

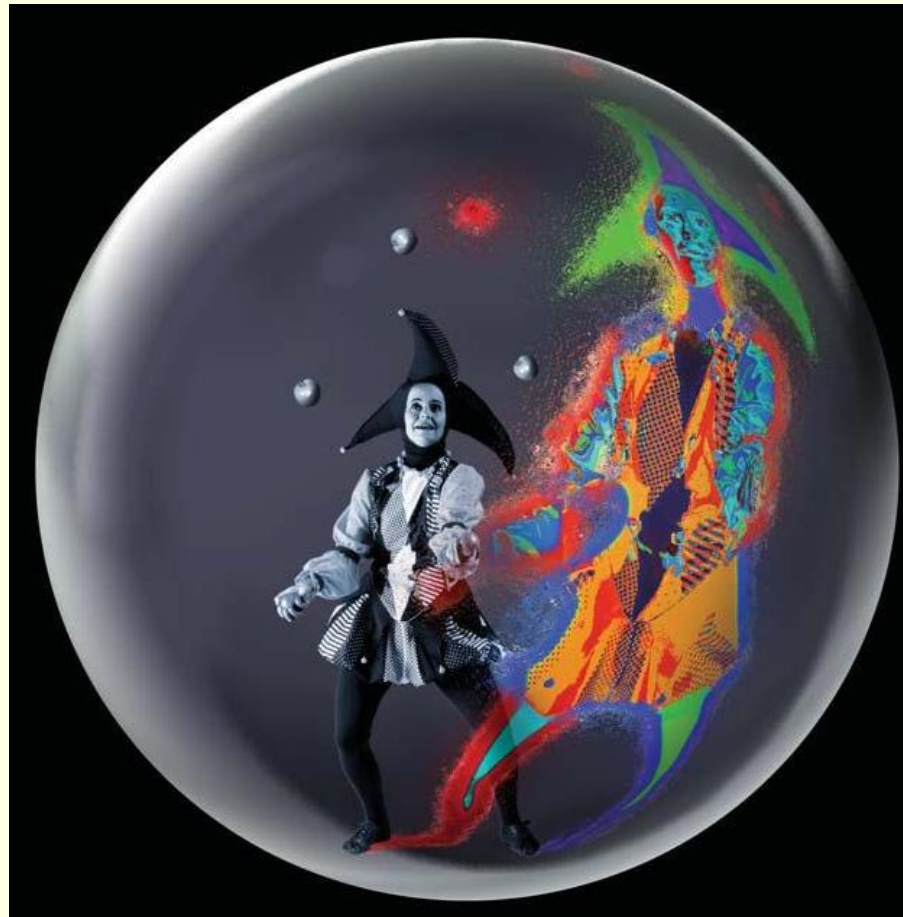
Seminar, December 7, 2012, Bratislava

# Gauge/gravity duality: an overview

**Z. Bajnok**

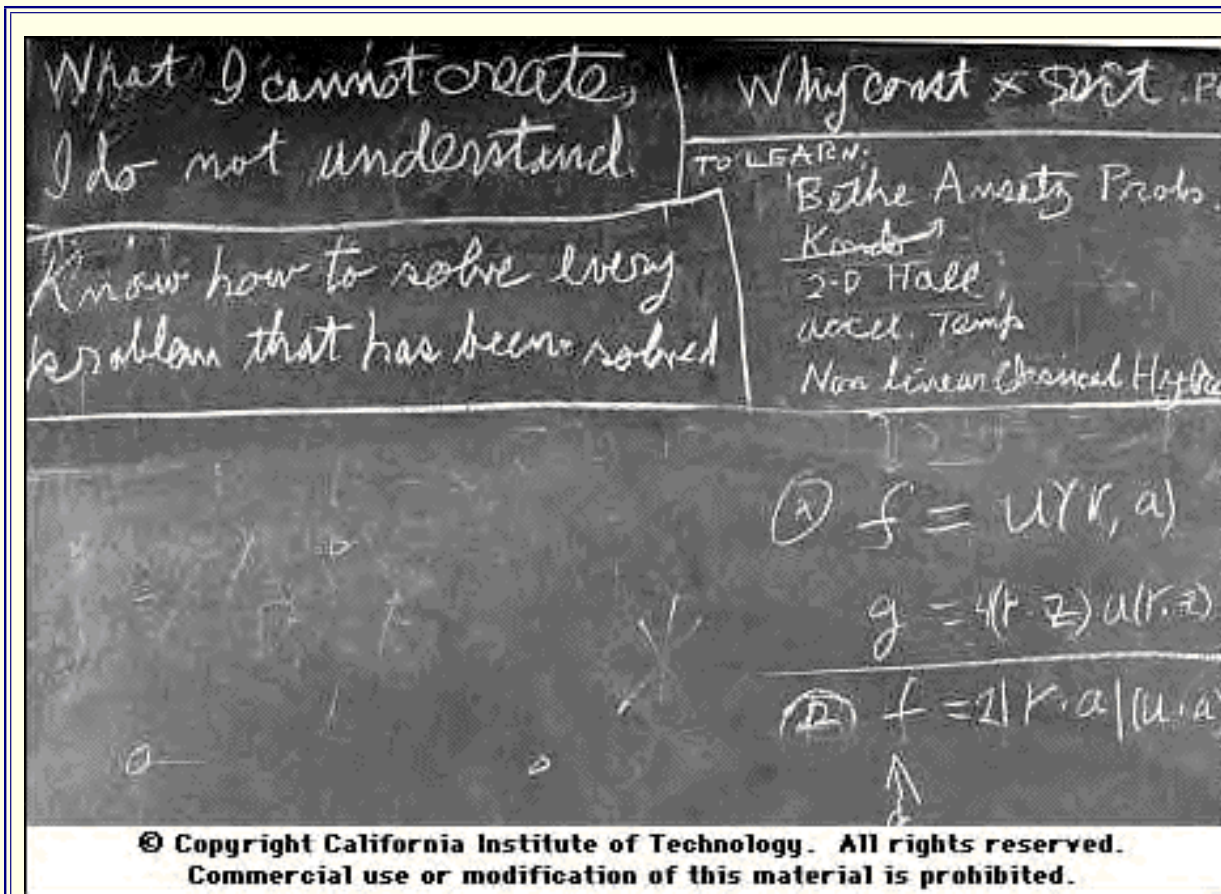
*MTA-Lendület Holographic QFT Group*

*Wigner Research Centre for Physics, Budapest*



The Illusion of Gravity - Juan Maldacena, Scientific American (2005)

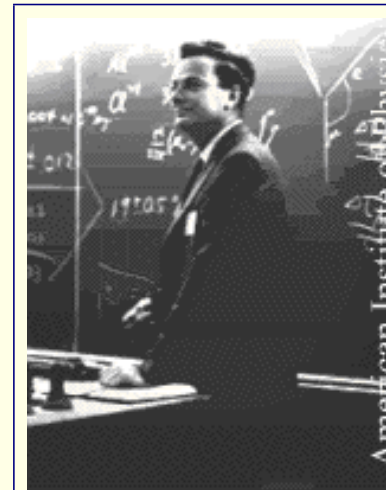
# Motivation: Feynman's legacy



What I cannot create I do not understand.

Know how to solve every problem that has been solved.

To learn: **Bethe Ansatz Probs.**, ← Kondo, 2D Hall, accel temp, Non linear classical Hydro

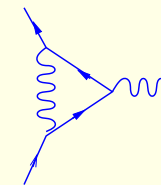


Richard P. Feynman  
(1918–1988)



1965

Quantumelectrodynamics:  
Feynman graphs



Strong interaction?

## Motivation: Organizing matter

# Motivation: Organizing matter

**Periodic Table of the Elements** © www.elementsdatabase.com

- hydrogen
- alkali metals
- alkali earth metals
- transition metals

- poor metals
- nonmetals
- noble gases
- rare earth metals

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac	104 Unq	105 Unp	106 Unh	107 Uns	108 Uno	109 Une	110 Unn								

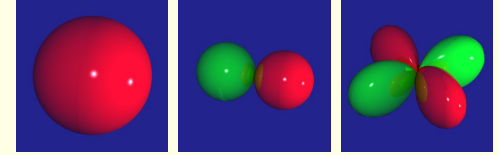
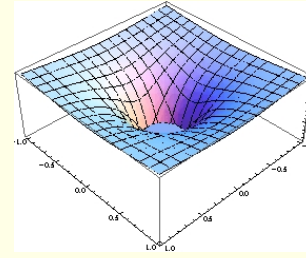
58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr



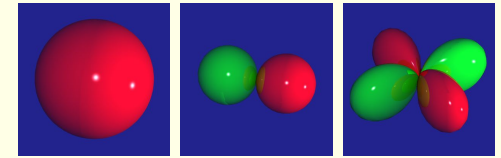
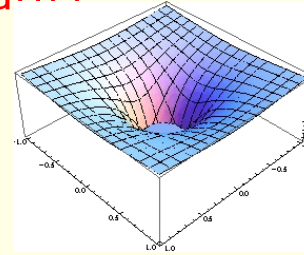
## Motivation: Organizing matter

Electric interaction (potential  $\Phi(r) = k\frac{Zq}{r}$ )

Quantum mechanics (Schrödinger eq.)  $H\Psi = \left(-\frac{(\hbar\nabla)^2}{2m} + \Phi(r)\right)\Psi = E\Psi$

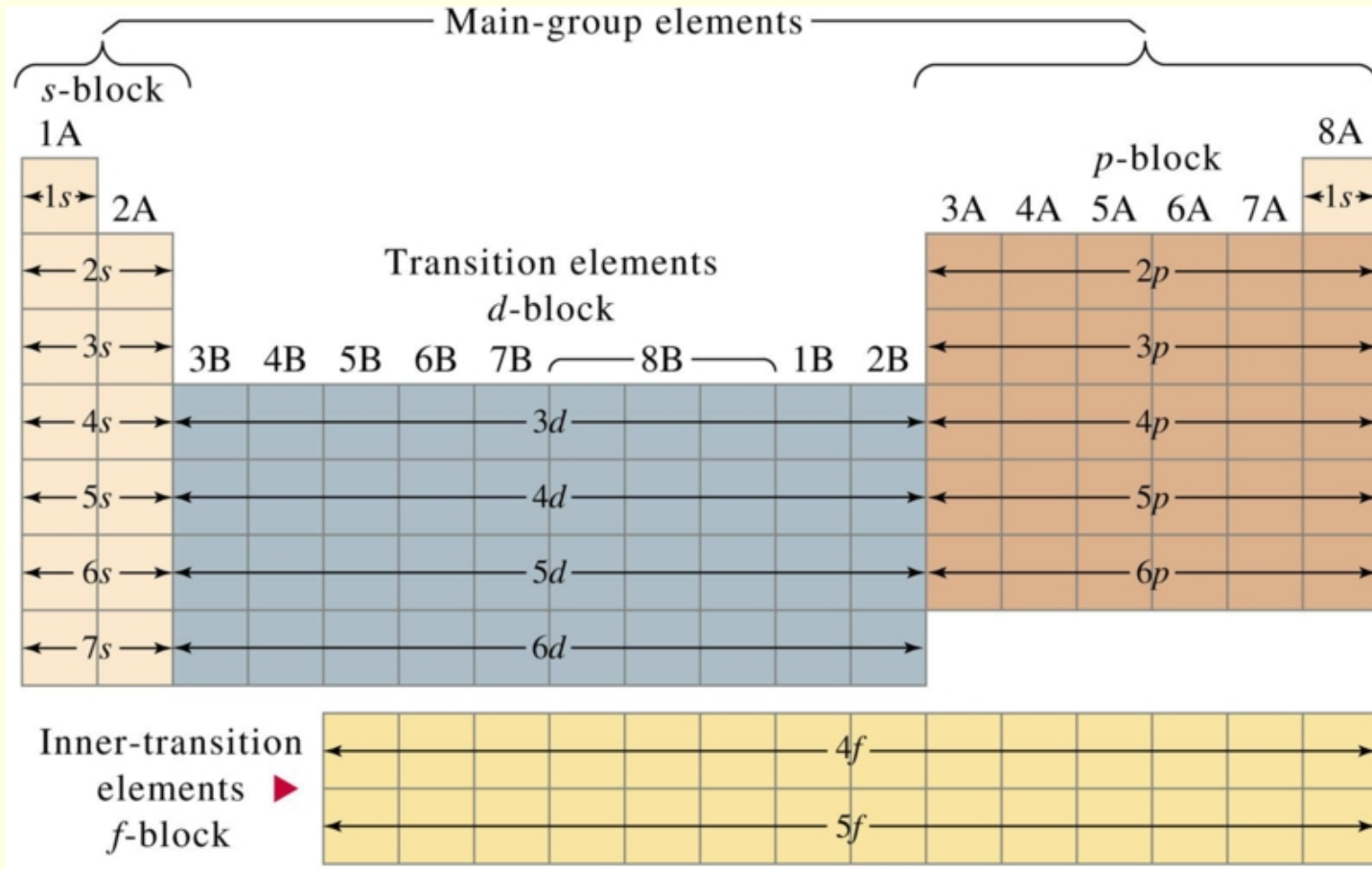


# Motivation: Organizing matter



Electric interaction (potential  $\Phi(r) = k\frac{Zq}{r}$ )

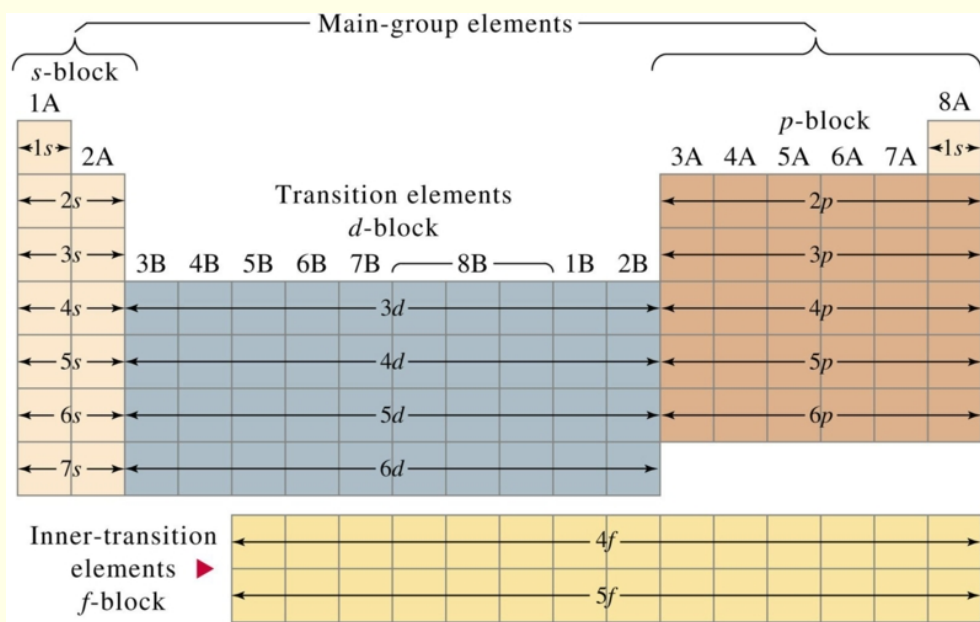
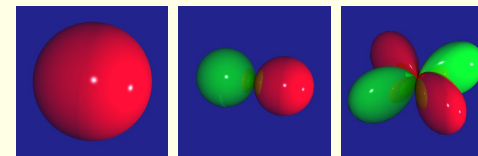
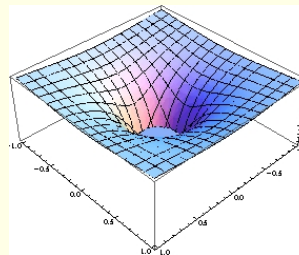
Quantum mechanics (Schrödinger eq.)  $H\Psi = \left(-\frac{(\hbar\nabla)^2}{2m} + \Phi(r)\right)\Psi = E\Psi$



# Motivation: Organizing matter

Electric interaction (potential  $\Phi(r) = k\frac{Zq}{r}$ )

Quantum mechanics (Schrödinger eq.)  $H\Psi = \left(-\frac{(\hbar\nabla)^2}{2m} + \Phi(r)\right)\Psi = E\Psi$



Periodic Table of the Elements

© www.elementsdatabase.com

Legend:

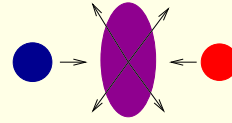
- hydrogen (green)
- alkali metals (yellow)
- alkali earth metals (light blue)
- transition metals (orange)
- poor metals (medium blue)
- nonmetals (white)
- noble gases (red)
- rare earth metals (grey)

1																	2
H																	He
3	4											5	6	7	8	9	10
Li	Be											B	C	N	O	F	Ne
11	12											13	14	15	16	17	18
Na	Mg											Al	Si	P	S	Cl	Ar
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
55	56	57	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
87	88	89	104	105	106	107	108	109	110								
Fr	Ra	Ac	Unq	Unp	Unh	Uns	Uno	Une	Uun								
		58	59	60	61	62	63	64	65	66	67	68	69	70	71		
		Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu		
		90	91	92	93	94	95	96	97	98	99	100	101	102	103		
		Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr		



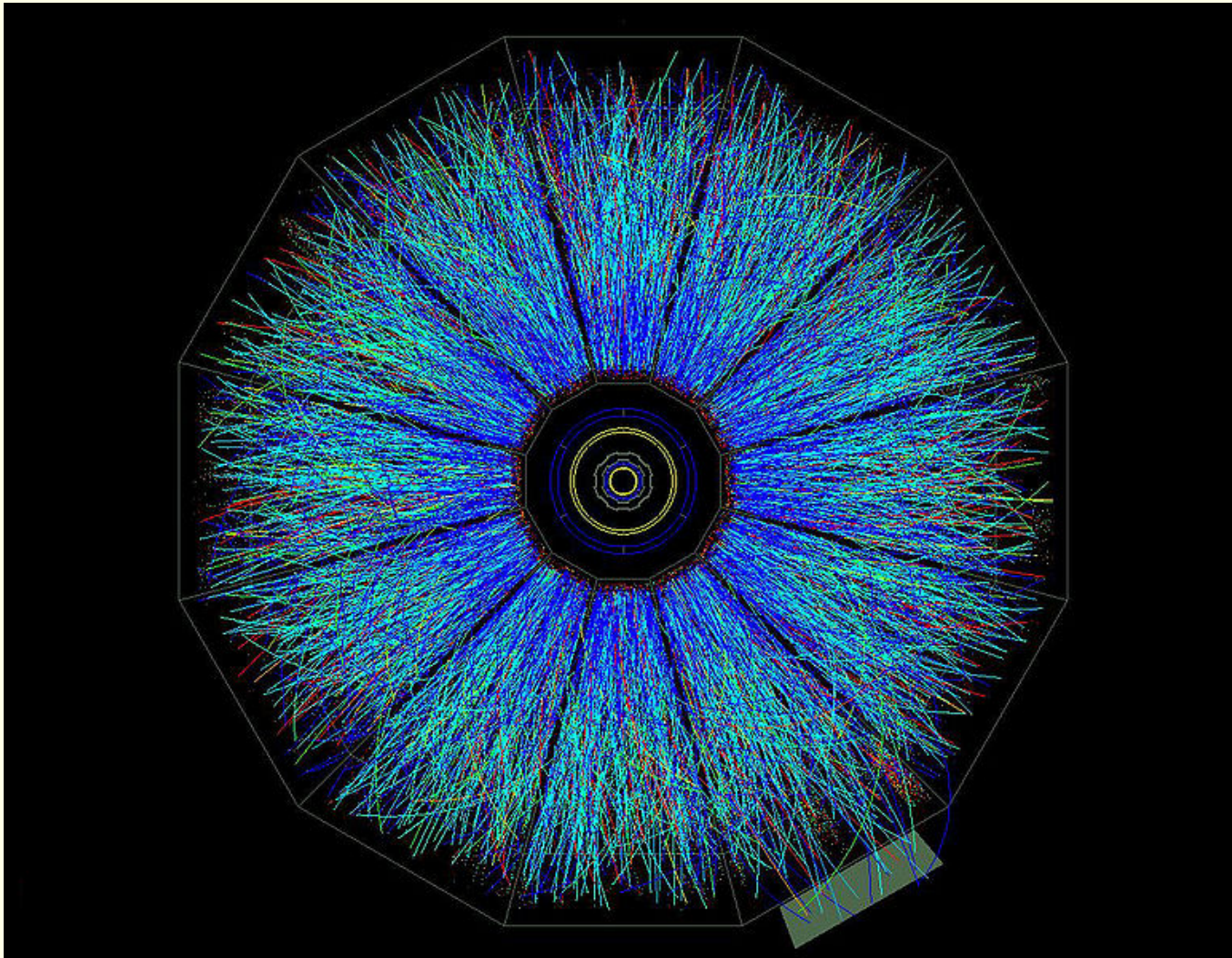
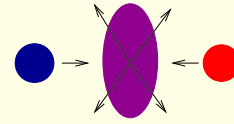
## Organizing matter II

Brookhaven: Relativistic heavy ion collider (gold ion)



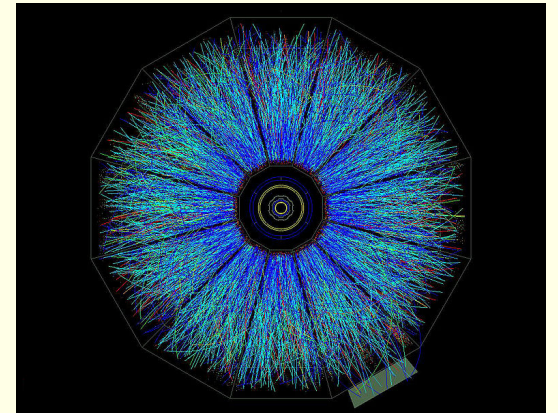
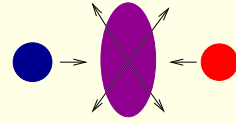
## Organizing matter II

Brookhaven: Relativistic heavy ion collider (gold ion)



## Organizing matter II

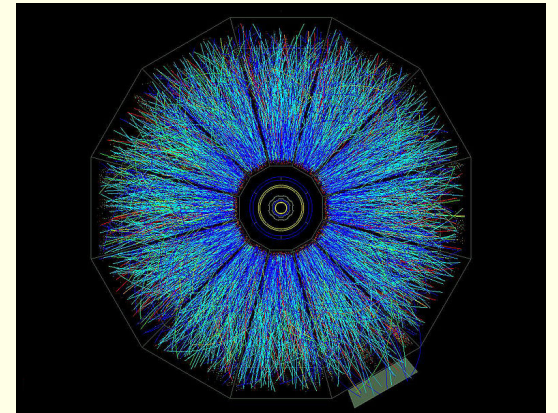
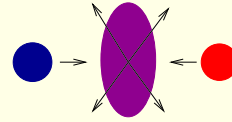
Brookhaven: Relativistic heavy ion collider (gold ion)



## Organizing matter II

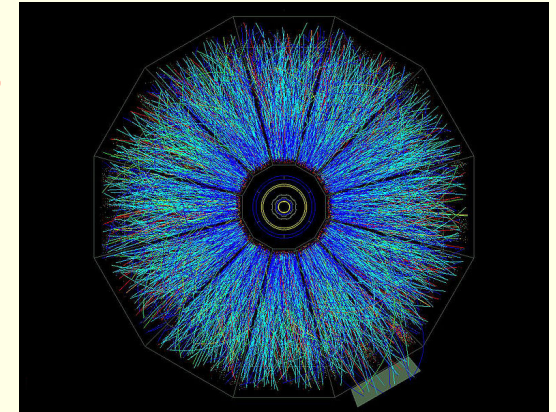
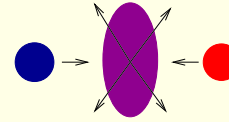
Brookhaven: Relativistic heavy ion collider (gold ion)

*Number of elementary particles*  $>$  number of atoms  $\rightarrow$  classification

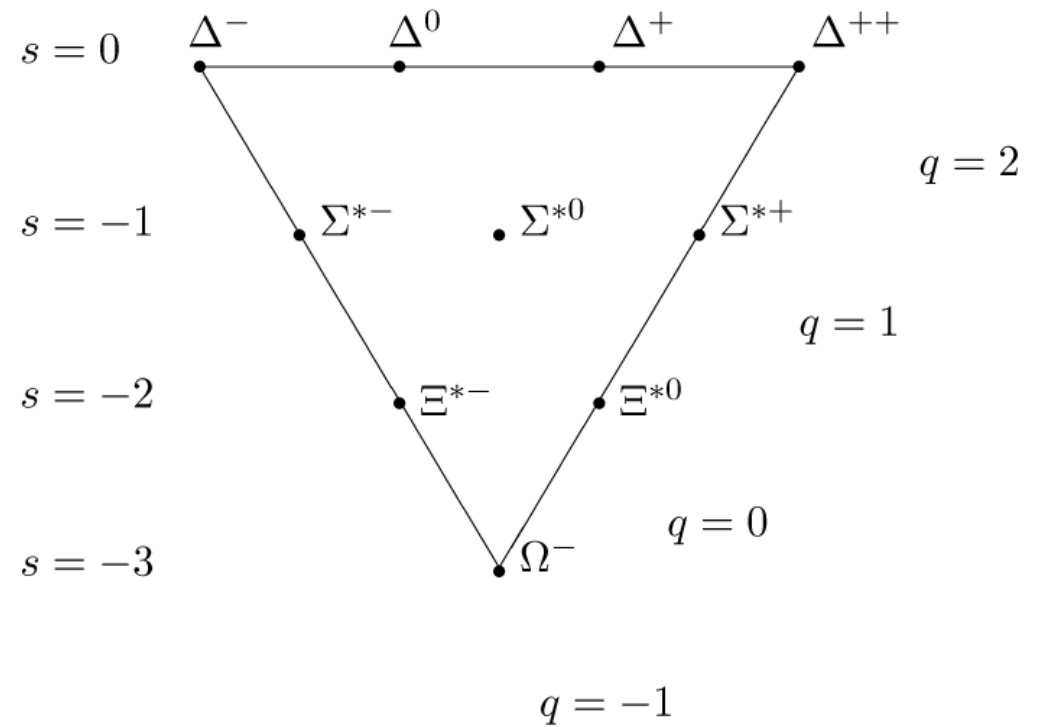
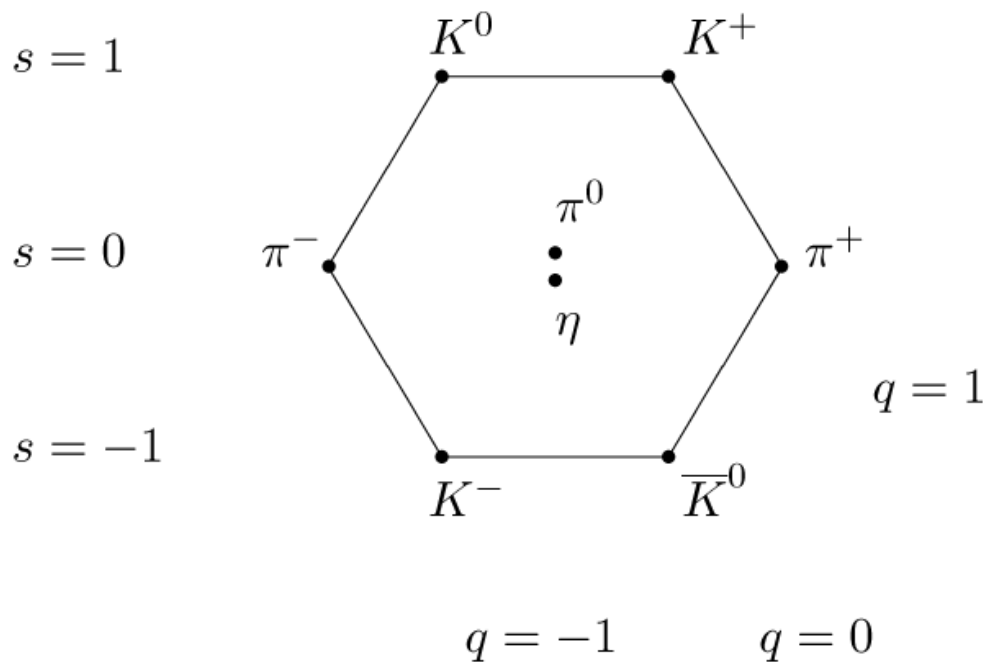


## Organizing matter II

Brookhaven: Relativistic heavy ion collider (gold ion)

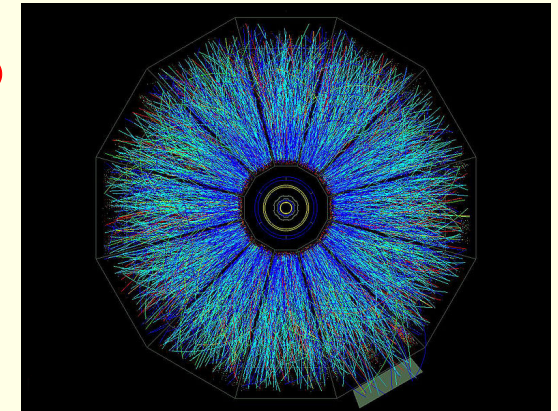
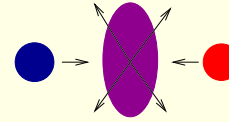


*Number of elementary particles* > number of atoms → classification



# Organizing matter II

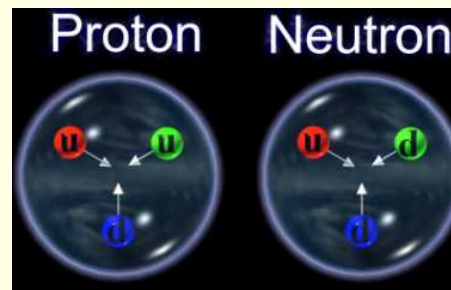
Brookhaven: Relativistic heavy ion collider (gold ion)



Number of elementary particles  $\gg$  number of atoms  $\rightarrow$  classification

Decay  $\rightarrow$  strong, weak interaction  $\rightarrow$  Standard Model

Interaction			
$\gamma$	photon	electromagnetic	<i>I.</i>
$W^{\pm}, Z$	weak bosons	weak	
$g$	gluon	strong	<i>II.</i>
$gr$	graviton	gravitational	



Leptons		Quarks		Bosons (Forces)	
name	charge	name	charge	name	charge
electron	$-\frac{1}{2}$	down	$-\frac{1}{3}$	photon	0
electron neutrino	0	up	$\frac{2}{3}$	gluon	0
muon	$-\frac{1}{2}$	charm	$\frac{2}{3}$	weak force	$\pm 1$
muon neutrino	0	strange	$-\frac{1}{3}$	weak force	0
tau	$-\frac{1}{2}$	bottom	$-\frac{1}{3}$		
tau neutrino	0	top	$\frac{2}{3}$		

Three Generations of Matter (Fermions)

mass  $\rightarrow$  2.4 MeV, 1.27 GeV, 171.2 GeV  
 charge  $\rightarrow$   $\frac{2}{3}$ ,  $\frac{2}{3}$ ,  $\frac{2}{3}$   
 spin  $\rightarrow$   $\frac{1}{2}$ ,  $\frac{1}{2}$ ,  $\frac{1}{2}$

Bosons (Forces)



# Quantum electrodynamics

Relativity theory:  $A_\mu = (\Phi, \underline{A})$

electric + magnetic int:  $F_{\mu\nu}$

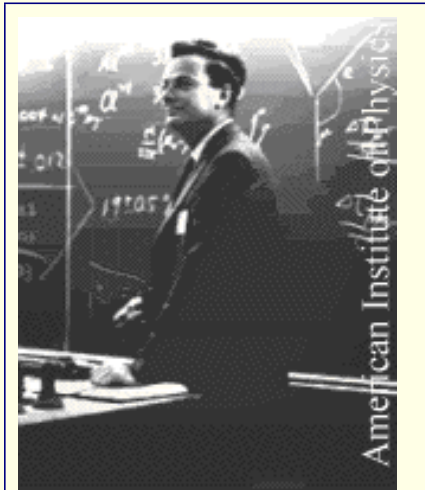
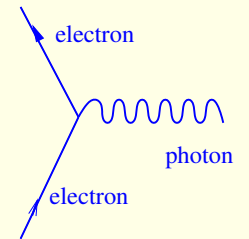
+ Quantum theory  $\rightarrow$  QED

$U(1)$  gauge theory:  $A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \Lambda(x)$

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}(i\partial - m)\Psi - e\bar{\Psi}\not{A}\Psi$$

experiment:  $\underline{\mu} = g \frac{e\hbar}{2mc} \underline{s}$  where  $g = 2(1 + a)$

Gabrielse et.al.:  $a = 1159652180.85(.76) \times 10^{-12}$



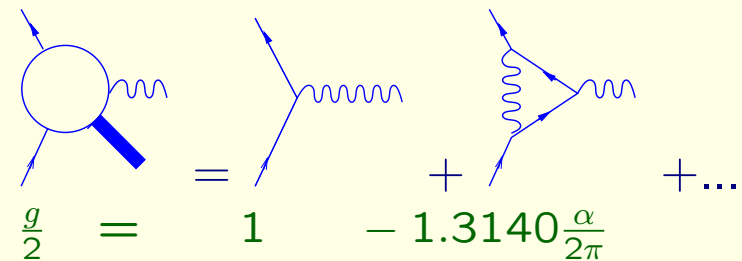
Feynman: *If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in.*

**Quantum gauge theory**

perturbation theory:

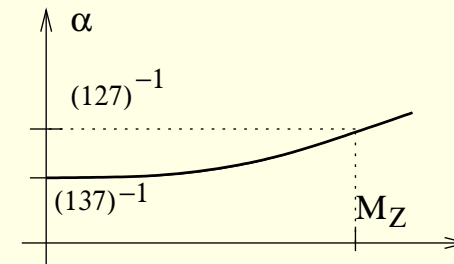
Feynman graphs

$$\frac{\alpha}{2\pi} = \frac{e^2}{2\pi\hbar c} = 0.001161$$



momentum-dependent coupling:

$$\beta(\alpha) = \mu \frac{\partial \alpha}{\partial \mu} > 0$$



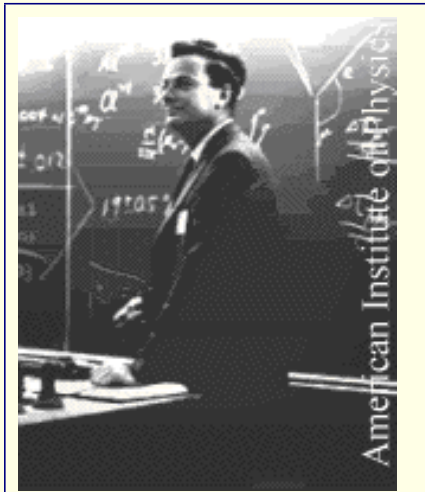
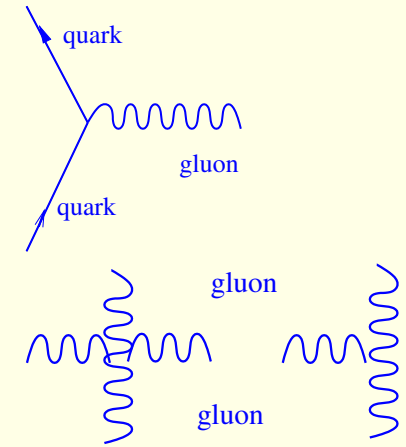
# Quantum Chromodynamics

photon  $A_\mu \leftrightarrow G_\mu^{1..8}$  gluon  $\rightarrow F_{\mu\nu}^{1..8}$

electron  $\Psi_e \leftrightarrow \Psi_{quark}$  quark

$SU(3)$  gauge theory:  $G_\mu \rightarrow g^{-1}G_\mu g + g^{-1}\partial_\mu g$

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}(i\partial - m)\Psi - g\bar{\Psi}G\Psi$$



Quantum gauge theory

asymptotic freedom

2004 Nobel Prize in Physics



David J. Gross H. David Politzer Frank Wilczek

experiments:

hadron spectrum

perturbation theory:

Feynman graphs

$$0.001 = \frac{\alpha}{2\pi} \leftrightarrow \frac{\alpha_s}{4\pi} = O(1)$$

momentum-dependent coupling:

$$\beta(\alpha_s) = \mu \frac{\partial \alpha_s}{\partial \mu} < 0$$

asymptotic freedom

confinement

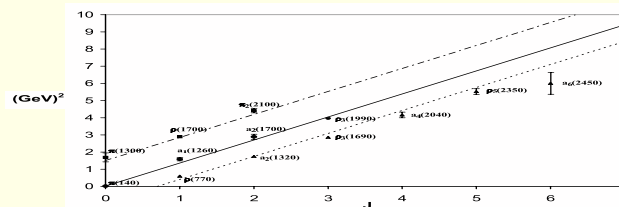
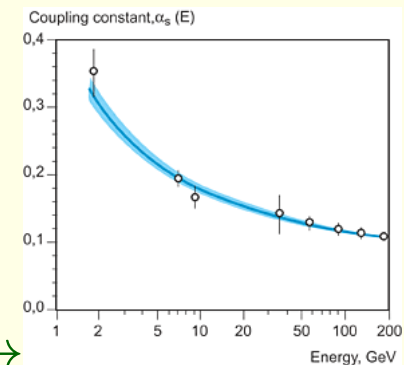
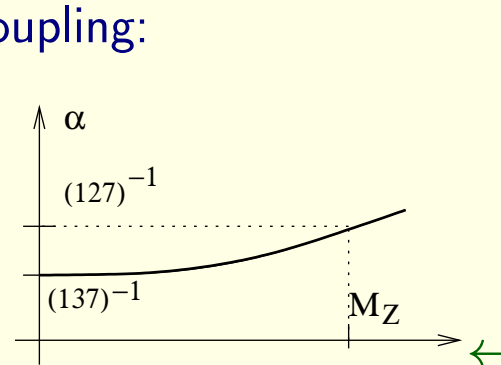
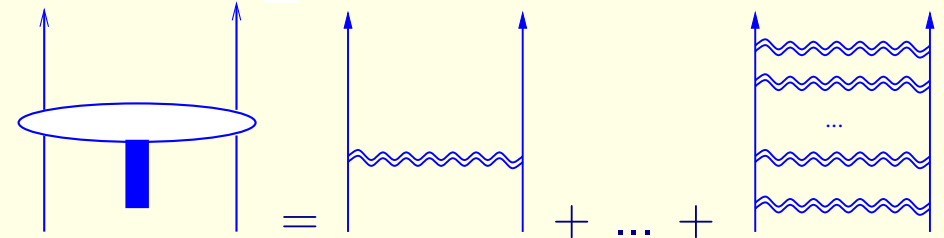


Fig. 1.





# CFT: maximally supersymmetric gauge theory

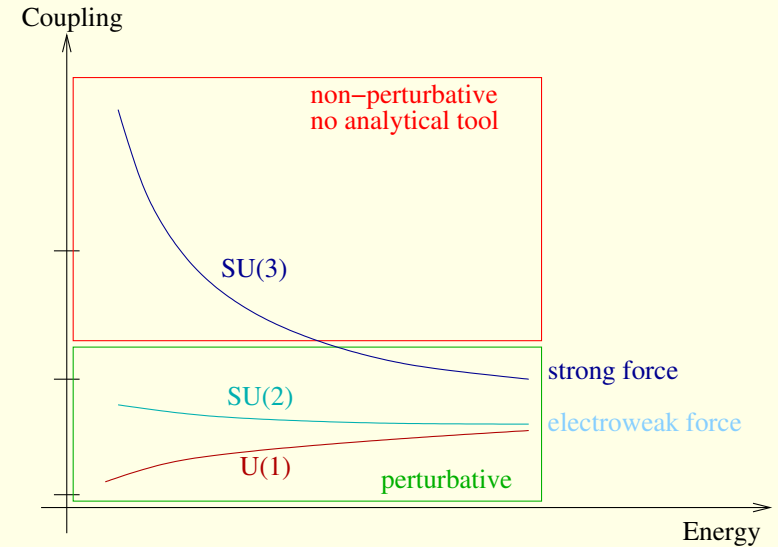
## Fundamental interactions

interaction	particles	gauge theory
electromagnetic	photon+electron	$U(1)$
electroweak	$W^\pm, Z, \mu, \nu$ +Higgs	$SU(2) \times U(1)$
strong	gluon+quarks	$SU(3)$

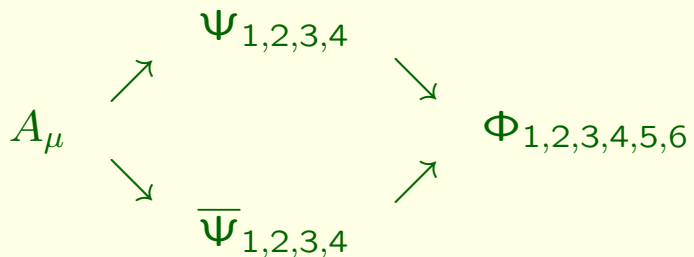
only analytical tool: perturbation theory

## maximally supersymmetric gauge theory

interaction	particles	gauge theory
$\mathcal{N} = 4$ supersymm.	gluon+quarks+scalars	$SU(N)$



all fields  $N^2 - 1$  component matrix



$$\mathcal{L} = \frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[ -\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \bar{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

no running  $\beta = 0 \rightarrow$  CFT

no confinement

supersymmetric

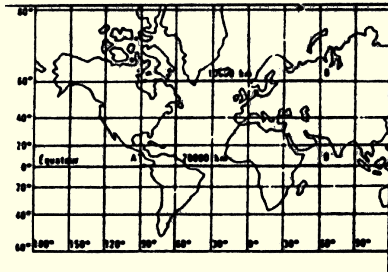
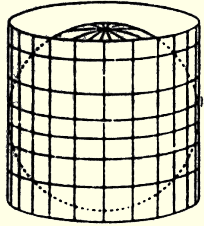
heavy ion collision:

finite T  $\rightarrow$  SUSY is broken

quark-gluon plasma is not confined

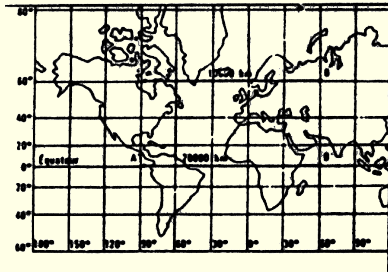
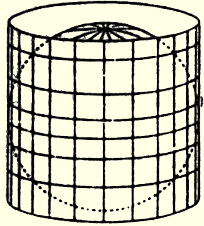
# AdS: string theory on Anti de Sitter $\supset$ gravitation

positively curved space

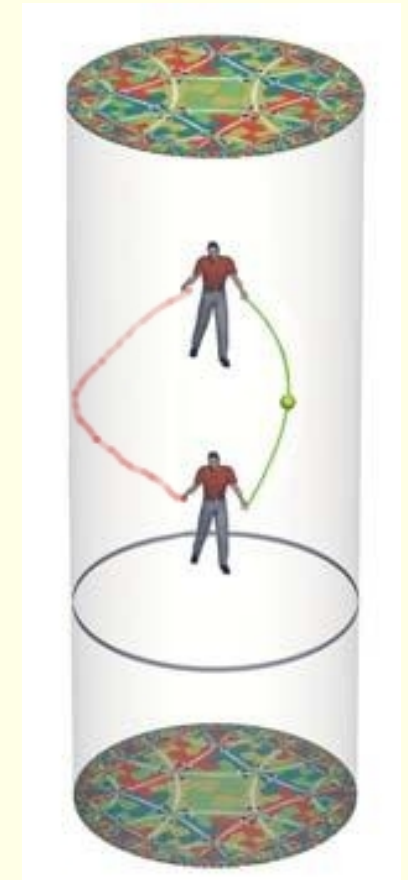


# AdS: string theory on Anti de Sitter $\supset$ gravitation

positively curved space

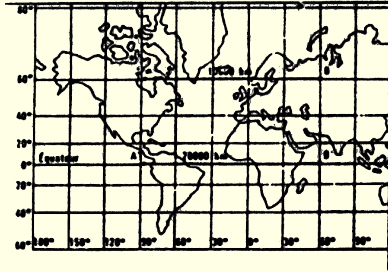
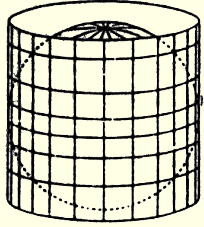


Anti de Sitter: negatively curved space

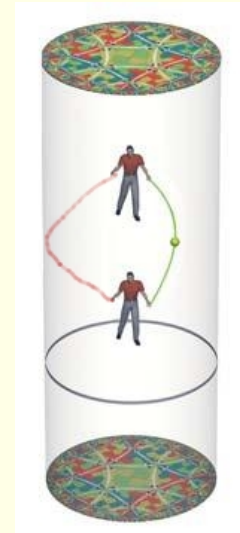


AdS: string theory on Anti de Sitter  $\supset$  gravitation

positively curved space

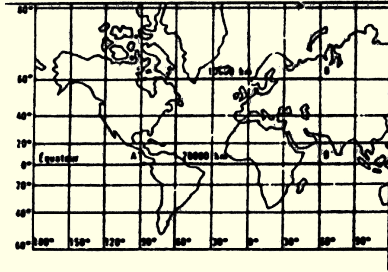
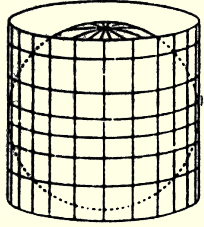


Anti de Sitter: negatively curved space



AdS: string theory on Anti de Sitter  $\supset$  gravitation

positively curved space

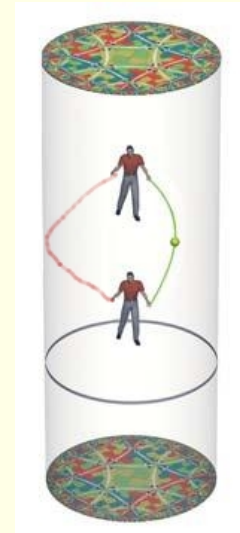
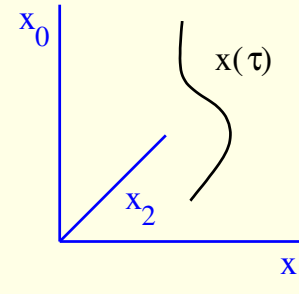


Anti de Sitter: negatively curved space



relativistic point particle:  $ds^2 = -dx_0^2 + dx_1^2 + \dots$

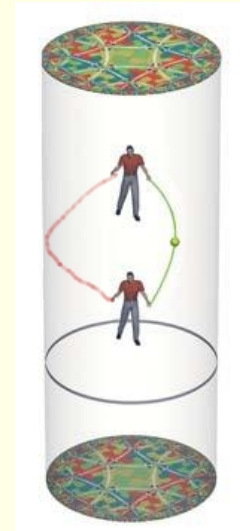
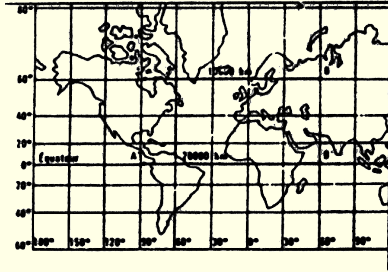
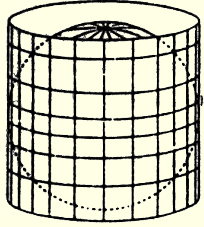
$S \propto \text{worldline} \propto \int ds = \int \sqrt{\dot{x} \cdot \dot{x}} d\tau$



# AdS: string theory on Anti de Sitter $\supset$ gravitation

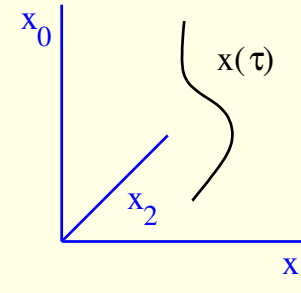
positively curved space

Anti de Sitter: negatively curved space



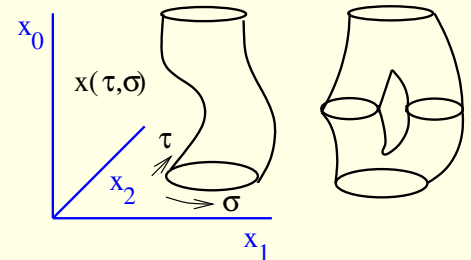
relativistic point particle:  $ds^2 = -dx_0^2 + dx_1^2 + \dots$

$S \propto \text{worldline} \propto \int ds = \int \sqrt{\dot{x} \cdot \dot{x}} d\tau$



relativistic string:  $ds^2 = -dx_0^2 + dx_1^2 + \dots$

$S \propto \text{worldsheet} \propto \int dA = \int \sqrt{(\dot{x} \cdot x')^2 - \dot{x}^2 x'^2} d\tau d\sigma$

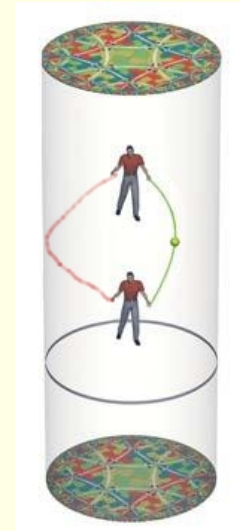
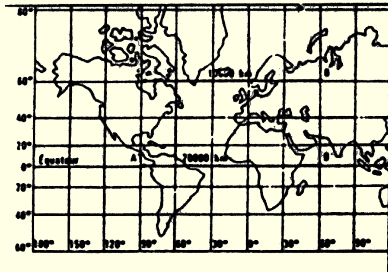
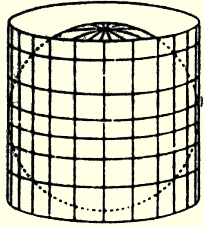




AdS: string theory on Anti de Sitter  $\supset$  gravitation

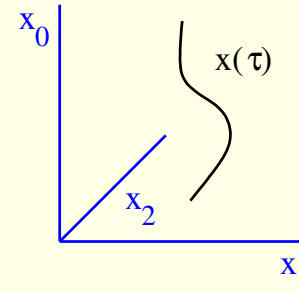
positively curved space

Anti de Sitter: negatively curved space



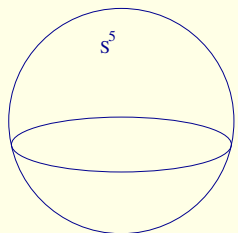
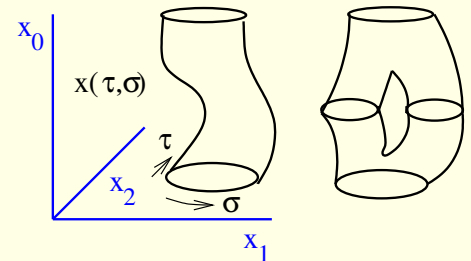
relativistic point particle:  $ds^2 = -dx_0^2 + dx_1^2 + \dots$

$S \propto \text{worldline} \propto \int ds = \int \sqrt{\dot{x} \cdot \dot{x}} d\tau$



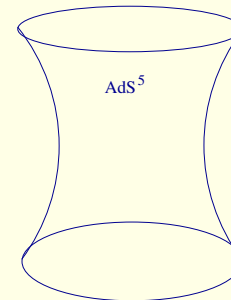
relativistic string:  $ds^2 = -dx_0^2 + dx_1^2 + \dots$

$S \propto \text{worldsheet} \propto \int dA = \int \sqrt{(\dot{x} \cdot x')^2 - \dot{x}^2 x'^2} d\tau d\sigma$



$$S^5 : Y_0^2 + Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 + Y_5^2 = R^2$$

$$AdS_5 : -X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 - X_5^2 = -R^2$$



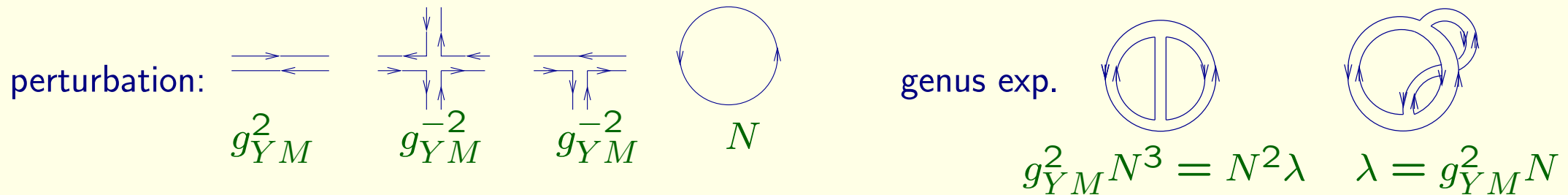
$$S = \frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left( \partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M \right) + \text{fermionok} \quad \text{supercoset } \frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$$

# CFT: Observables

maximally supersymmetric gauge theory	
$A$	$\Psi_{1,2,3,4}$ $\Phi_{1,2,3,4,5,6}$ fields $SU(N)$ matrices
$\bar{\Psi}_{1,2,3,4}$	
$\mathcal{S} = \frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[ -\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\bar{\Psi}\not{D}\Psi + V \right]$	
$V(\Phi, \Psi) = \frac{1}{4}[\Phi, \Phi]^2 + \bar{\Psi}[\Phi, \Psi]$	

observables
parameters: $g_{YM}, N$
observables: partition function
gauge-invariant operators
$\mathcal{O}(x) = \text{Tr}(A^{L_1}\Psi^{L_2}\Phi^{L_3}..)$
correlators: $\langle \mathcal{O}_1(x)\mathcal{O}_2(0) \rangle$

correlators:  $\langle \mathcal{O}_1(x)\mathcal{O}_2(0) \rangle = \int [dA...] e^{-i\mathcal{S}} \mathcal{O}_1(x)\mathcal{O}_2(0) = \langle \mathcal{O}_1(x)\mathcal{O}_2(0)e^{-iV} \rangle_0$



partition func.  $Z(\lambda, \frac{1}{N}) = N^2 \sum_g (\frac{1}{N})^{2g} \sum_n \alpha(g, n) \lambda^n$  string interactions? (t' Hooft)

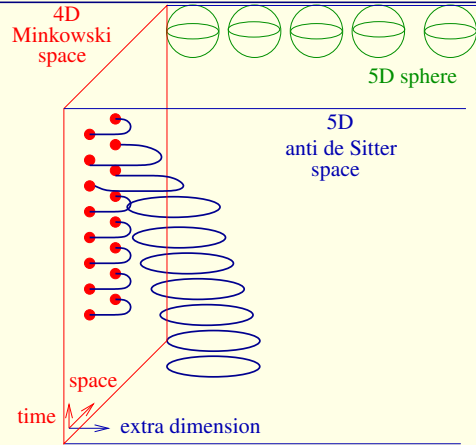
conformal field theory:  $\langle \mathcal{O}_i(x)\mathcal{O}_j(0) \rangle = \frac{\delta_{ij}}{|x|^{2\Delta_i}}$  scale dim.:  $\Delta_i$  Konishi op.  $\mathcal{O}_K = \text{Tr}(\Phi_i^2)$

$$\Delta_K(\lambda) = 2 + 6\frac{\lambda}{4\pi^2} - 24\frac{\lambda^2}{(4\pi^2)^2} + 168\frac{\lambda^3}{(4\pi^2)^3} - (1410 + 144\zeta(3) + \frac{1}{2}(324 + 864\zeta(3) - 1440\zeta(5)))\frac{\lambda^4}{(4\pi^2)^4}$$



# AdS/CFT correspondence (Maldacena 1998)

## II<sub>B</sub> superstring on AdS<sub>5</sub> × S<sup>5</sup>



$$\sum_1^6 Y_i^2 = R^2 \quad - + + + + - = -R^2$$

$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} (\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M) + \dots$$

≡

## N = 4 D=4 SU(N) SYM

$$\frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[ -\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i\bar{\Psi} \not{D}\Psi + V \right]$$

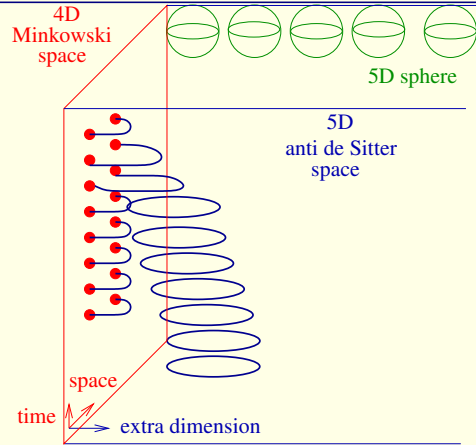
$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

$$\beta = 0 \text{ superconformal } \frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$$

$$\text{gaugeinvariants: } \mathcal{O} = \text{Tr}(\Phi^2), \det(\ )$$

# AdS/CFT correspondence (Maldacena 1998)

$II_B$  superstring on  $AdS_5 \times S^5$



$$\sum_1^6 Y_i^2 = R^2 \quad - + + + + - = -R^2$$

$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} (\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M) + \dots$$

$\equiv$

$\mathcal{N} = 4$  D=4  $SU(N)$  SYM

$$\frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[ -\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i\bar{\Psi} \not{D}\Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

$\beta = 0$  superconformal  $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

gaugeinvariants:  $\mathcal{O} = \text{Tr}(\Phi^2), \det(\dots)$

Couplings:  $\sqrt{\lambda} = \frac{R^2}{\alpha'}$ ,  $g_s = \frac{\lambda}{N} \rightarrow 0$

2D QFT

String energy levels:  $E(\lambda)$

$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$$

Dictionary

strong  $\leftrightarrow$  weak

$\Downarrow$

$\lambda = g_{YM}^2 N$ ,  $N \rightarrow \infty$  planar limit

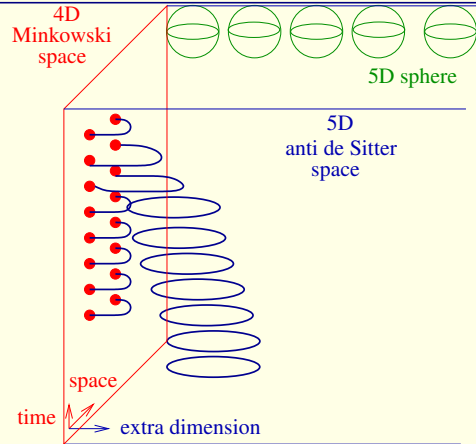
$$\langle \mathcal{O}_n(x) \mathcal{O}_m(0) \rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$$

Anomalous dim  $\Delta(\lambda)$

$$\Delta(\lambda) = \Delta(0) + \lambda \Delta_1 + \lambda^2 \Delta_2 + \dots$$

# AdS/CFT correspondence (Maldacena 1998)

$II_B$  superstring on  $AdS_5 \times S^5$



$$\sum_1^6 Y_i^2 = R^2 \quad - + + + + - = -R^2$$

$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} (\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M) + \dots$$

$\mathcal{N} = 4$  D=4  $SU(N)$  SYM

$$\frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[ -\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i\bar{\Psi} \not{D}\Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

$\beta = 0$  superconformal  $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

gaugeinvariants:  $\mathcal{O} = \text{Tr}(\Phi^2), \det(\ )$

Dictionary

Couplings:  $\sqrt{\lambda} = \frac{R^2}{\alpha' l}, g_s = \frac{\lambda}{N} \rightarrow 0$

2D QFT

String energy levels:  $E(\lambda)$

$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$$

strong  $\leftrightarrow$  weak



$\lambda = g_{YM}^2 N, N \rightarrow \infty$  planar limit

$$\langle \mathcal{O}_n(x) \mathcal{O}_m(0) \rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$$

Anomalous dim  $\Delta(\lambda)$

$$\Delta(\lambda) = \Delta(0) + \lambda \Delta_1 + \lambda^2 \Delta_2 + \dots$$

2D integrable QFT

spectrum:  $Q = 1, 2, \dots, \infty$  dispersion:  $\epsilon_Q(p) = \sqrt{Q^2 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}}$

Exact scattering matrix:  $S_{Q_1 Q_2}(p_1, p_2, \lambda)$

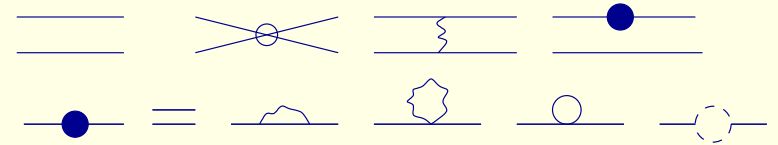
# CFT: Integrability

Perturbative correlator:  $\langle \mathcal{O}_1(x) \mathcal{O}_2(0) \rangle = \langle \mathcal{O}_1(x) \mathcal{O}_2(0) e^{-i(\frac{1}{4}[\Phi, \Phi]^2 + \bar{\Psi}[\Phi, \Psi])} \rangle_0$

Conformal (scale invariant) field theory:  $= \frac{\delta_{ij}}{|x|^{2\Delta(\lambda)}} = \frac{1}{|x|^{2\Delta(0)}} \left[ 1 + \lambda \Delta_1 \log \frac{1}{|x|^2} + \dots \right]$

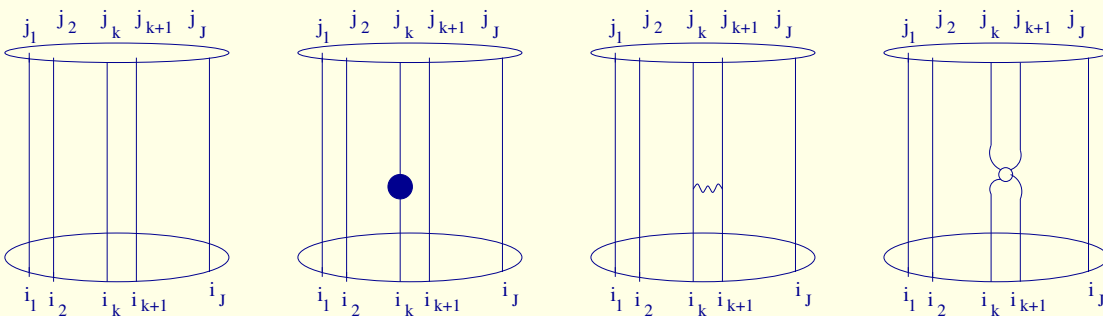
Scalar sector:  $Z_1 = \Phi_1 + i\Phi_2, Z_2 = \Phi_3 + i\Phi_4$  SUSY st:  $\mathcal{O} = \text{Tr} [Z_i^J] \rightarrow \Delta_{\mathcal{O}}(\lambda) = J$

Operator mixing:  $\mathcal{O}_1 = \text{Tr} [Z_1 Z_1 Z_2 Z_2] \leftrightarrow |\uparrow\uparrow\downarrow\downarrow\rangle$   
 $\mathcal{O}_2 = \text{Tr} [Z_1 Z_2 Z_1 Z_2] \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle$



diagonalize the 1-loop mixing matrix:  $\mathcal{O}_{\pm} = \mathcal{O}_1 \pm \mathcal{O}_2 \rightarrow \Delta_{\mathcal{O}_+}(\lambda) = 4$   
 $\Delta_{\mathcal{O}_-}(\lambda) = 4 + 6 \frac{\lambda}{4\pi^2}$

generic state at size  $J$ :  $\mathcal{O}_{i_1 \dots i_J} = \text{Tr} [Z_{i_1} \dots Z_{i_J}] \leftrightarrow |i_1 \dots i_J\rangle$



$$\Delta = J\mathbb{I} + \frac{\lambda}{8\pi^2} \sum_{k=1}^J (\mathbb{I} - \mathbb{P}_{k,k+1})$$

Heisenberg spin chain

## CFT: Integrability + Bethe Ansatz

Mixing matrix on the subspace  $\text{Tr} [Z_{i_1} \dots Z_{i_J}]$  of dim  $2^J$ : Minahan-Zarembo 2002

$$\Delta = H_0 + \lambda H_1 + \lambda^2 H_2 + \dots = J \mathbb{I} + \frac{\lambda}{8\pi^2} H_{XXX} + \lambda^2 H_2 + \dots$$

$H_2$ : next-to-nearest neighbour integrable!  $\rightarrow$  use Bethe ansatz

1. choose a groundstate:  $Z = Z_1 \rightarrow \text{Tr} [Z^J] = \text{Tr} [ZZZZZ \dots ZZZZ] \leftrightarrow |\uparrow \dots \uparrow\rangle$

2. excitations  $Z \dots Z X Z \dots X$  with SUSY multiplet  $X = Z_2, Z_3, \Psi_a^\alpha, \Psi_a^{\dot{\alpha}}, D_\mu$

3. plane wave:  $\sum_n e^{ipn} \text{Tr} (\overbrace{Z \dots Z}^n X Z \dots ZZ)$

4. scattering states:  $\sum_{n_1 n_2 a_1 a_2} e^{ip_1 n_1 + ip_2 n_2} \text{Tr} (\underbrace{Z \dots Z}_{n_1} \overbrace{X_{a_1} Z \dots Z}^{n_2} X_{a_2} Z \dots Z) + S(12)_{a_1 a_2}^{b_1 b_2} \Sigma$

symmetry completely fixes the S-matrix for any  $\lambda$  (satisfies unitarity, crossing, Yang-Baxter)

Bethe ansatz follows from S-matrix: Shastry's Hubbard S-matrix

## AdS/CFT correspondence: confirmation

supersymmetric **BPS** operators

$$V(\Phi, \Psi) = \frac{1}{4}[\Phi, \Phi]^2 + \bar{\Psi}[\Phi, \Psi]$$

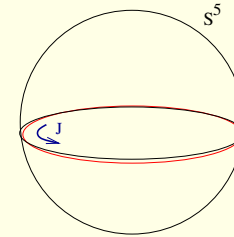
$$Z = \Phi_1 + i\Phi_2, X = \Phi_3 + i\Phi_4$$

$$\mathcal{O}_{BPS} = \text{Tr}(Z^J) \leftrightarrow |\uparrow\uparrow \dots \uparrow\rangle$$

$$\Delta_{BPS} = J$$

weak  $\leftrightarrow$  strong

**BPS** string configuration



$$E_{BPS}(\lambda) = J$$

## AdS/CFT correspondence: confirmation

supersymmetric **BPS** operators

$$V(\Phi, \Psi) = \frac{1}{4}[\Phi, \Phi]^2 + \bar{\Psi}[\Phi, \Psi]$$

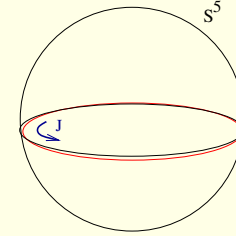
$$Z = \Phi_1 + i\Phi_2, X = \Phi_3 + i\Phi_4$$

$$\mathcal{O}_{BPS} = \text{Tr}(Z^J) \leftrightarrow |\uparrow\uparrow \dots \uparrow\rangle$$

$$\Delta_{BPS} = J$$

weak  $\leftrightarrow$  strong

**BPS** string configuration



$$E_{BPS}(\lambda) = J$$

2D integrable QFT

$$\text{supersymmetric groundstate } E_0(J) = \Delta(\lambda) - J = 0$$

## AdS/CFT correspondence: confirmation

supersymmetric **BPS** operators

$$V(\Phi, \Psi) = \frac{1}{4}[\Phi, \Phi]^2 + \bar{\Psi}[\Phi, \Psi]$$

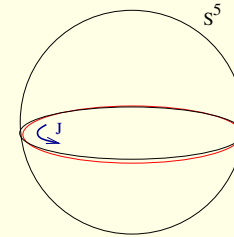
$$Z = \Phi_1 + i\Phi_2, X = \Phi_3 + i\Phi_4$$

$$\mathcal{O}_{BPS} = \text{Tr}(Z^J) \leftrightarrow |\uparrow\uparrow \dots \uparrow\rangle$$

$$\Delta_{BPS} = J$$

weak  $\leftrightarrow$  strong

**BPS** string configuration



$$E_{BPS}(\lambda) = J$$

2D integrable QFT

$$\text{supersymmetric groundstate } E_0(J) = \Delta(\lambda) - J = 0$$

Nontrivial anomalous dimension

supersymmetric theory: Excited state

$$\mathcal{O}_K = \text{Tr}(ZXZX + \dots) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + \dots$$

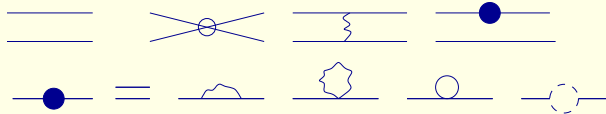


## Confirmation: excited state - Konishi operator

nonsupersymmetric operator: Konishi

$$\mathcal{O}_K = \text{Tr}(ZXZX + \dots) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + \dots$$

operator mixing



$$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \dots + \lambda^4\Delta_4 + \dots$$

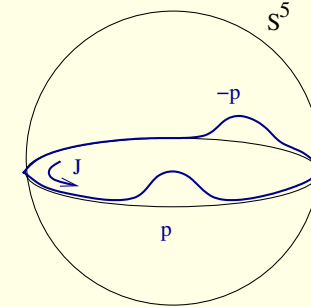


[Fiamberti ..'08]

$$\Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$$

≡

string configuration



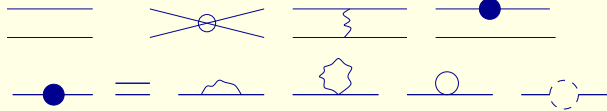
moving bumps (sine-Gordon) [Hofman .. '07]

string action = saddle point + loop corr.


$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$$

## Confirmation: excited state - Konishi operator

nonsupersymmetric operator: Konishi  
 $\mathcal{O}_K = \text{Tr}(ZXZX + \dots) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + \dots$   
 operator mixing



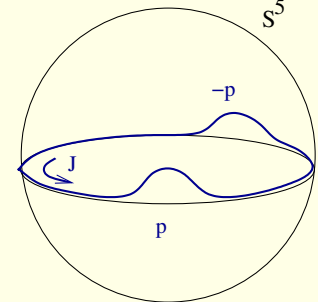
$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \dots + \lambda^4\Delta_4 + \dots$



[Fiamberti ..'08]  
 $\Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$

$\equiv$

string configuration



moving bumps (sine-Gordon) [Hofman .. '07]  
 string action = saddle point + loop corr.  
 $E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$

## two particle state

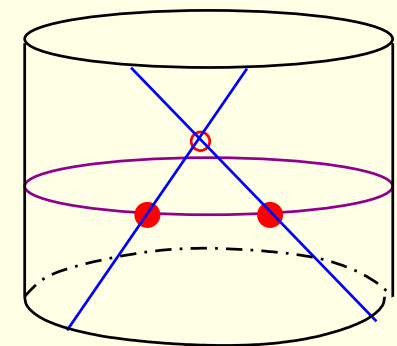
$$E = E_{BA} + E_{FSC}$$

Bethe Ansatz:  $e^{ipJ} S(p, -p) = 1$

$$E_{BA} = 2E(p, \lambda) = 2\sqrt{1 + \frac{\lambda}{\pi^2} (\sin \frac{p}{2})^2}$$

$$E_{FSC} = \sum_Q \int \frac{dq}{2\pi} S_{Q1}(q, p) S_{Q1}(q, -p) e^{-\epsilon_Q L}$$

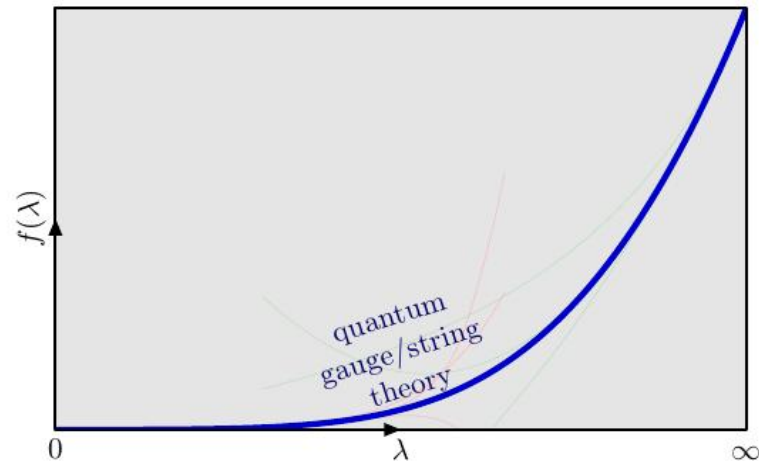
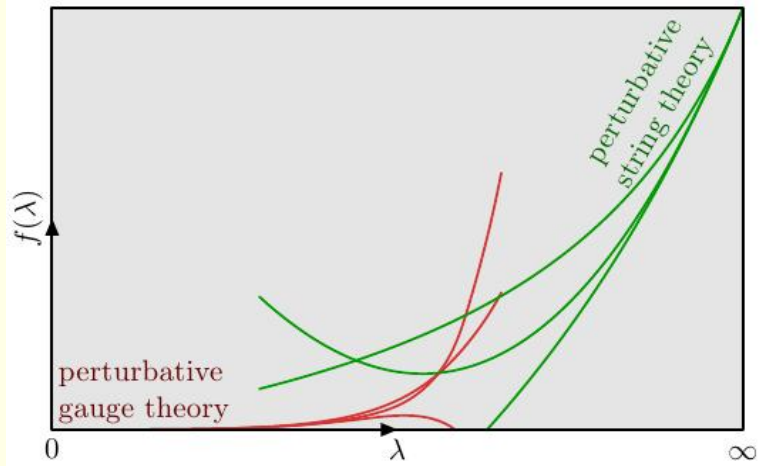
$E_4 = \Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$  [Z.B., R. Janik '09]



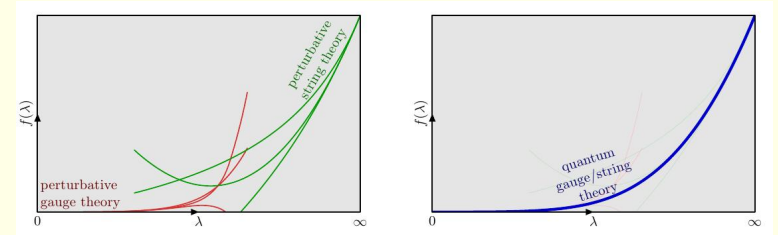
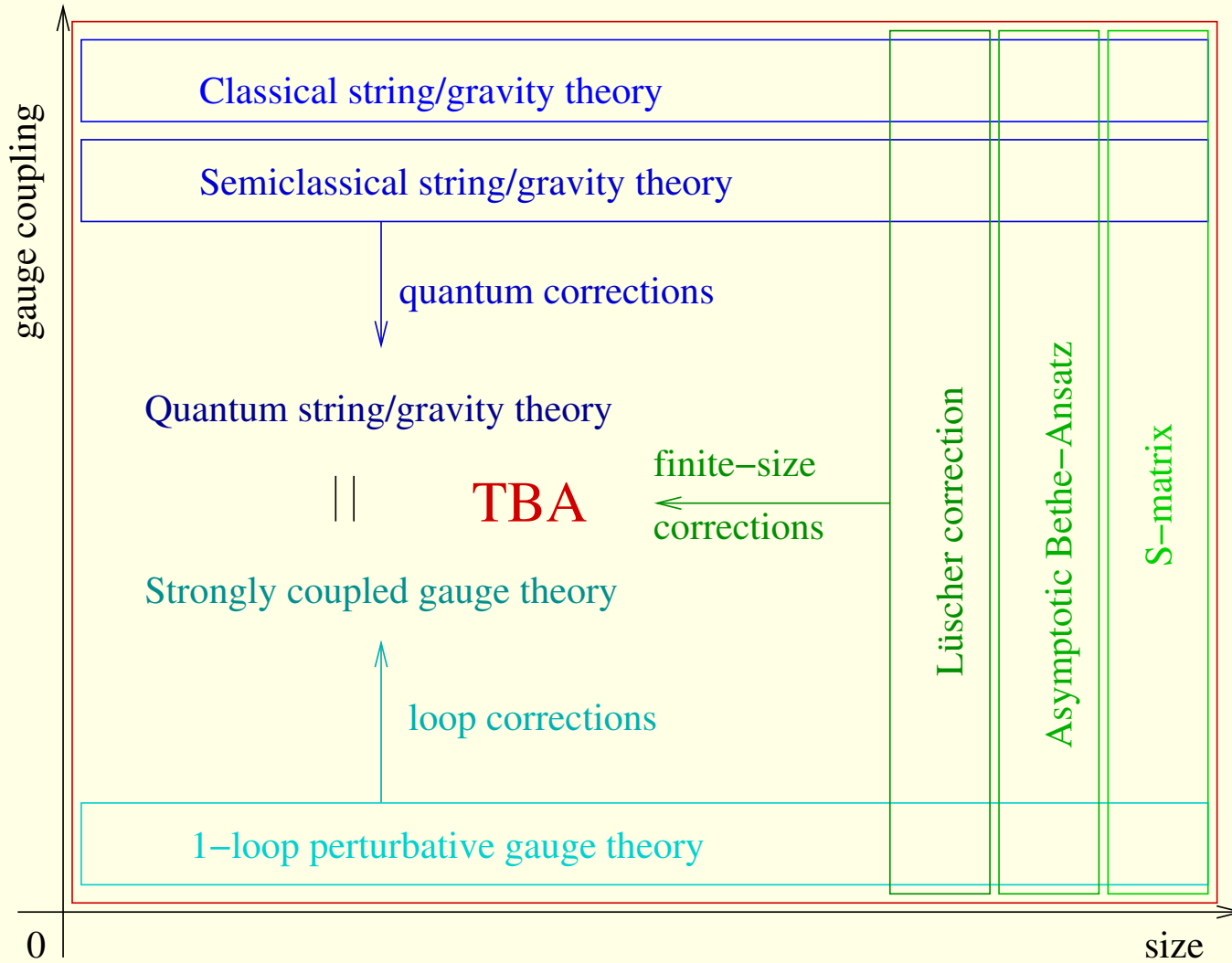
## AdS/CFT spectral problem

## AdS/CFT spectral problem

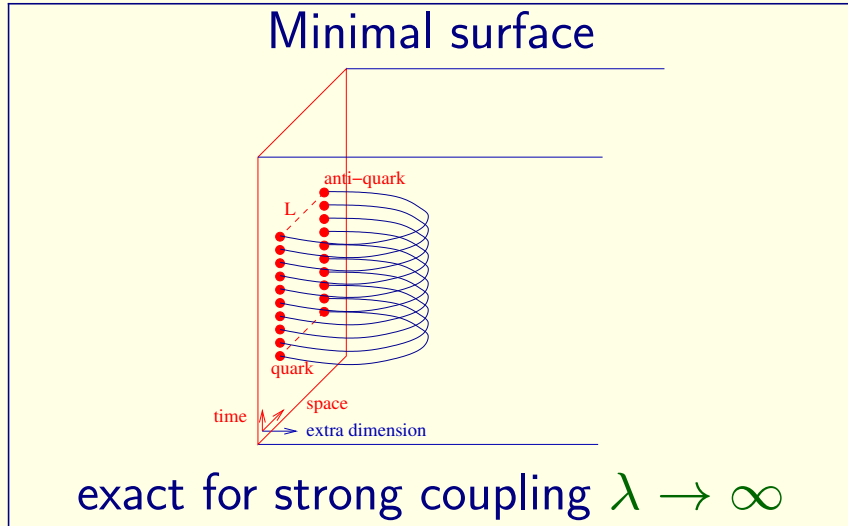
Konishi dimension:  $\text{Tr}(ZXZX - ZZXX)$



# AdS/CFT spectral problem



## AdS/CFT correspondence: applications



$\equiv$

quark-antiquark potential

Wilson loop:  $\langle \oint_C A_\mu dx^\mu \rangle$   
non-perturbative

$$V(r) = -\frac{4\pi^2 \sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4} \frac{1}{r}$$

# AdS/CFT correspondence: applications

### Minimal surface

exact for strong coupling  $\lambda \rightarrow \infty$

quark-antiquark potential

≡

Wilson loop:  $\langle \oint_C A_\mu dx^\mu \rangle$   
 non-perturbative  

$$V(r) = -\frac{4\pi^2 \sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4} \frac{1}{r}$$

### growing black hole

metric  $\delta g(x, 0) \propto \langle T_{\mu\nu} \rangle$   

$$ds^2 = \frac{1}{z^2} (g(x, z)_{\mu\nu} dx^\mu dx^\nu + dz^2)$$
  
 Einstein equation  

$$R_{ab} - \frac{1}{2} g_{ab} R - 6g_{ab} = 0$$
  
 growing black hole  

$$g_{tt} = -\frac{(1-z^4/z_0^4)^2}{(1+z^4/z_0^4)^2}; \quad g_{xx} = 1 + \frac{z^4}{z_0^4}$$

≡

### Heavy ion collision: expansion

$\langle T_{\mu\nu} \rangle$  matter distribution  
 relativistic hydrodynamics  
 $\partial_\mu T^{\mu\nu} = 0$  and  $T^\mu_\mu = 0$   
 viscous quark-gluon plasma  
 expansion in time: perfect fluid +  $\frac{\eta}{s} = \frac{1}{4\pi} + \dots$