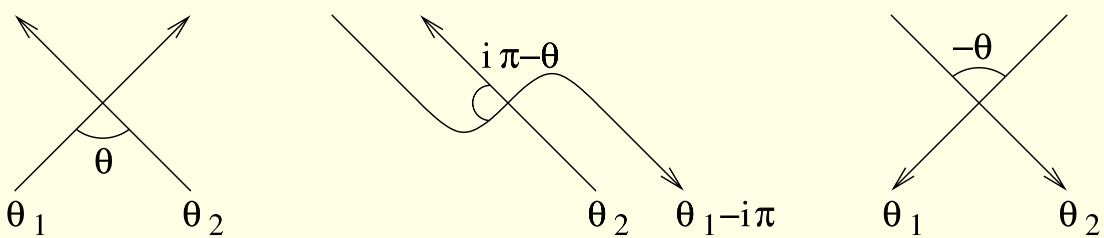


Infinite volume $\rightarrow$ crossing symmetry, $\theta \rightarrow i\pi - \theta$ in rapidity $(E(\theta), p(\theta)) = m(\cosh\theta, \sinh\theta)$



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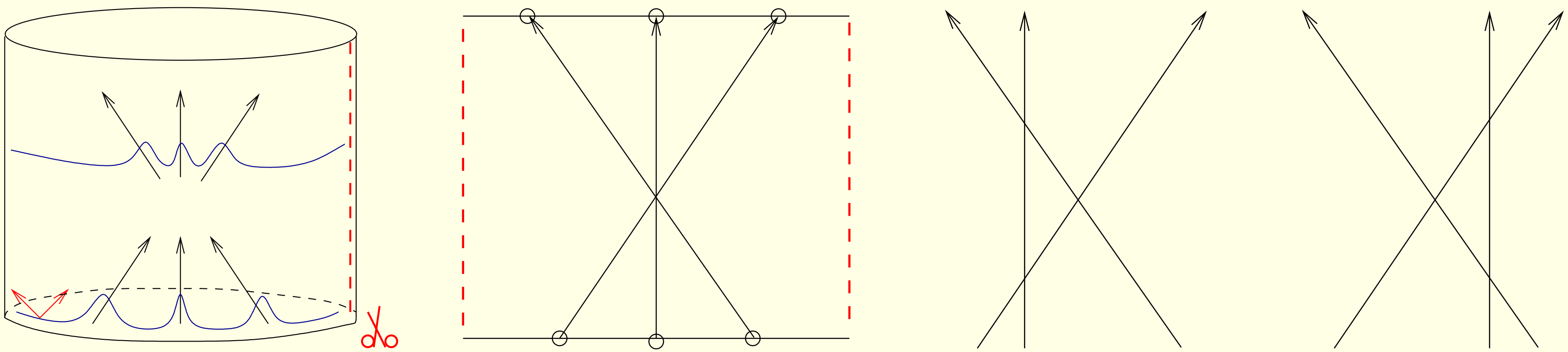

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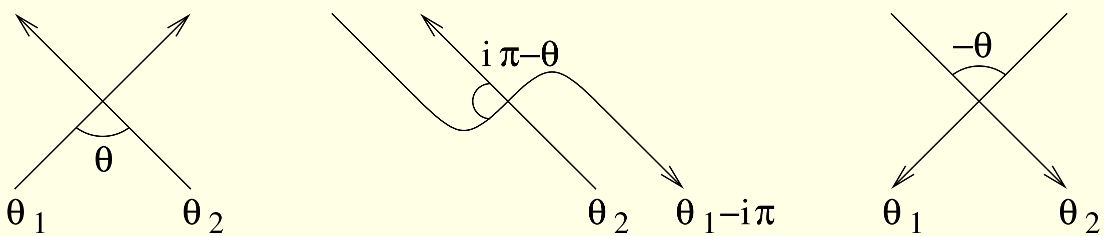

Simple solution:
sinh-Gordon

$$S(\theta) = \frac{\sinh\theta - ia}{\sinh\theta + ia} \qquad a = \frac{\pi b^2}{8\pi + b^2}$$

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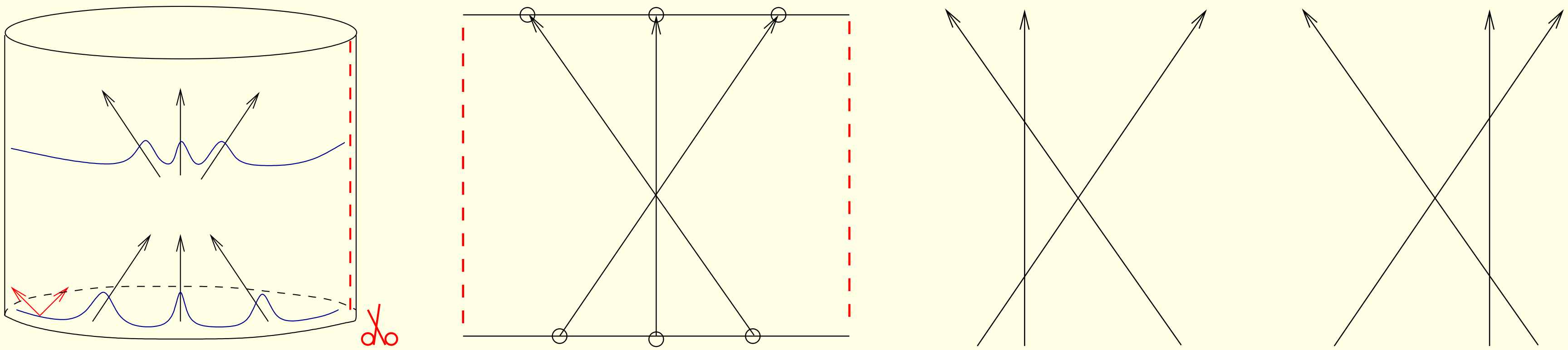

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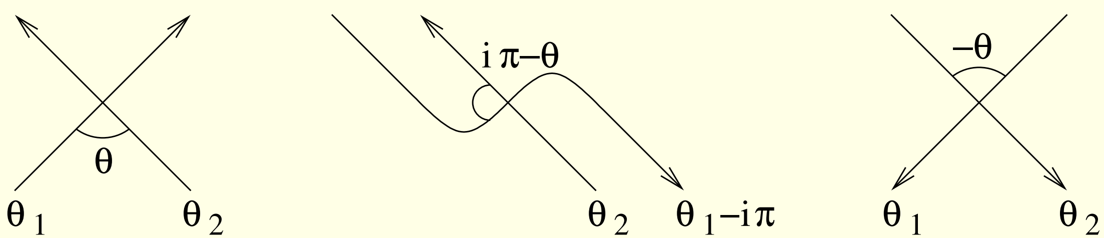

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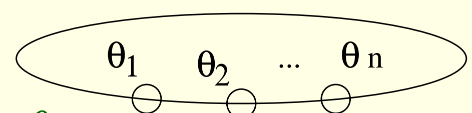
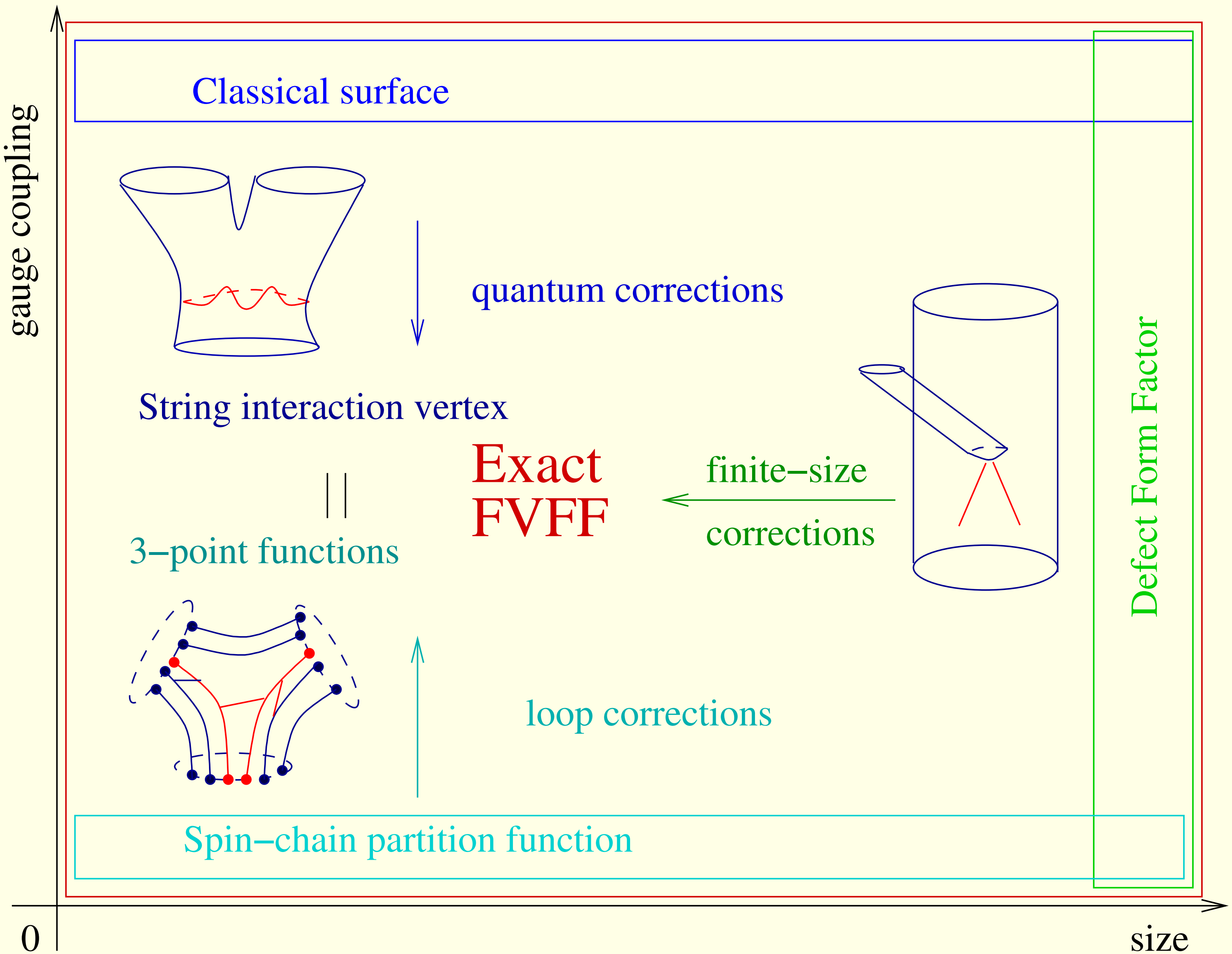
Finite volume spectrum [Bethe-Yang] upto $O(e^{-mL})$ i.e. polynomial in L^{-1} :



$$e^{i\Phi_1} = e^{ip_1 L} S(\theta_1 - \theta_2) \dots S(\theta_1 - \theta_n) = 1 \qquad E_n(L) = \sum_i m \cosh\theta_i$$

String interaction, 3pt functions



Form factor bootstrap

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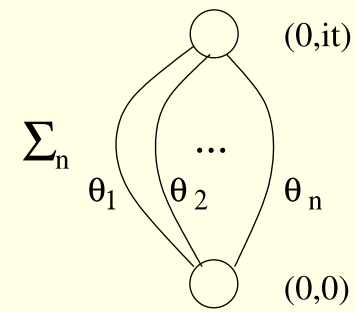

Correlation functions: [Smirnov, Karowszki] $\langle 0|\mathcal{O}(it)\mathcal{O}(0)|0\rangle =$

$\sum_n \frac{1}{n!}\int\frac{d\theta_1}{2\pi}\ldots\int\frac{d\theta_n}{2\pi}|\langle 0|\mathcal{O}(0)|\theta_1,\ldots,\theta_n\rangle|^2 e^{-m(\sum_i \cosh\theta_i)t}$

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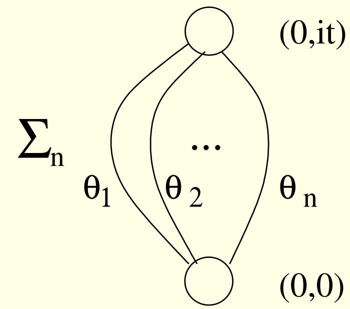

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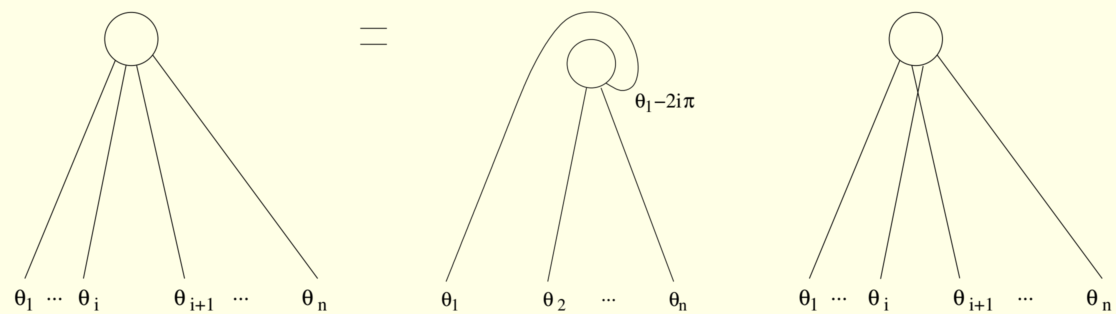

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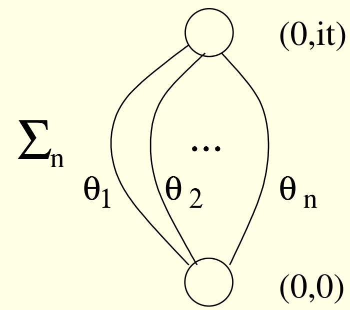

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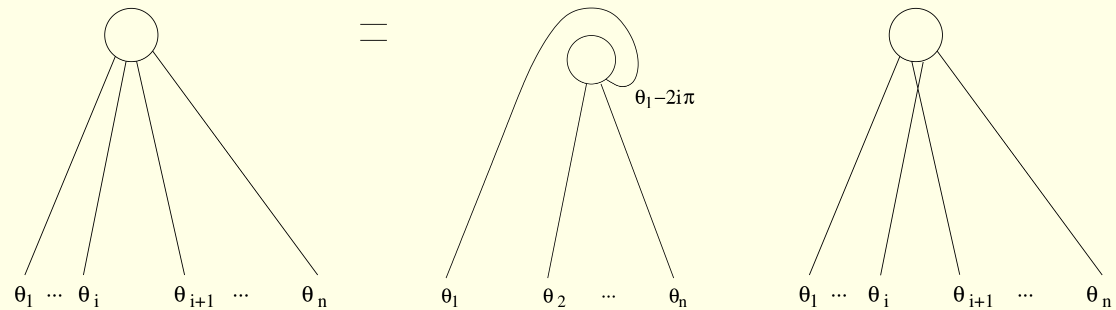

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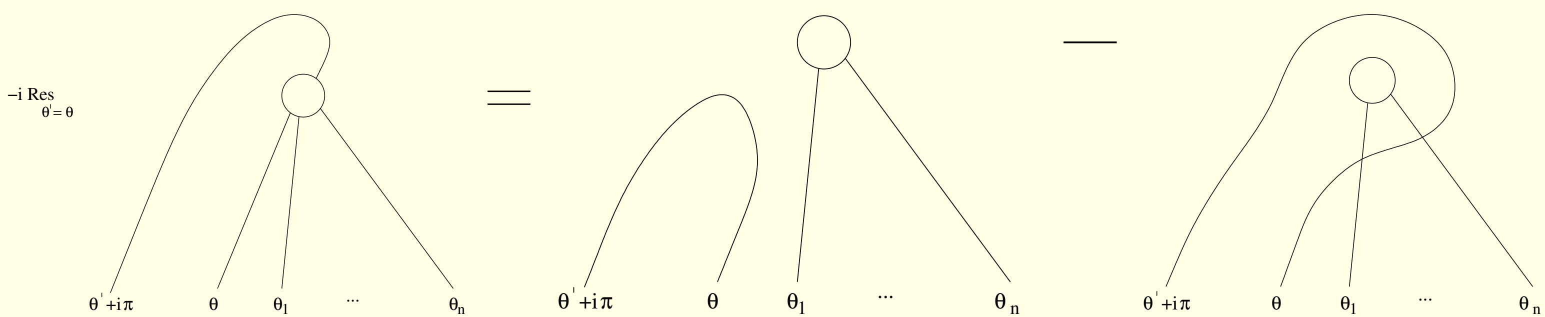
Singularity stucture



$$-i\text{Res}_{\theta'=\theta}\langle 0|\mathcal{O}|\theta' + i\pi,\theta,\theta_1\dots,\theta_n\rangle = (1 - \prod_i S(\theta-\theta_i))\langle 0|\mathcal{O}|\theta_1,\dots,\theta_n\rangle$$

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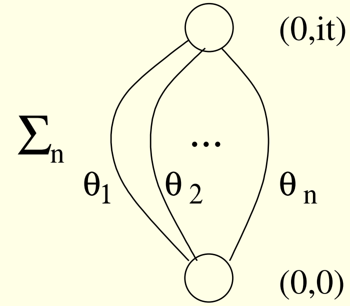

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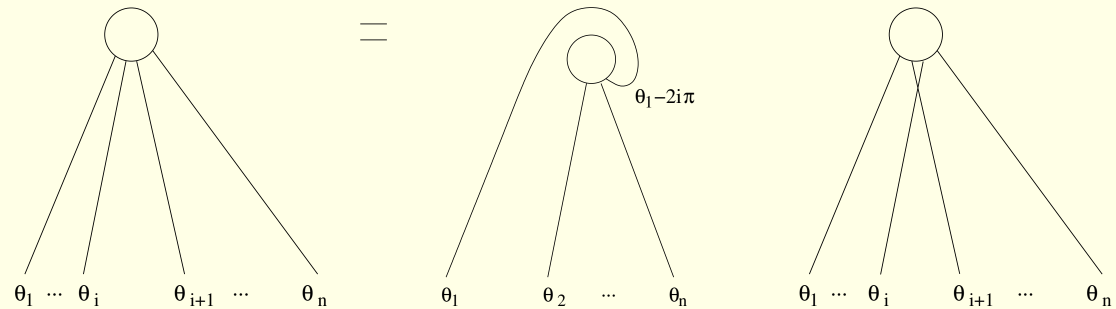

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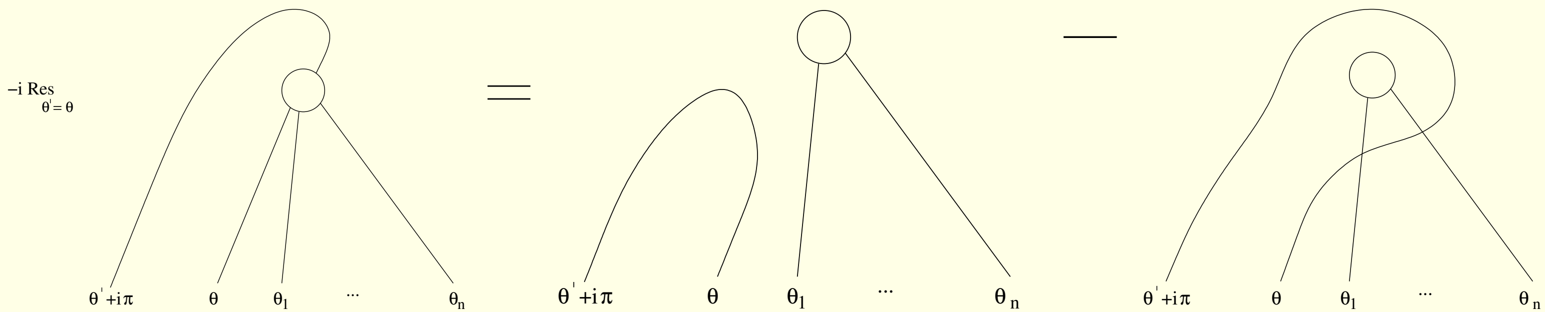

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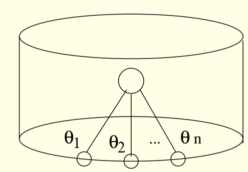
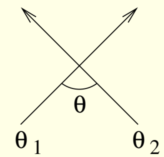
Solution for sinh-Gordon: $\langle 0|\mathcal{O}|\theta_1,\theta_2\rangle = e^{(D+D^{-1})^{-1}\log S}$; $Df(\theta)=f(\theta+i\pi)$

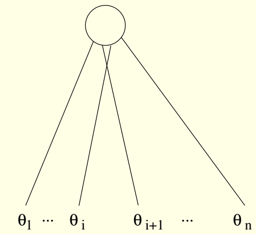
Finite volume form factors: polynomial in L^{-1}: $\langle 0|\mathcal{O}|\theta_1,\dots,\theta_n\rangle_L = \frac{\langle 0|\mathcal{O}|\theta_1,\dots,\theta_n\rangle}{\sqrt{\det[\frac{\partial\Phi_i}{\partial\theta_j}]}}$

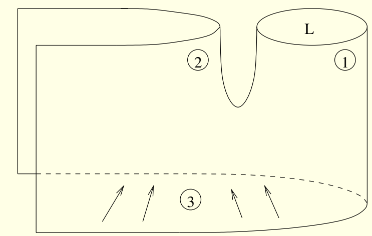


Outline

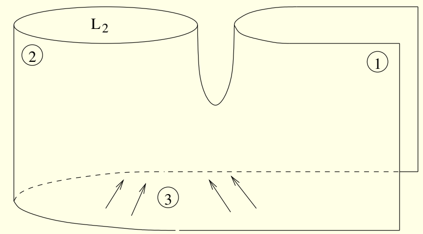
Motivation: the S-matrix and the FF bootstrap



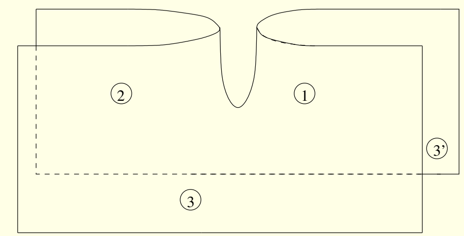




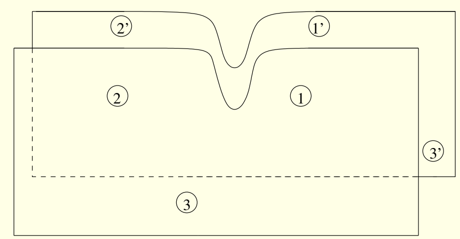
Cutting the stringvertex: decompactifying 2 & 3: Form factor axioms



Decompactifying 1 & 3: the bootstrap program

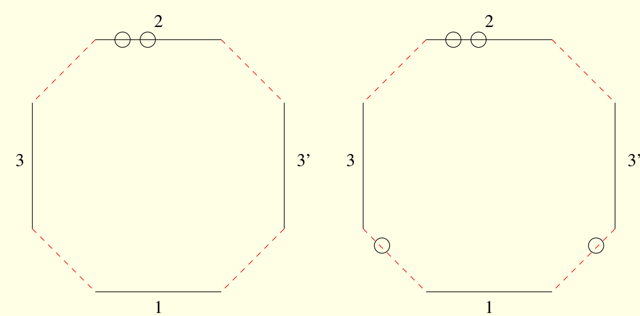


Decompactifying all volumes: Octagon axioms



Cutting one more: hexagon axioms

PP wave limit of AdS/CFT: Solving the FF axioms



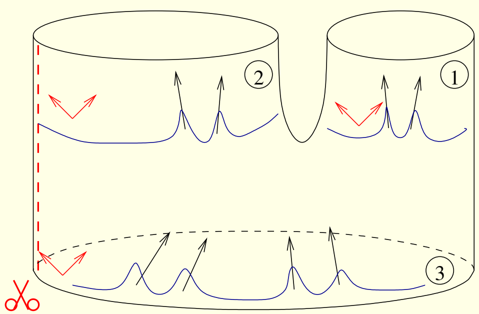
Summing up the octagon

Conclusion

Decompactification limit of the string vertex

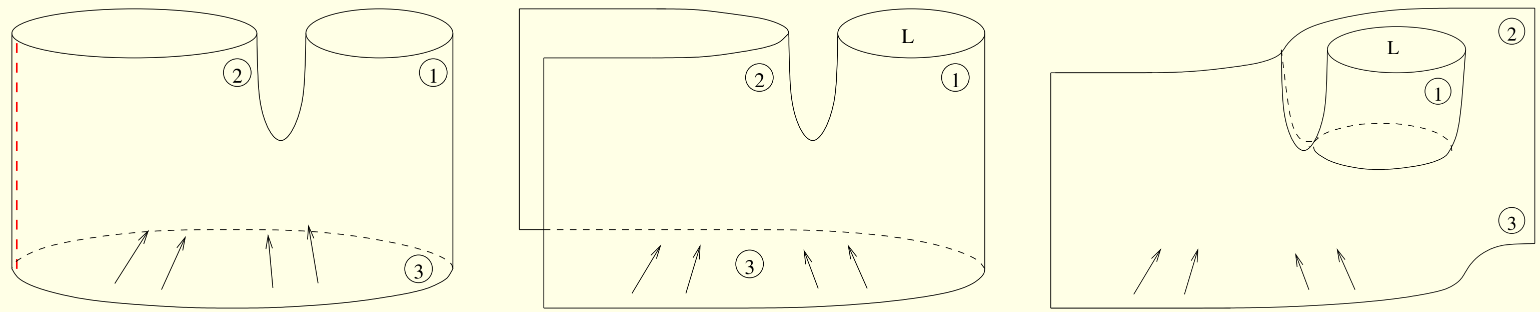
Decompactification limit of the string vertex

Decompactify string 2 & 3:



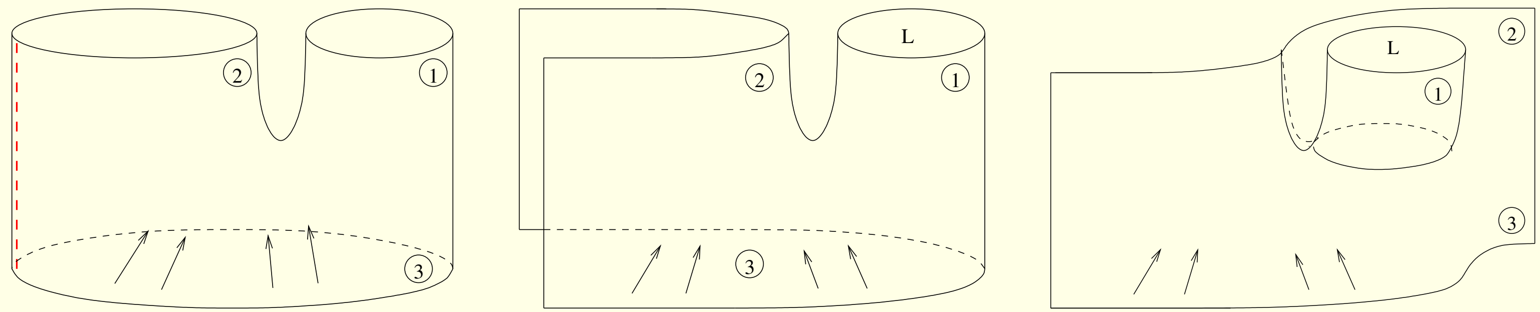
Decompactification limit of the string vertex

Decompactify string 2 & 3:

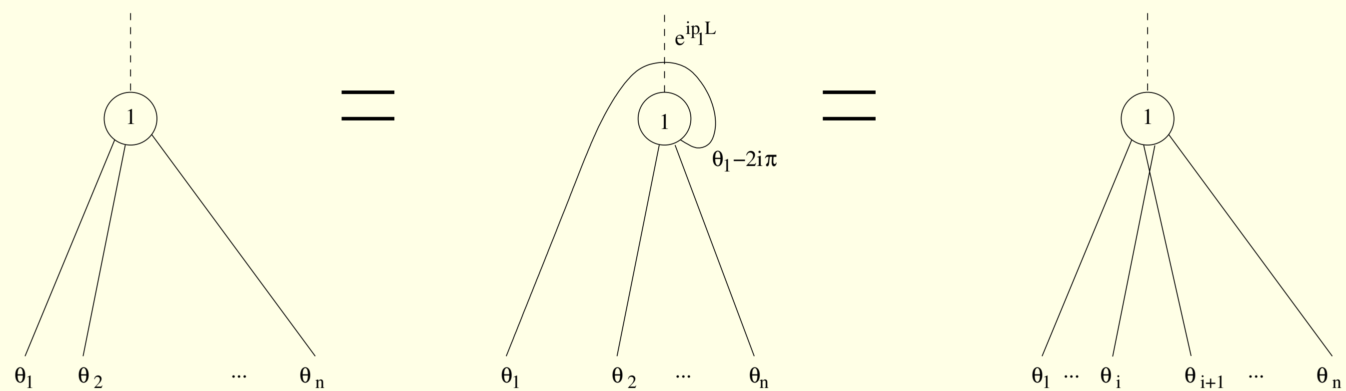


Decompactification limit of the string vertex

Decompactify string 2 & 3:



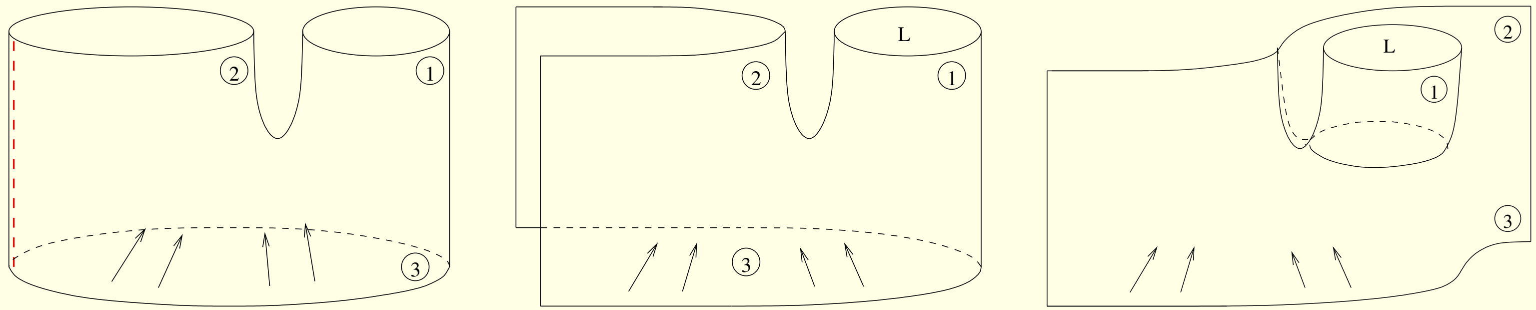
Form factor equations:



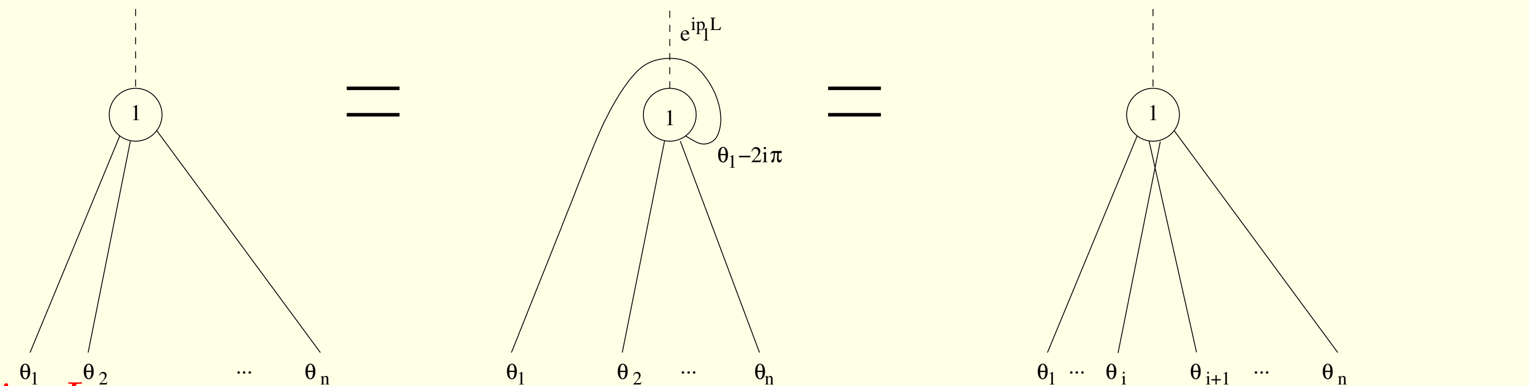
$$N_L(\theta_1,\ldots,\theta_n\rangle = e^{-ip_1L}N_L(\theta_2,\ldots,\theta_n,\theta_1-2i\pi\rangle = S(\theta_i-\theta_{i+1})N_L(\ldots,\theta_{i+1},\theta_i,\ldots)$$

Decompactification limit of the string vertex

Decompactify string 2 & 3:

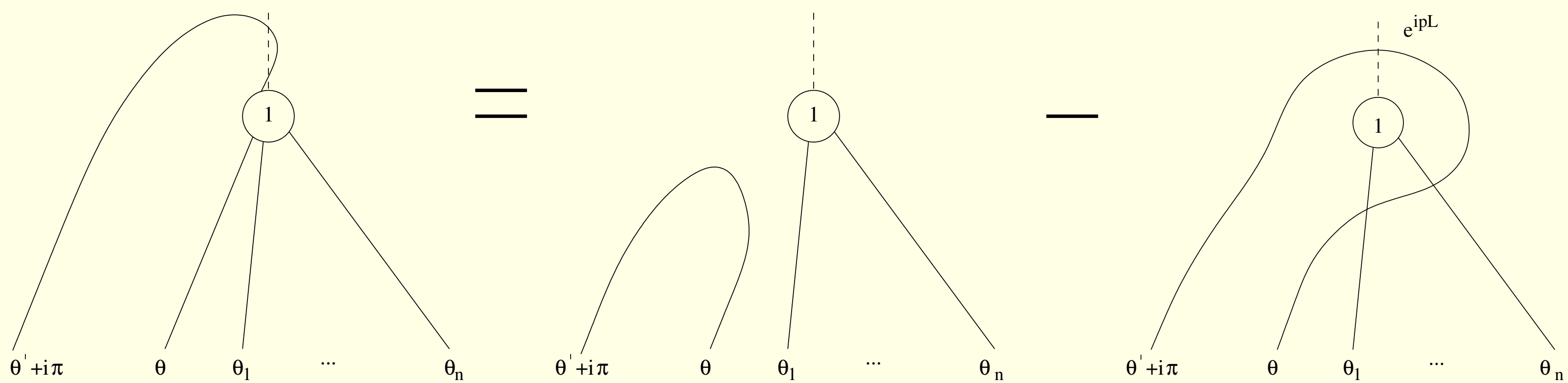


Form factor equations:



$$N_L(\theta_1,\dots,\theta_n\rangle = e^{-ip_1L}N_L(\theta_2,\dots,\theta_n,\theta_1-2i\pi\rangle = S(\theta_i-\theta_{i+1})N_L(\dots,\theta_{i+1},\theta_i,\dots)$$

Singularity stucture



$$-i\mathrm{Res}_{\theta'=\theta}N_L(\theta'+i\pi,\theta,\theta_1\dots,\theta_n) = (1-e^{ipL}\prod_i S(\theta-\theta_i))N_L(\theta_1,\dots,\theta_n)$$

Other decompactification: the bootstrap program

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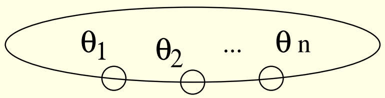
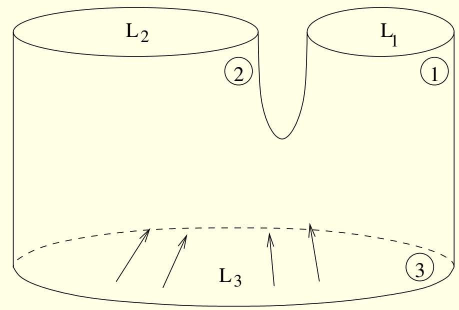
The full large volume amplitude $O(e^{-mL})$

$$e^{ip_1L}S(\theta_1-\theta_2)\dots S(\theta_1-\theta_n)=1$$





Other decompactification: the bootstrap program

The full large volume amplitude $O(e^{-mL})$

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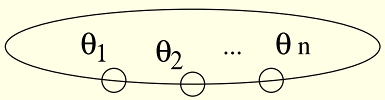
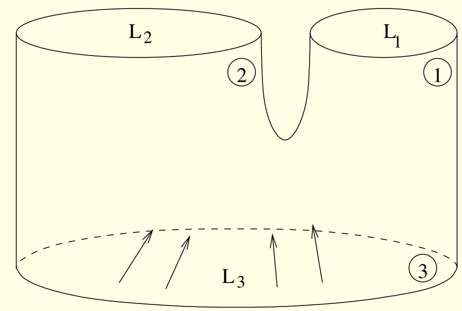
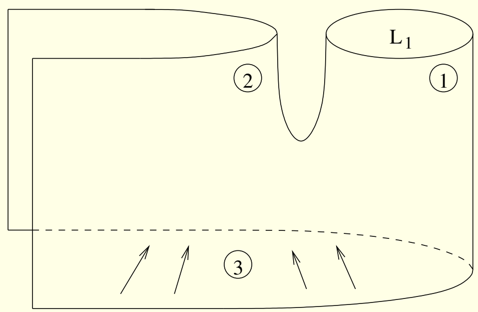




Decompactify

string 2 & 3:

$N_{L_1}(\theta_1, \dots, \theta_n)$



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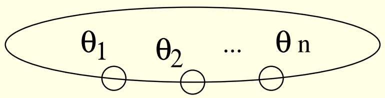
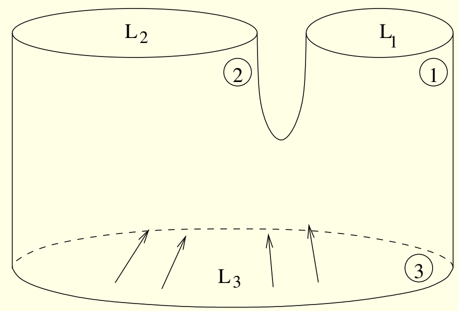
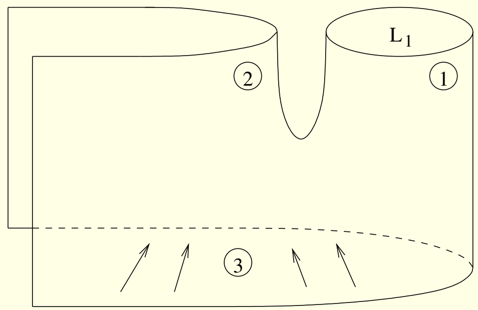
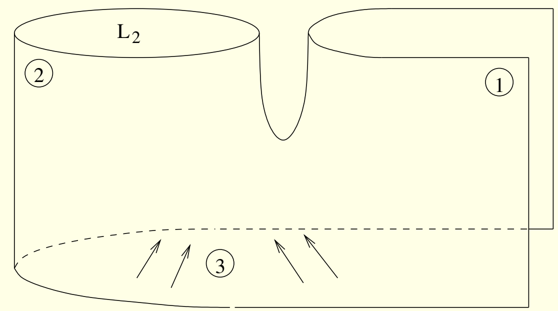




Decompactify string 2 & 3: $N_{L_1}(\theta_1, \ldots, \theta_n)$



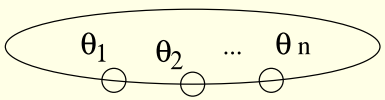
Decompactify string 1 & 3: $N_{L_2}(\theta_1, \ldots, \theta_n)$

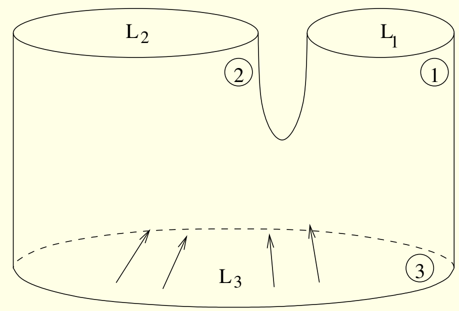


Other decompactification: the bootstrap program

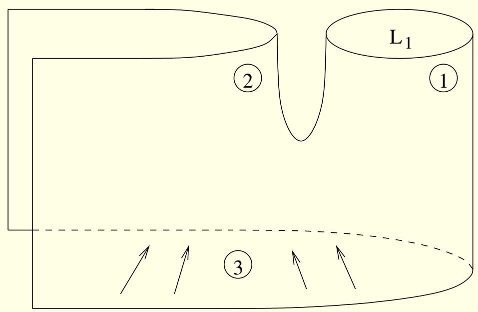
The full large volume amplitude $O(e^{-mL})$

$$e^{ip_1 L} S(\theta_1 - \theta_2) \dots S(\theta_1 - \theta_n) = 1$$

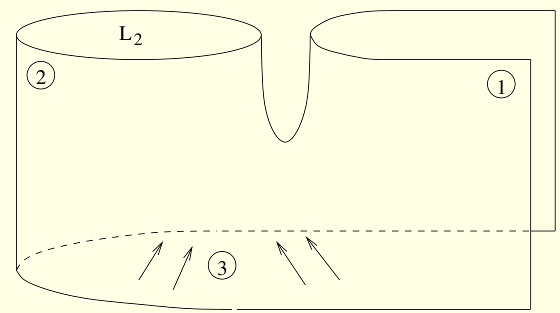




Decompactify string 2 & 3: $N_{L_1}(\theta_1, \dots, \theta_n)$



Decompactify string 1 & 3: $N_{L_2}(\theta_1, \dots, \theta_n)$



Finite (large volume) and infinite volume amplitudes are the same (upto normalization).

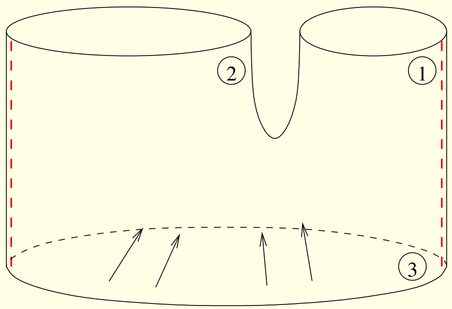
Find the relevant solutions by matching the two in the large L_1, L_2 limit:

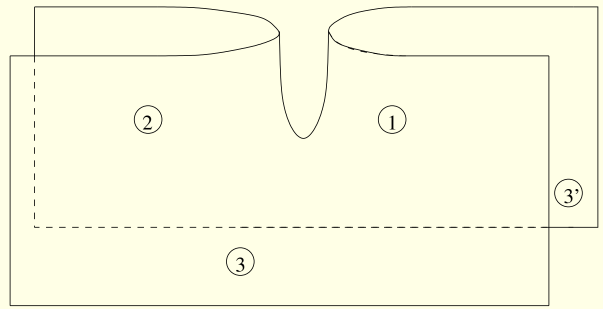
$$N_{L_1}(\theta_1, \dots, \theta_n) \propto N_{L_1,L_2}(\theta_1, \dots, \theta_{n)}) \propto N_{L_2}(\theta_1, \dots, \theta_n)$$

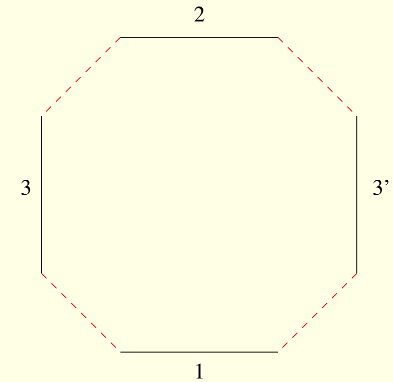
Decompactifying all volumes $L_1 = \infty$: octagon

Decompactifying all volumes $L_1 = \infty$: octagon

Decompactify all volumes

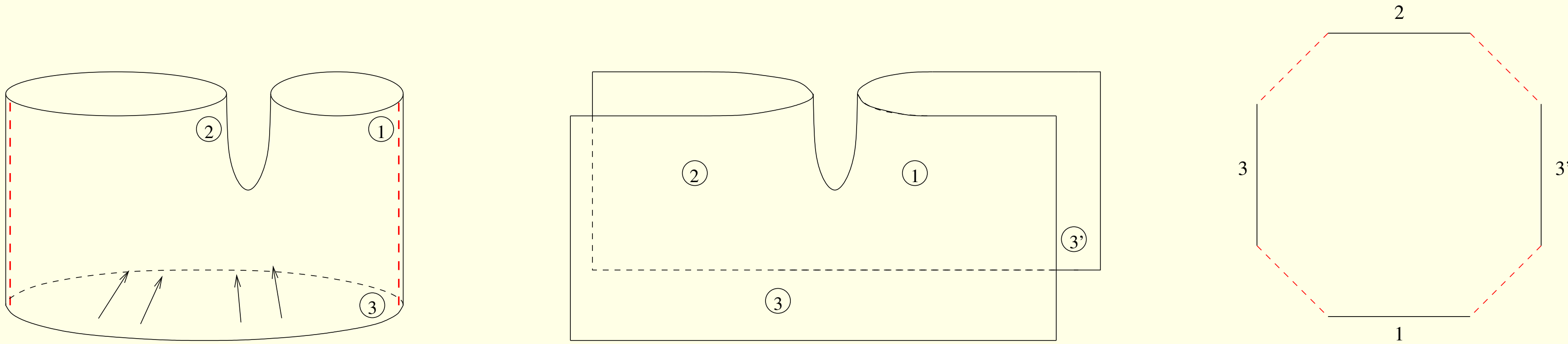




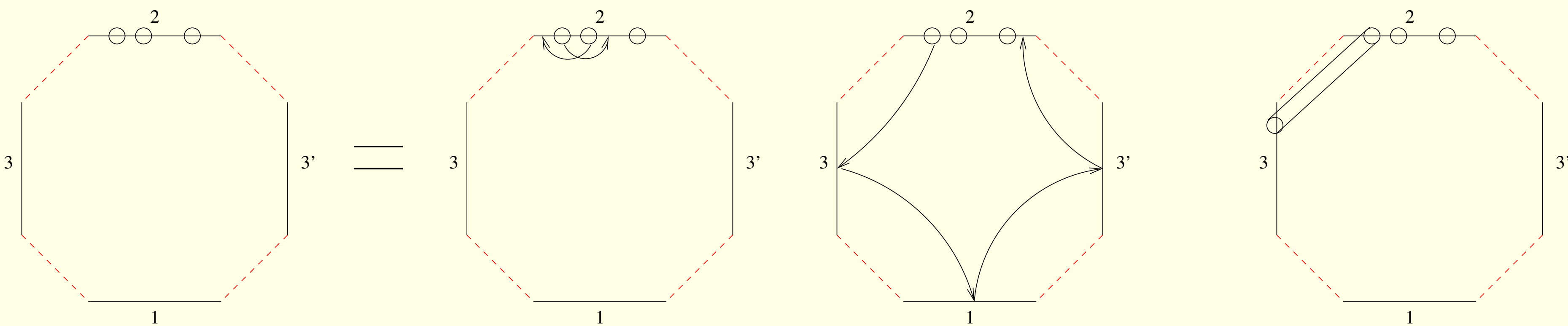


Decompactifying all volumes $L_1 = \infty$: octagon

Decompactify all volumes



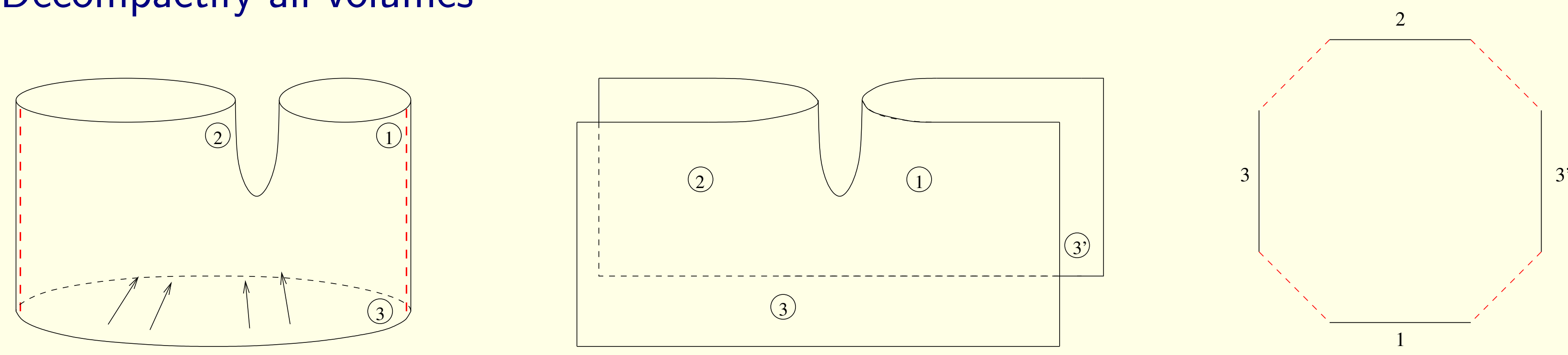
Octagon axioms:



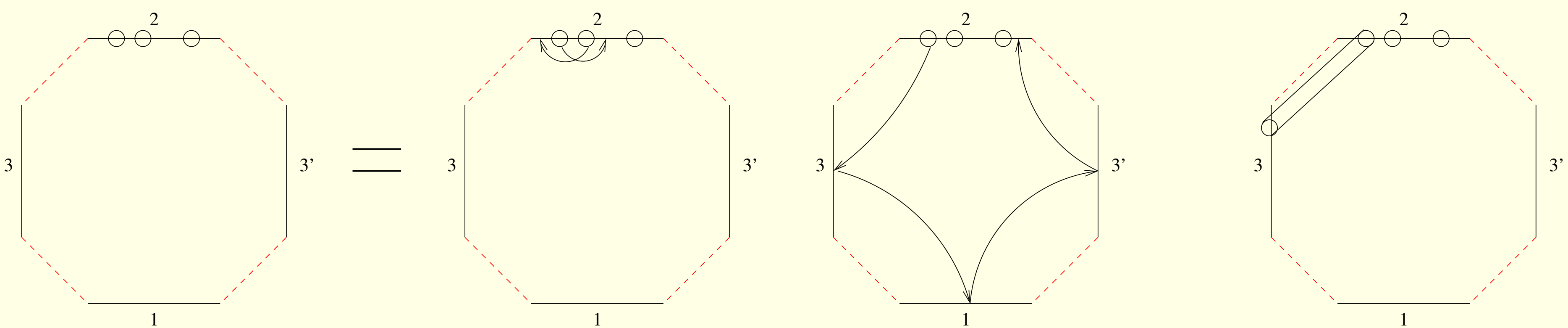
$$O(\theta_i, \dots, \theta_n) = S(\theta_i, \theta_{i+1}) O(\dots, \theta_{i+1}, \theta_i, \dots) = O(\theta_2, \dots, \theta_n, \theta_1 - 4i\pi)$$

Decompactifying all volumes $L_1 = \infty$: octagon

Decompactify all volumes



Octagon axioms:



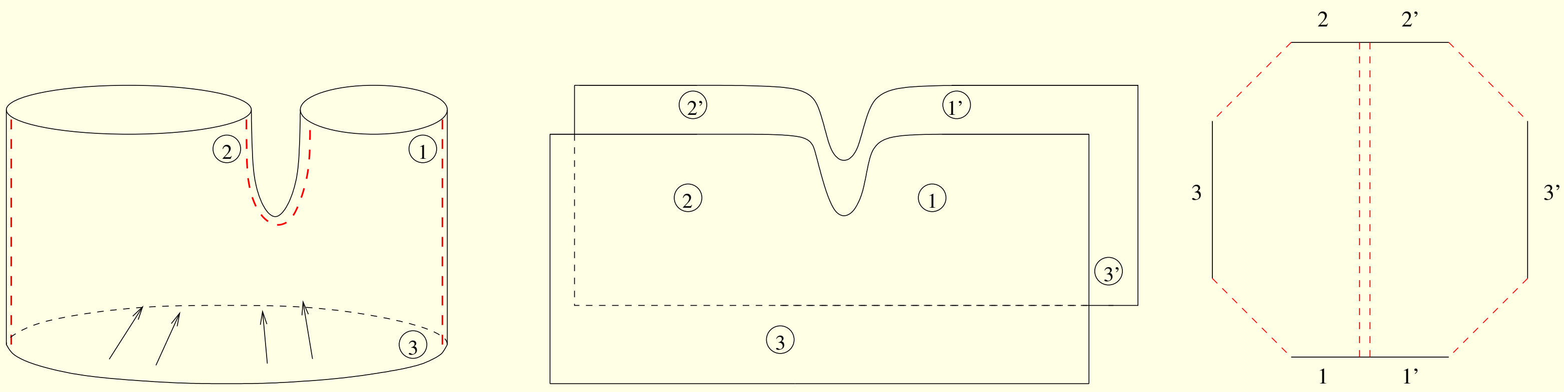
$$O(\theta_i, \dots, \theta_n) = S(\theta_i, \theta_{i+1}) O(\dots, \theta_{i+1}, \theta_i, \dots) = O(\theta_2, \dots, \theta_n, \theta_1 - 4i\pi)$$

Kinematical singularity $-i\text{Res}_{\theta'=\theta} O(\theta' + i\pi, \theta, \theta_1, \dots, \theta_n) = O(\theta_1, \dots, \theta_n)$

Cutting one more: two hexagons

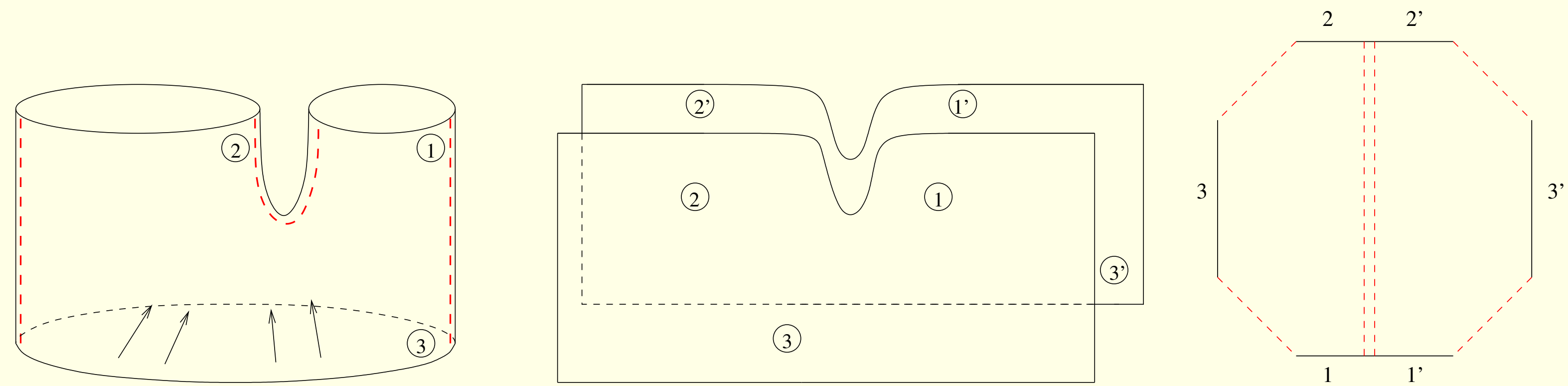
Cutting one more: two hexagons

Decompactify all volumes

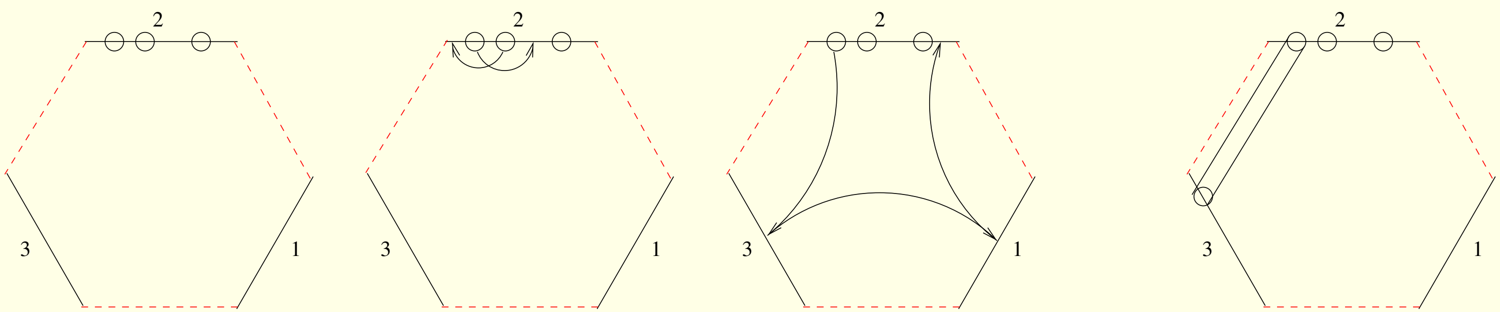


Cutting one more: two hexagons

Decompactify all volumes



Hexagon axioms: [Basso,Komatsu,Vieira '15]

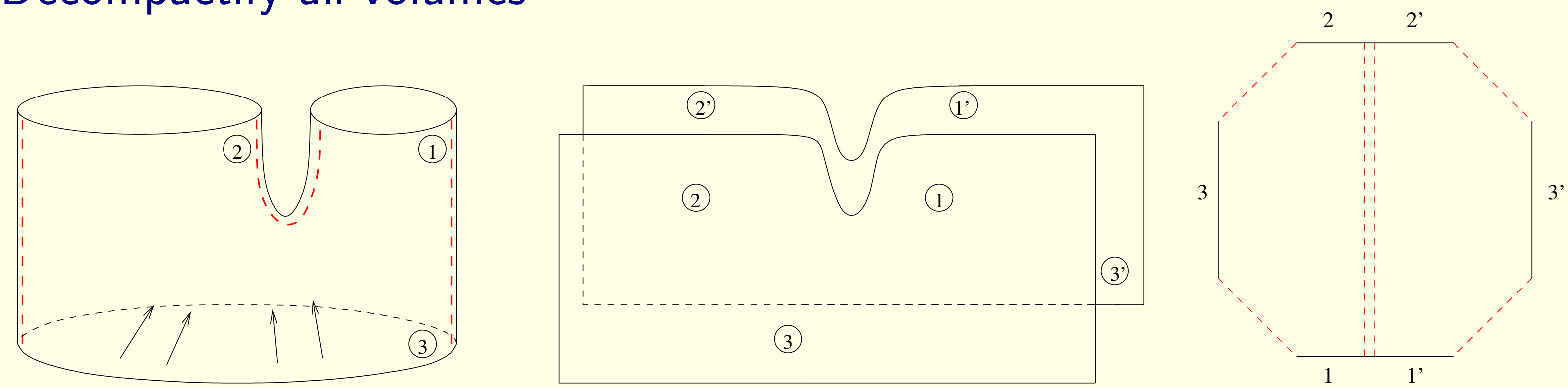


$$h(\theta_i,\ldots,\theta_n)=S(\theta_i,\theta_{i+1})h(\ldots,\theta_{i+1},\theta_i,\ldots)=h(\theta_2,\ldots,\theta_n,\theta_1-3i\pi)$$

Kinematical singularity $-i\mathrm{Res}_{\theta'=\theta}h(\theta'+i\pi,\theta,\theta_1,\ldots,\theta_n)=h(\theta_1,\ldots,\theta_n)$

Cutting one more: two hexagons

Decompactify all volumes



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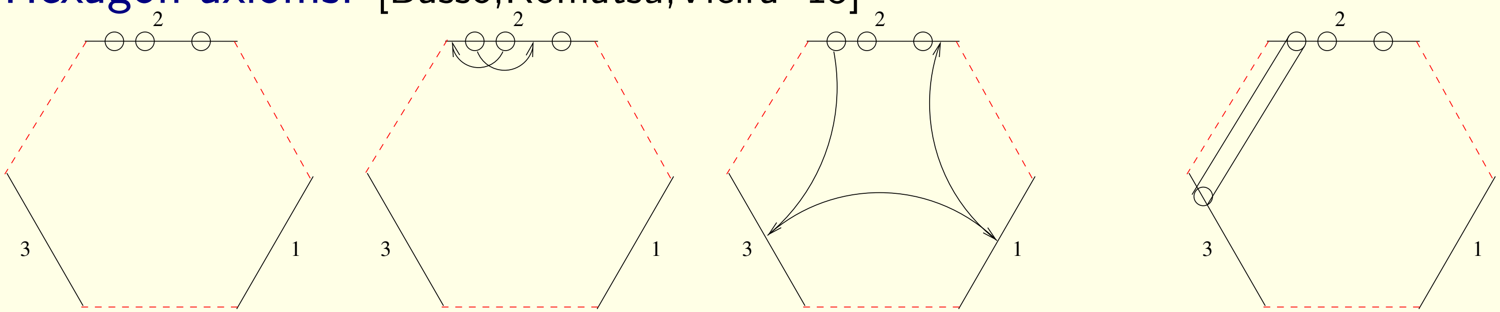


$$h(\theta_i,\ldots,\theta_n) = S(\theta_i,\theta_{i+1})h(\ldots,\theta_{i+1},\theta_i,\ldots) = h(\theta_2,\ldots,\theta_n,\theta_1-3i\pi)$$

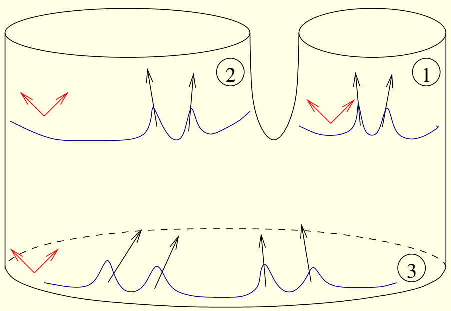
Kinematical singularity $-i\text{Res}_{\theta'=\theta}h(\theta'+i\pi,\theta,\theta_1,\ldots,\theta_n) = h(\theta_1,\ldots,\theta_n)$

Complete solution: $h(\theta_1,\theta_2) \propto \sigma(\theta_1,\theta_2)S_{\text{Beisert}}(\theta_1,\theta_2)$

Comparision of the different approaches

Comparision of the different approaches

Ultimate goal:

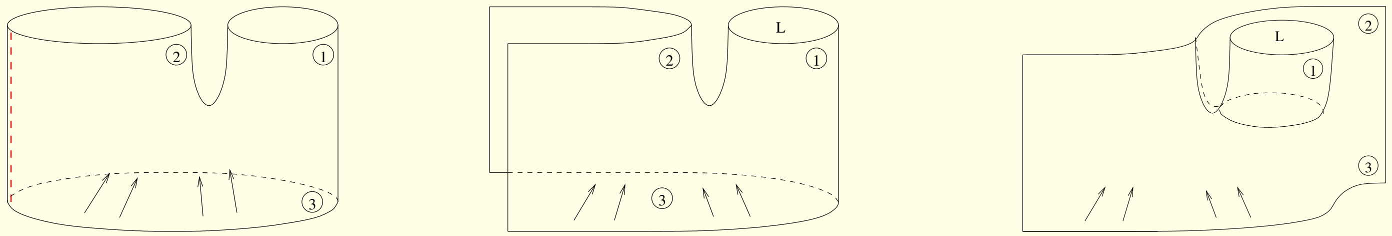


Comparision of the different approaches

1 cut:

nonlocal

form factors



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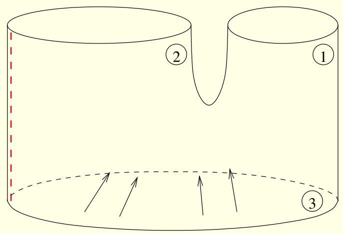
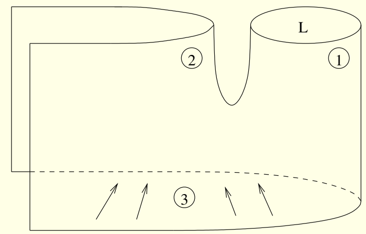
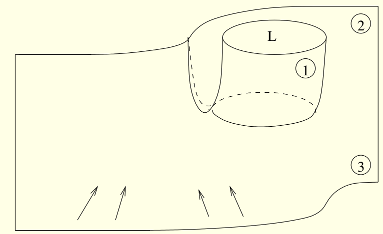
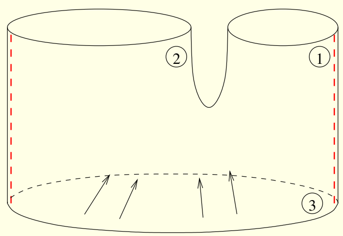
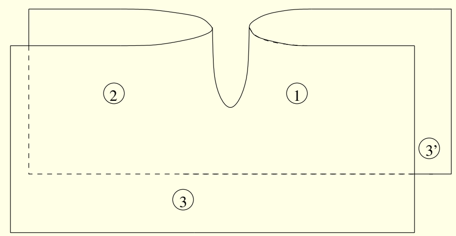


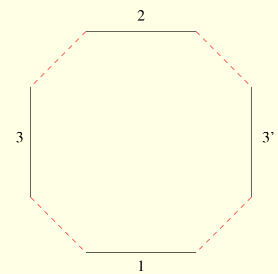




2 cuts
octagon

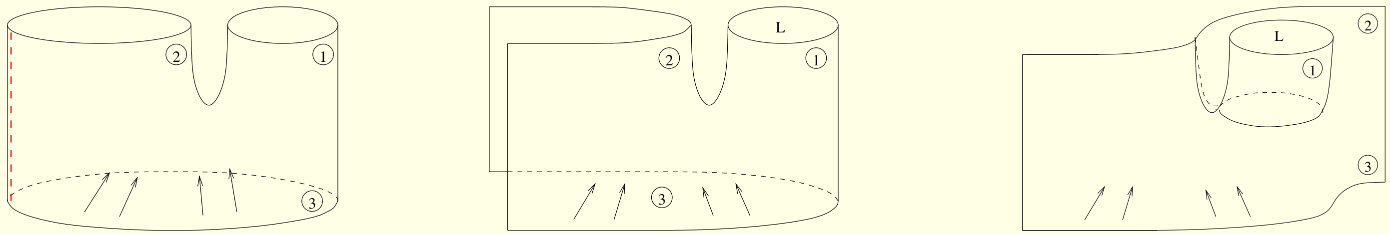




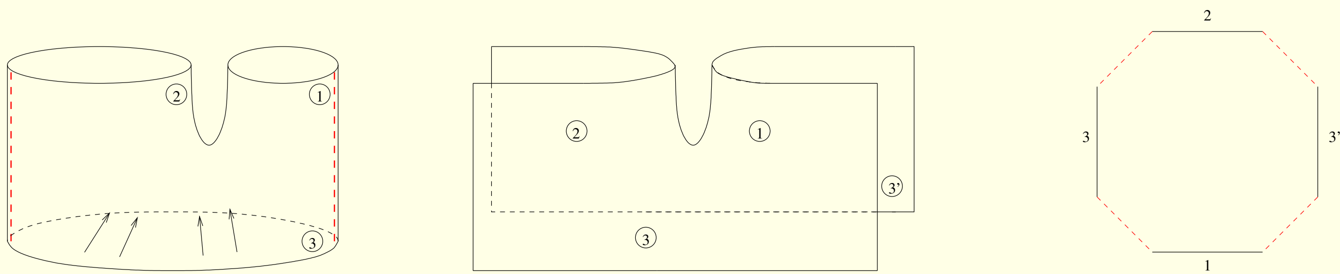


Comparision of the different approaches

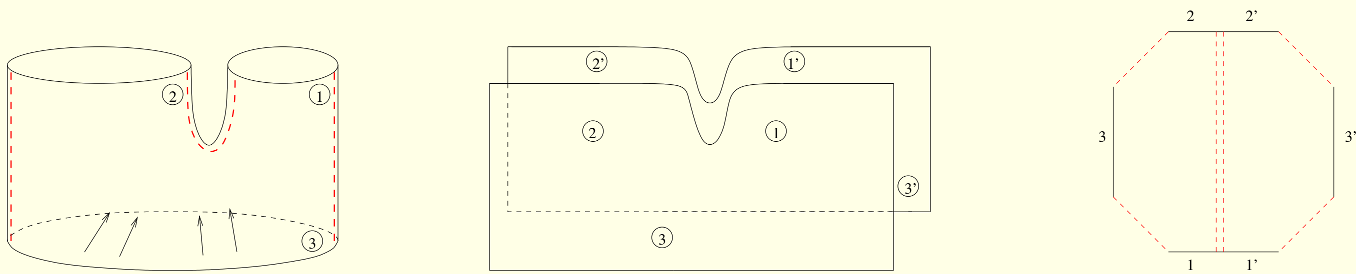
1 cut:
nonlocal
form factors



2 cuts
octagon



3 cuts
hexagon



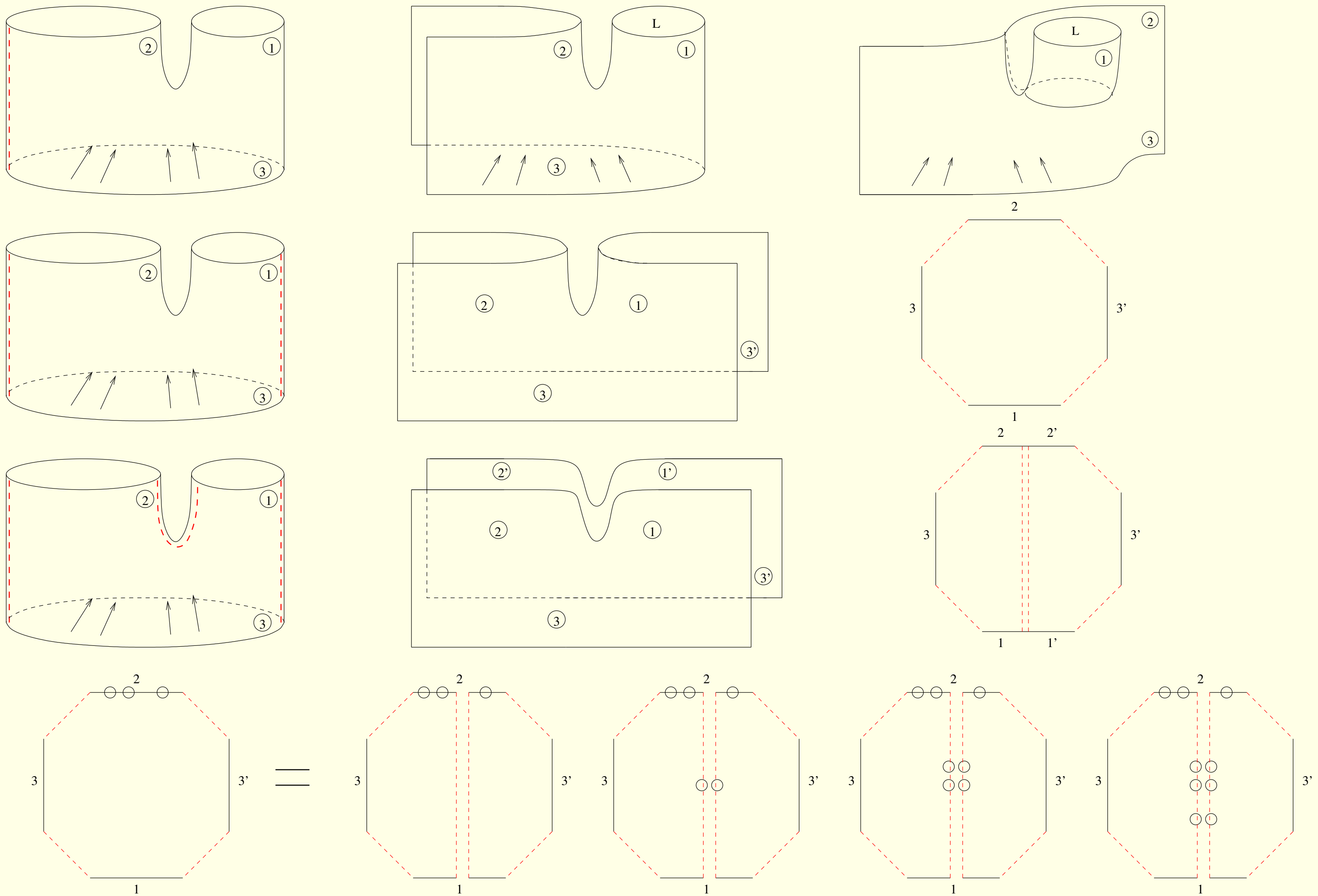
Comparision of the different approaches

1 cut:
nonlocal
form factors

2 cuts
octagon

3 cuts
hexagon

sewing back



$$O(\theta_1,\theta_2,\theta_3)=h(\theta_1,\theta_2)h(\theta_3)+\ldots+\int\frac{du}{2\pi}\mu(u)h(\theta_1,\theta_2,u-i\tfrac{\pi}{2})h(u+i\tfrac{\pi}{2},\theta_3)e^{-E(u)l}+$$

Perturbative checks: [Eden & Sfondrini] [Basso et. al] HHH: [Jiang, Komatsu, Kostov, Serban]

Free massive boson: pp-wave limit of strings on $AdS_5 \times S^5$

Free massive boson: pp-wave limit of strings on $AdS_5 \times S^5$

1 cut:
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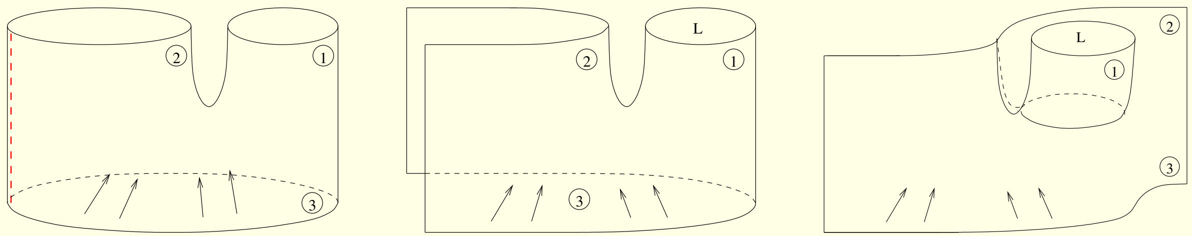
form factors



Free massive boson: pp-wave limit of strings on $AdS_5 \times S^5$

1 cut:
nonlocal
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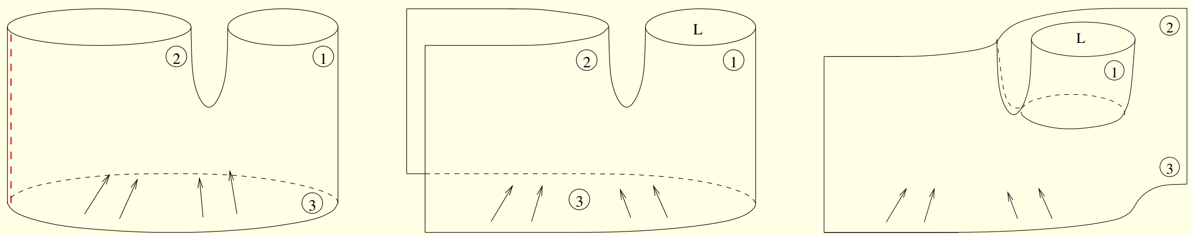
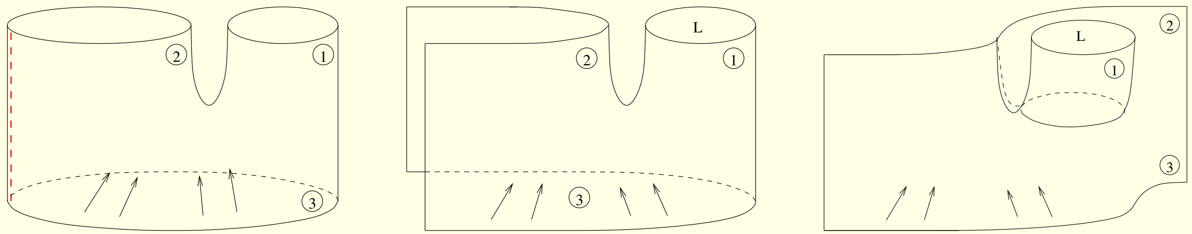


2 particles in string 3: $$N_L(\theta_1, \theta_2) = N_L(\theta_2, \theta_1) = e^{-ip_1 L} N_L(\theta_2, \theta_1 - 2i\pi)$$

kinematical singularity: $$-i\mathrm{Res}_\epsilon N_L(\theta + i\pi + \epsilon, \theta) = (1 - e^{ipL})$$

Free massive boson: pp-wave limit of strings on $AdS_5 \times S^5$

1 cut:
nonlocal
form factors



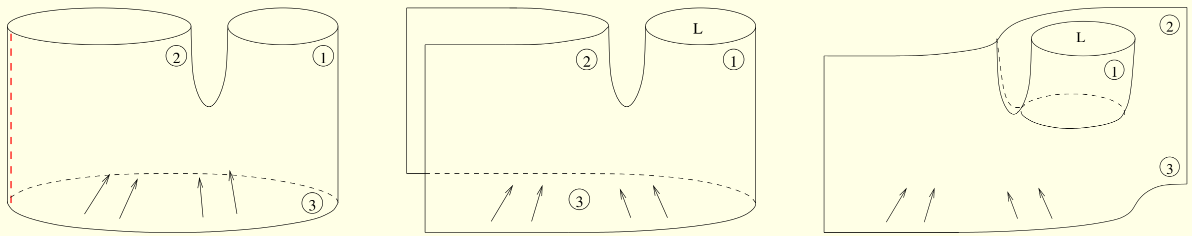
2 particles in string 3: $$N_L(\theta_1,\theta_2)=N_L(\theta_2,\theta_1)=e^{-ip_1L}N_L(\theta_2,\theta_1-2i\pi)$$

kinematical singularity: $$-i\text{Res}_\epsilon N_L(\theta+i\pi+\epsilon,\theta)=(1-e^{ipL})$$

Solve the first two by: $$N_L(\theta_1,\theta_2)=\frac{e^{-\theta_1\frac{p_1}{2\pi}L}e^{-\theta_2\frac{p_2}{2\pi}L}}{\cosh\frac{\theta_1+\theta_2}{2}}n(\theta_1)n(\theta_2)$$

Free massive boson: pp-wave limit of strings on $AdS_5 \times S^5$

1 cut:
nonlocal
form factors



2 particles in string 3: $N_L(\theta_1,\theta_2) = N_L(\theta_2,\theta_1) = e^{-ip_1 L} N_L(\theta_2,\theta_1 - 2i\pi)$

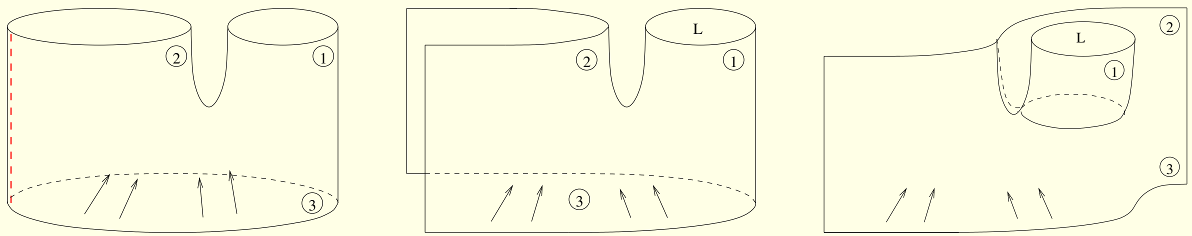
kinematical singularity: $-i\mathrm{Res}_\epsilon N_L(\theta + i\pi + \epsilon, \theta) = (1 - e^{ipL})$

Solve the first two by: $N_L(\theta_1,\theta_2) = \dfrac{e^{-\theta_1 \frac{p_1}{2\pi} L} e^{-\theta_2 \frac{p_2}{2\pi} L}}{\cosh \frac{\theta_1+\theta_2}{2}} n(\theta_1) n(\theta_2)$

kin. singularity: $n(\theta) n(\theta + i\pi) = \sinh\theta \sin\frac{pL}{2}$ zeros at $\theta = \frac{2\pi n}{L}$ and $\theta = \frac{2\pi n}{L} + i\pi$

Free massive boson: pp-wave limit of strings on $AdS_5 \times S^5$

1 cut:
nonlocal
form factors



2 particles in string 3: $N_L(\theta_1,\theta_2) = N_L(\theta_2,\theta_1) = e^{-ip_1 L} N_L(\theta_2, \theta_1 - 2i\pi)$

kinematical singularity: $-i\mathrm{Res}_\epsilon N_L(\theta + i\pi + \epsilon, \theta) = (1 - e^{ipL})$

Solve the first two by: $$N_L(\theta_1,\theta_2) = \frac{e^{-\theta_1 \frac{p_1}{2\pi} L} e^{-\theta_2 \frac{p_2}{2\pi} L}}{\cosh \frac{\theta_1+\theta_2}{2}} n(\theta_1) n(\theta_2)$$

kin. singularity: $n(\theta)n(\theta+i\pi) = \sinh\theta \sin\frac{pL}{2}$ zeros at $\theta = \frac{2\pi n}{L}$ and $\theta = \frac{2\pi n}{L} + i\pi$

$n(\theta) = \sinh\frac{\theta}{2} \sin\frac{pL}{2}\, \Gamma_{\frac{mL}{2\pi}}(m \sinh\theta)$ where Γ_μ removes zeros at $\theta = \frac{2\pi n}{L} + i\pi$

$\Gamma_\mu(z) = z^{-1} e^{-\omega_z(\gamma + \log\frac{\mu}{2e})} \prod \frac{n}{\omega_n + \omega_z} e^{-\frac{\omega_n}{z}}$ and $\omega_z = \sqrt{\mu^2 + z^2}$

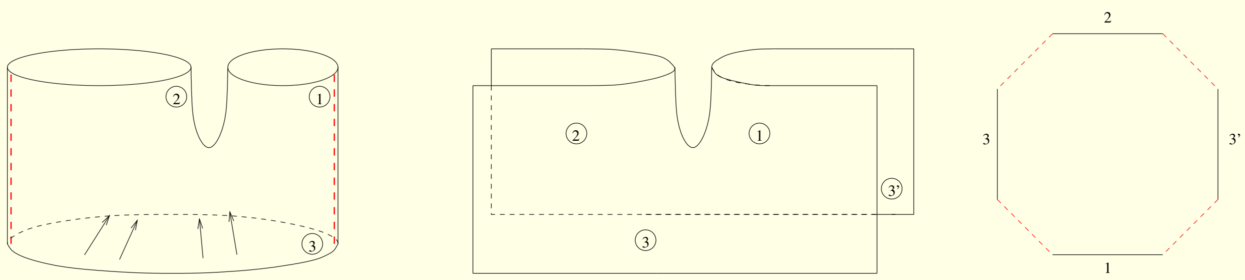
[Spradlin et al '02,Lucietti et al '03]

Free massive boson: summing up the octagons

Free massive boson: summing up the octagons

2 cuts

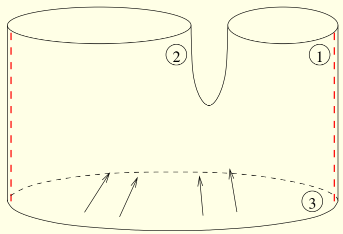
octagon

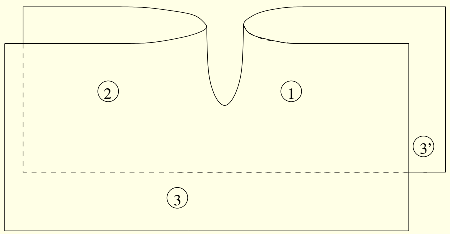


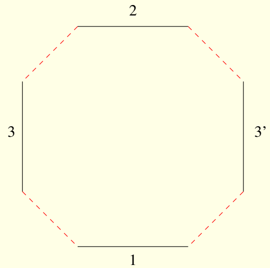
Free massive boson: summing up the octagons

2 cuts

octagon







2 particles: $O(\theta_1, \theta_2) = O(\theta_2, \theta_1) = O(\theta_2, \theta_1 - 4i\pi)$

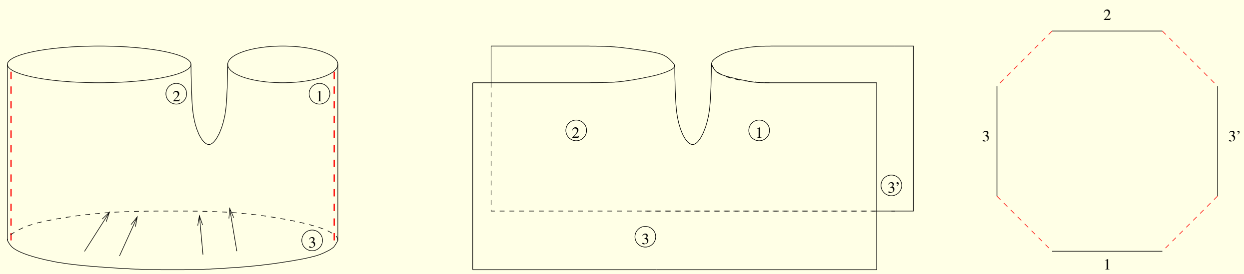
kinematical singularity: $-i\text{Res}_\epsilon O(\theta + i\pi + \epsilon, \theta) = 1$

$\rightarrow \quad O(\theta_1, \theta_2) = \frac{1}{\cosh \frac{\theta_1 - \theta_2}{2}}$

Free massive boson: summing up the octagons

2 cuts

octagon



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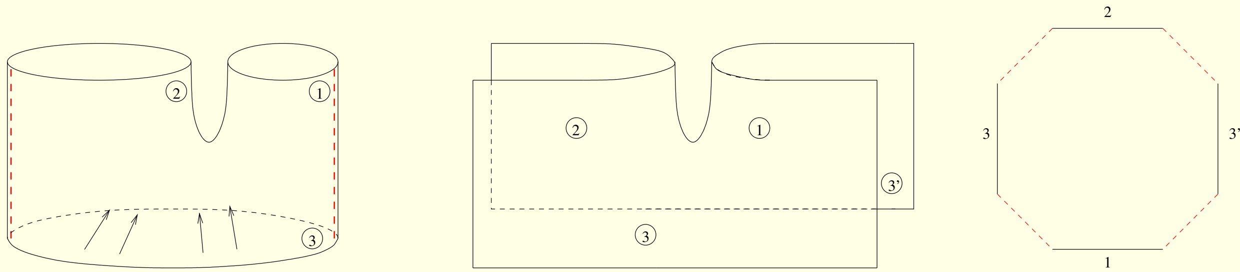
Multiparticle solution: $O(1,2,3,4) = O(1,2)O(3,4) + O(1,3)O(2,4) + O(1,4)O(2,3)$

(Wick theorem)

Free massive boson: summing up the octagons

2 cuts
octagon



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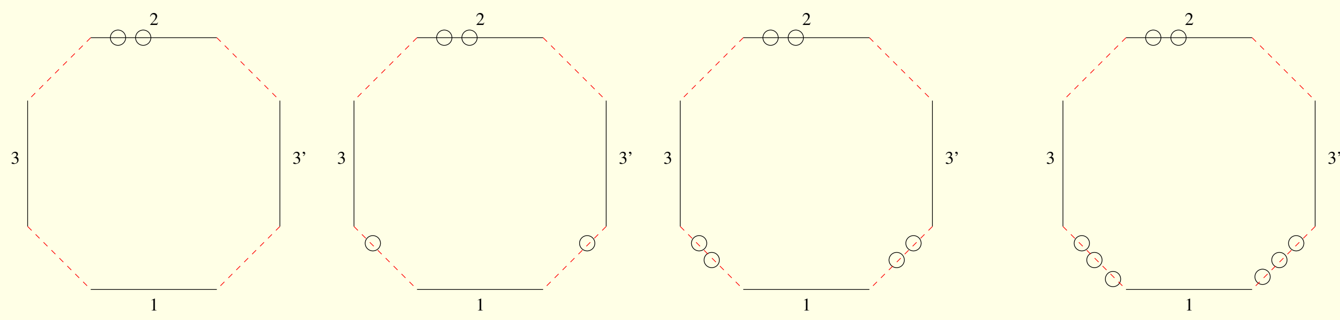
Summing up
virtual corrections



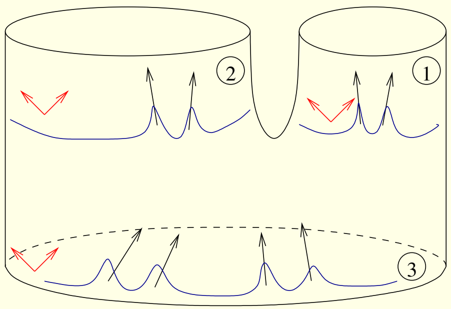
$$N_L(\theta_1, \theta_2) = O(\theta_1, \theta_2) + \int \frac{du}{2\pi} O(\theta_1, \theta_2, u - 3i\tfrac{\pi}{2}, u + 3i\tfrac{\pi}{2})_c \, e^{-m \cosh u L} + \dots$$

Agrees with the expansion of $N_L(\theta_1, \theta_2)$!

Conclusion

Conclusion

Ultimate goal:

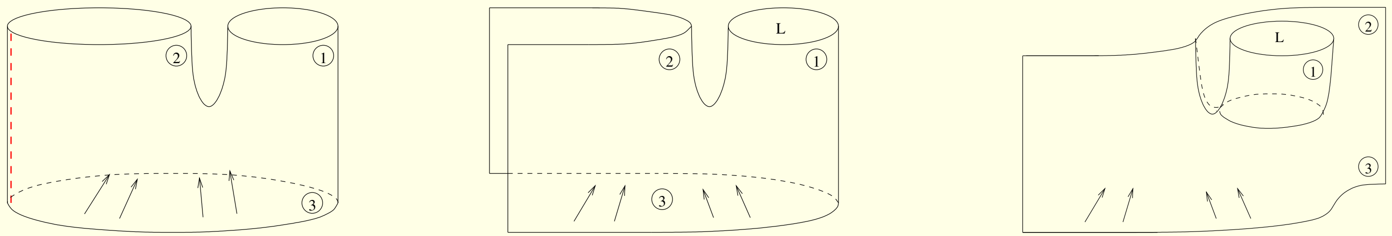


Conclusion

1 cut:

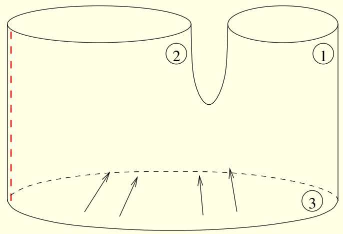
nonlocal

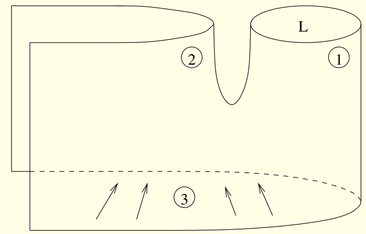
factors

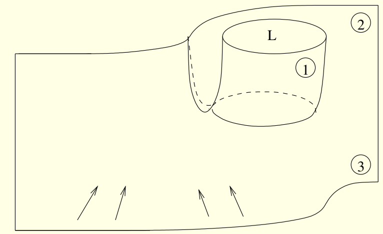


Conclusion

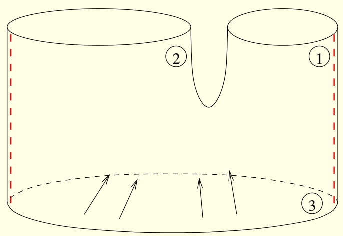
1 cut:
nonlocal
factors

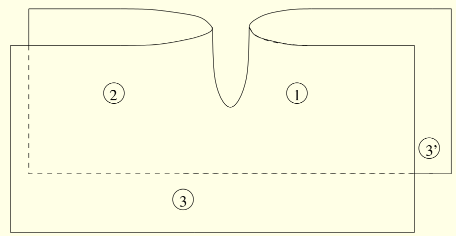


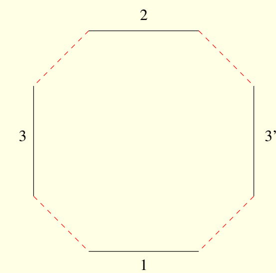




2 cuts
octagon







Conclusion

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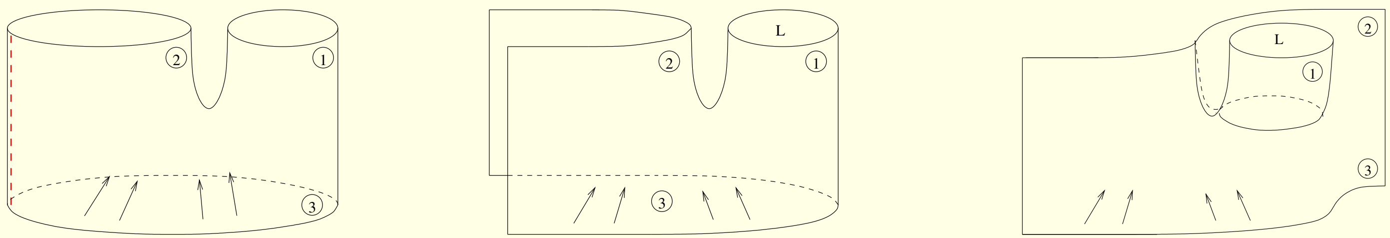

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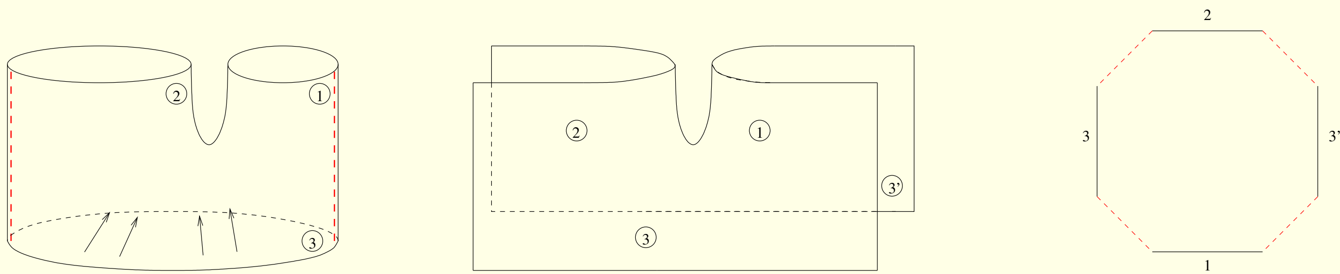
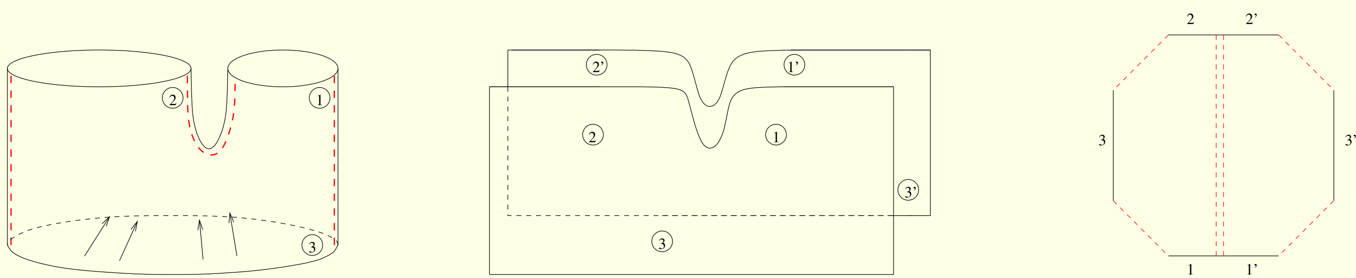
octagon



3 cuts
hexagon



Conclusion

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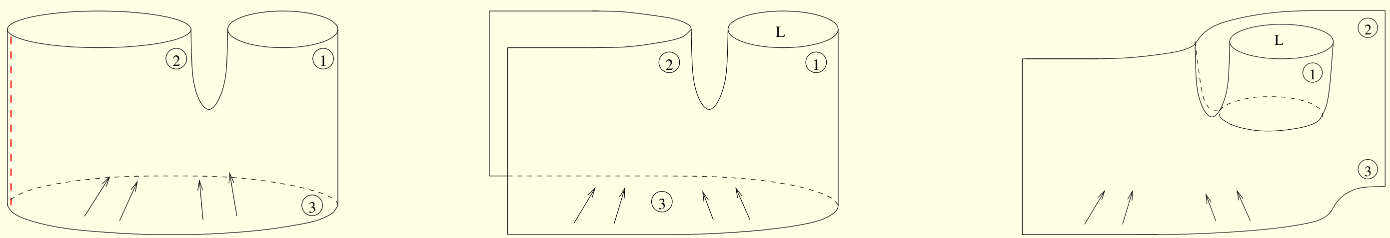

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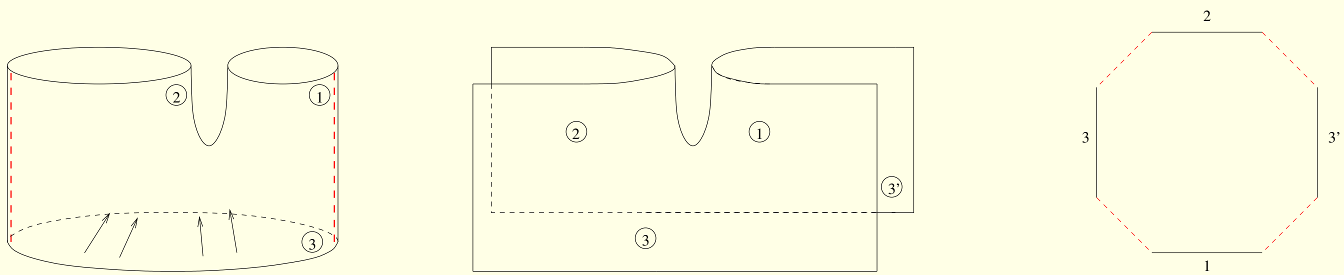

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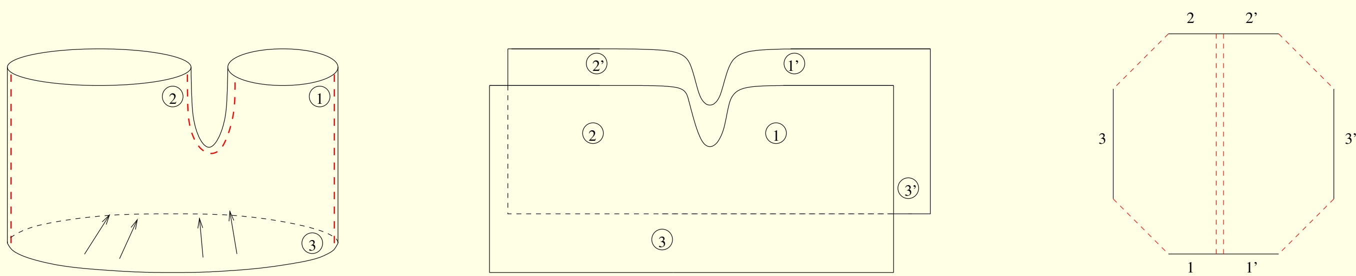
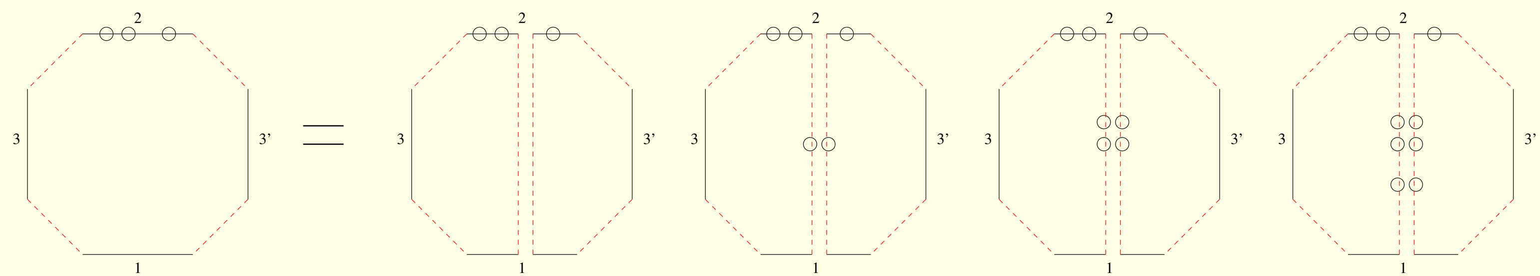


sewing back



$$O(\theta_1,\theta_2,\theta_3) = h(\theta_1,\theta_2)h(\theta_3) + \ldots + \int \frac{du}{2\pi}\mu(u)h(\theta_1,\theta_2,u-i\tfrac{\pi}{2})h(u+i\tfrac{\pi}{2},\theta_3)e^{-E(u)l} +$$

The string vertex for $L_1 = 0$: diagonal form factor

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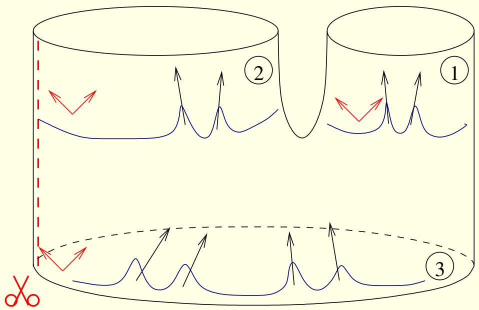
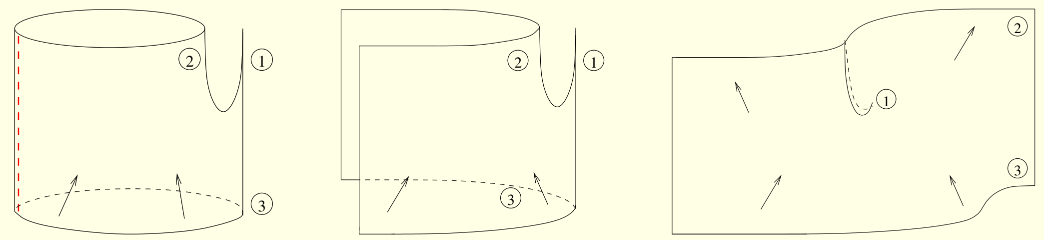
Decompactify string 2 & 3 but $L_1 = 0$:



The string vertex for $L_1 = 0$: diagonal form factor

Decompactify string 2 & 3 and $L_1 = 0$:



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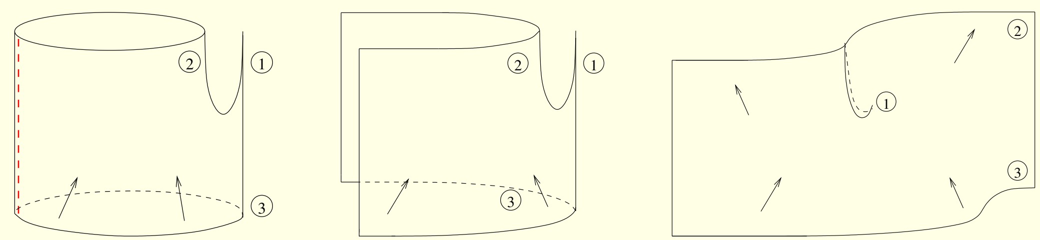

Local operator form factor equations:

$$N_0(\theta_1, \dots, \theta_n\rangle = N_0(\theta_2, \dots, \theta_n, \theta_1 - 2i\pi\rangle = S(\theta_i - \theta_{i+1}) N_0(\dots, \theta_{i+1}, \theta_i, \dots)$$

$$-i\mathrm{Res}_{\theta'=\theta} N_0(\theta' + i\pi, \theta, \theta_1 \dots, \theta_n) = (1 - \textstyle\prod_i S(\theta - \theta_i)) N_0(\theta_1, \dots, \theta_n)$$

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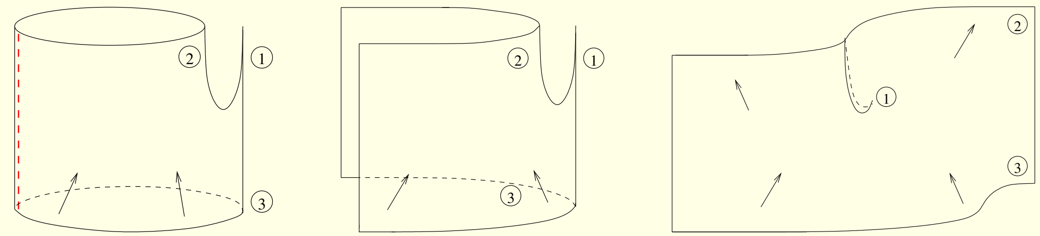

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HeavyHeavyLight 3pt function strong coupling prescription

[Costa et al., Zarembo]: $C_{HHL} = \int_{\text{world sheet}} \mathcal{V}(X[\text{heavy solution}(\sigma, \tau)]) d^2\sigma$

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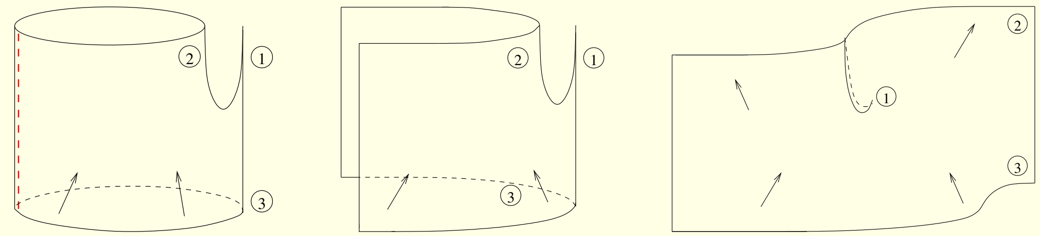

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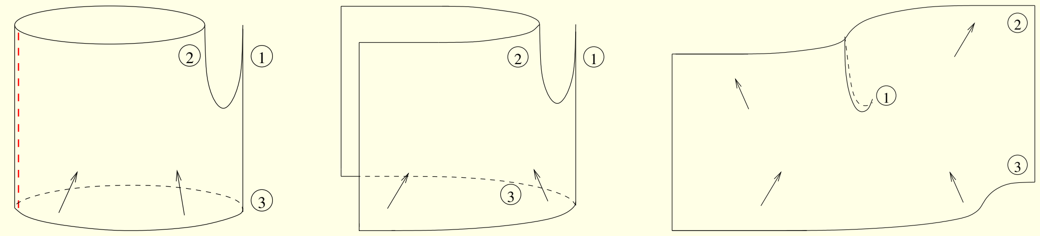

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(classical) diagonal form factors: ${}_L\langle \theta_2, \theta_1 | \mathcal{V} | \theta_1, \theta_2 \rangle_L = \frac{F_2^s(\theta_1,\theta_2) + \rho_1(\theta_1) F_1^s(\theta_2) + \rho_1(\theta_2) F_1^s(\theta_1)}{\rho_2(\theta_1,\theta_2)}$

$$e^{i\Phi_k} = 1 \quad ; \quad \Phi_k = p_k L - i \sum_{j:j\neq k} \log S(\theta_k, \theta_j) \quad ; \qquad \rho_n(\theta_1, ..., \theta_n) = \det \left[\frac{\partial \Phi_j}{\partial \theta_i} \right]$$

diagonal form factor $F_2^s(\theta_1, \theta_2) = \lim_{\epsilon \to 0} N_0(\bar\theta_2, \bar\theta_1, \theta_1 + \epsilon, \theta_2 + \epsilon)$

Explicitly checked at weak coupling [Hollo, Jiang, Petrovskii], AdS Form factors [McLoughlin, Klose]