Matrix Workshop: Integrability in Low-Dimensional Quantum systems

Creswick 2017 June-July

AdS/CFT correspondence from an integrable point of view

Z. Bajnok

MTA Wigner Research Center for Physics,

Holographic QFT Group, Budapest

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Prologue: QFT as the continuum limit of XXZ

Sine-Gordon / massive Thirring duality

Anti-de Sitter / Conformal Field Theory duality

Prologue: QFT as a continuum limit

Consider the inhomogenous XXZ spin chain

$$R(\lambda) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sinh(\lambda)}{\sinh(\lambda - i\gamma)} & \frac{\sinh(-i\gamma)}{\sinh(\lambda - i\gamma)} & 0 \\ 0 & \frac{\sinh(-i\gamma)}{\sinh(\lambda - i\gamma)} & \frac{\sinh(\lambda)}{\sinh(\lambda - i\gamma)} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\vec{\xi} = \{ \ \xi_-, \ \xi_+, \ \ldots, \ \xi_+ \ \},$$

$$T(\lambda|\vec{\xi}) = R_{01}(\lambda - \xi_1)\, R_{02}(\lambda - \xi_2) \ldots \ldots R_{0N}(\lambda - \xi_N) = \begin{pmatrix} A(\lambda) & B(\lambda) \\ C(\lambda) & D(\lambda) \end{pmatrix}$$

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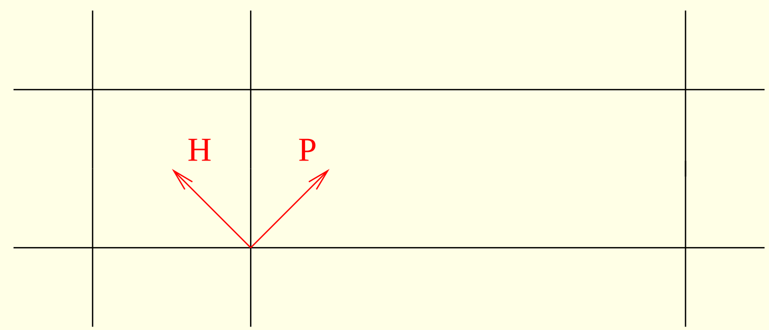
Integrability: $\mathcal{T}(\lambda|\vec{\xi}) = \mathrm{Tr}_0\, T(\lambda|\vec{\xi})$ commute $\left[\mathcal{T}(\lambda|\vec{\xi}), \mathcal{T}(\lambda'|\vec{\xi})\right] = 0$

conserved charges $U_\pm = \mathcal{T}(\xi_\pm|\vec{\xi}) = e^{i\frac{2}{a}(H \pm P)}$



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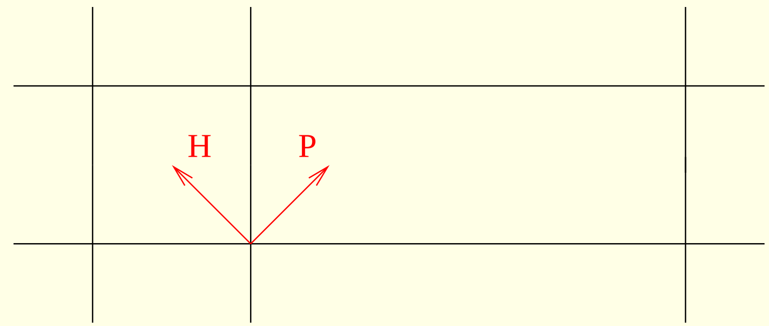


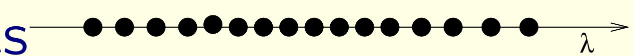
Eigenvectors: $B(\lambda_1)\, B(\lambda_2) \ldots B(\lambda_m)\, |0\rangle$,

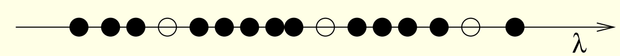
Bethe Ansatz:

$$\prod_{i=1}^{N} \frac{\sinh(\lambda_a - \xi_i - i\gamma)}{\sinh(\lambda_a - \xi_i)} \prod_{b=1}^{m} \frac{\sinh(\lambda_a - \lambda_b + i\gamma)}{\sinh(\lambda_a - \lambda_b - i\gamma)} = -1$$

Alternating inhomogeneities

$$\xi_\pm = \pm\frac{\gamma}{\pi}\Lambda - i\frac{\gamma}{2}$$

Grd st. $\frac{N}{2}$ roots 

Exc. st. +holes: 

Prologue: QFT as a continuum limit

Counting function $(-1)^{\delta}\, e^{i\, Z_{\lambda}(\lambda)} = \prod_{i=1}^{N} \frac{\sinh(\lambda-\xi_i-i\gamma)}{\sinh(\lambda-\xi_i)} \prod_{b=1}^{m} \frac{\sinh(\lambda-\lambda_b+i\gamma)}{\sinh(\lambda-\lambda_b-i\gamma)}$

Bethe Ansatz: $e^{i\, Z_{\lambda}(\lambda_a)} = -1$ take $\delta = 0$ and redefine $Z_N(\theta) = Z_{\lambda}(\frac{\gamma}{\pi}\theta)$, which satisfies

$$Z_N(\theta) = \frac{N}{2}\left\{ \begin{array}{c} \arctan\left[\sinh(\theta-\Lambda)\right] + \\ \arctan\left[\sinh(\theta+\Lambda)\right] \end{array} \right\} + \sum_{k=1}^{m_H} \chi(\theta-\theta_k) + 2\Im m \int_{-\infty}^{\infty} \frac{d\theta'}{2\pi i}\, G(\theta-\theta'-i\eta) \ln\left(1+e^{i\, Z_N(\theta'+i\eta)}\right)$$

[Klümper, Pearce, Destri, de Vega,...]

$$G(\theta) = -i\partial_{\theta} \log S(\theta) = \int_{-\infty}^{\infty} d\omega\, e^{-i\,\omega\theta}\, \frac{\sinh(\frac{(p-1)\,\pi\omega}{2})}{2\cosh(\frac{\pi\omega}{2})\, \sinh(\frac{p\,\pi\,\omega}{2})}$$

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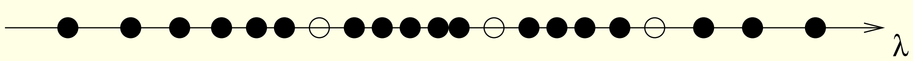
QFT = Scaled continuum limit $N \to \infty$: $\Lambda = \ln \frac{4}{\mathcal{M}\, a} = \ln \frac{2\, N}{\mathcal{M}\, L} \to \infty$

$$Z(\theta) = \mathcal{M} L \sinh\theta - i \sum_{k=1}^{m_H} \log S(\theta-\theta_k) + \int_{-\infty}^{\infty} \frac{d\theta'}{2\pi i} G(\theta-\theta'-i\eta) \ln\left(1 + e^{i\, Z(\theta'+i\eta)}\right)$$

$$E \pm P = \mathcal{M} \sum_{k=1}^{m_H} e^{\pm\theta_k} \mp 2\mathcal{M}\, \Re e \int_{-\infty}^{\infty} \frac{d\theta}{2\pi i}\, e^{\pm\theta+i\eta} \ln\left(1 + e^{i\, Z(\theta+i\eta)}\right)$$



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large volume equation: $e^{iZ(\theta_k)} = e^{i\mathcal{M}L \sinh\theta_k} \prod_{j=1}^{m_H} S(\theta_k-\theta_j) = -1$

relativistic energy spectrum: $E = \sum_{j=1}^{m_H} \mathcal{M} \cosh\theta_k$

Sine-Gordon/massive Thirring duality

$$\mathcal{L}_{SG} = \frac{1}{2}\partial_\nu \Phi \partial^\nu \Phi + \frac{m^2}{\beta^2} : \cos(\beta\Phi) : \qquad 0 < \beta^2 < 8\pi,$$

strong-weak duality:

$$1 + \frac{g}{4\pi} = \frac{4\pi}{\beta^2} = \frac{p+1}{2p}$$

$$\mathcal{L}_{MT} = \bar{\Psi}(i\gamma_\nu \partial^\nu + m_0)\Psi - \frac{g}{2}\bar{\Psi}\gamma^\nu \Psi \bar{\Psi}\gamma_\nu \Psi$$

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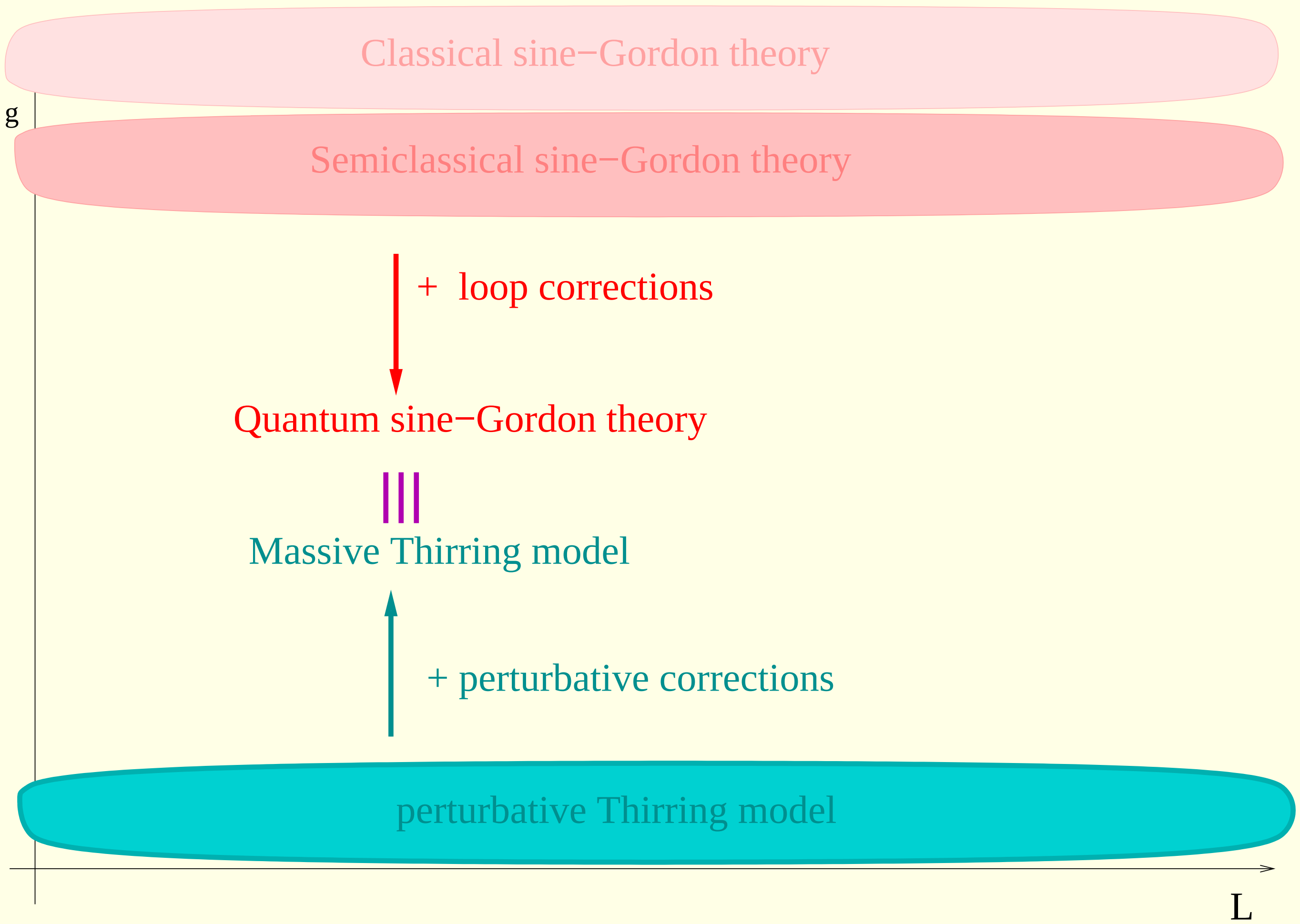

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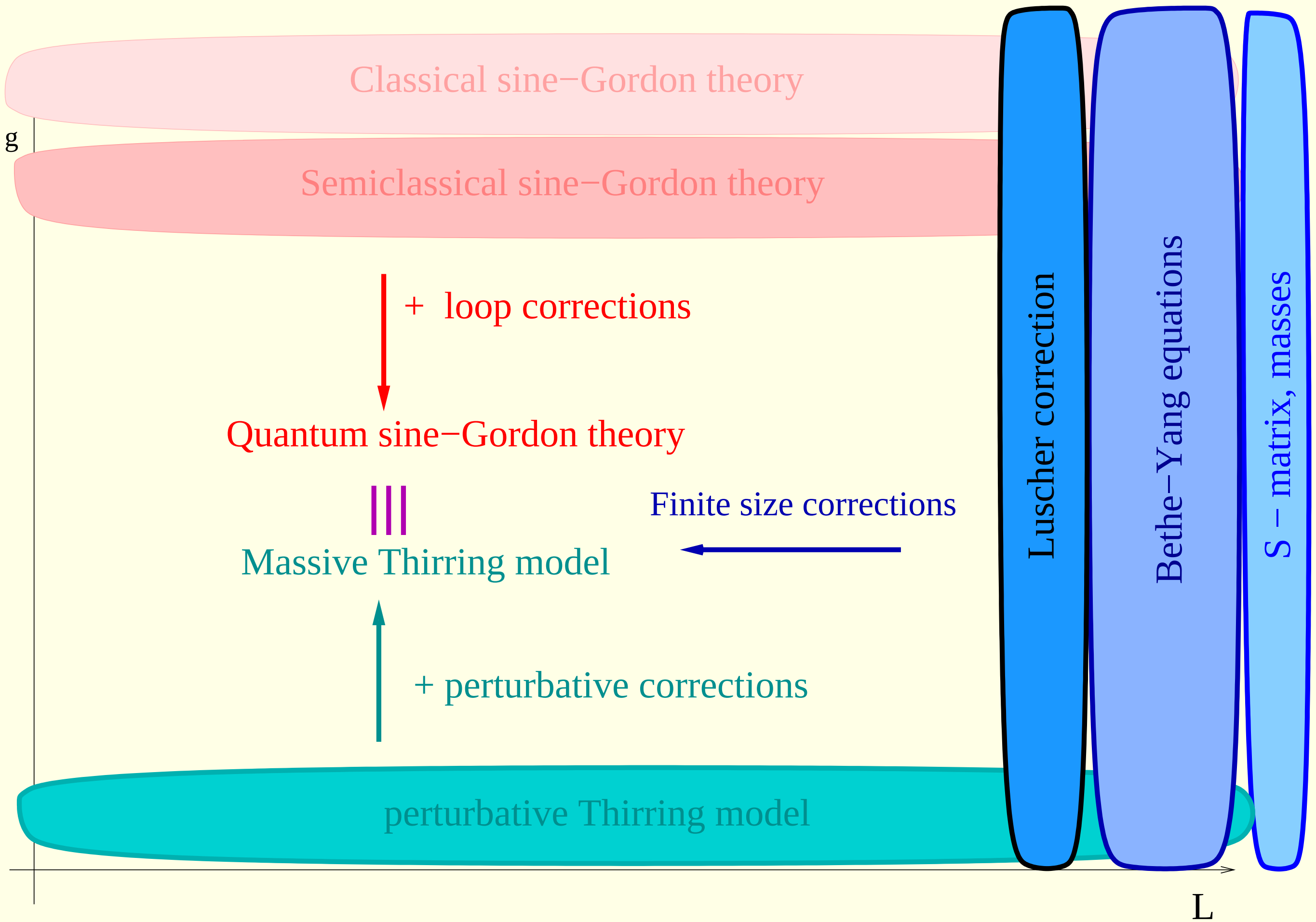

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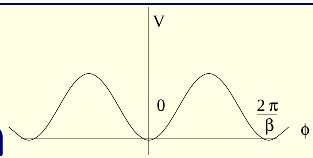
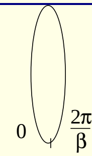
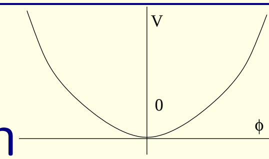
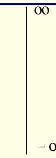
Classical integrable models: sin(e/h)-Gordon theory

sine-Gordon  target 	$\beta \leftrightarrow ib$	sinh-Gordon  target 
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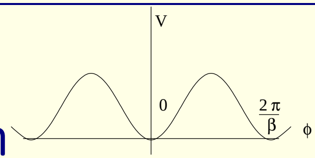
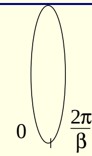
<td>sine-Gordon  target </td>
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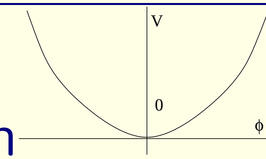
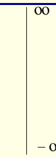
<td>sinh-Gordon  target </td>
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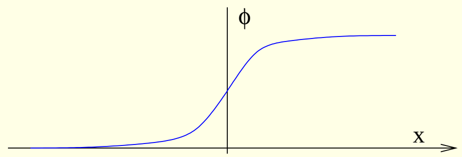
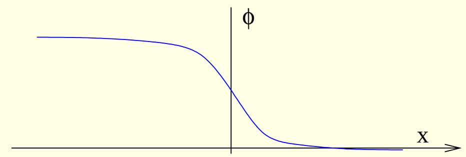
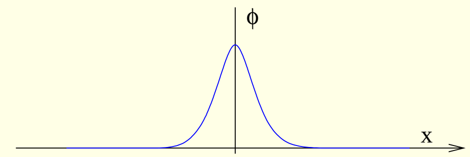
Classical
finite energy
solutions:
sine-Gordon theory







soliton	anti-soliton	breather

Classical integrable models: sin(e/h)-Gordon theory

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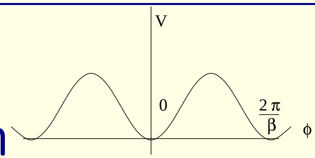
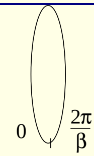
<td>sine-Gordon 

target 
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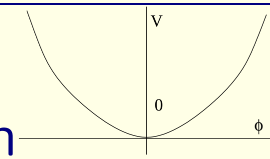
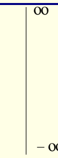
<td>sinh-Gordon 

target 
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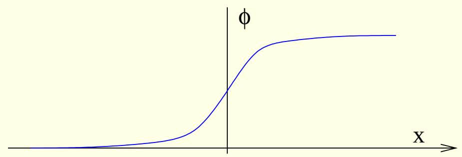
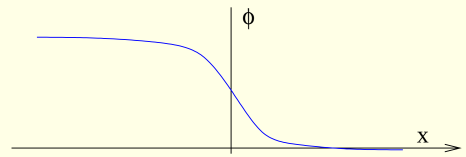
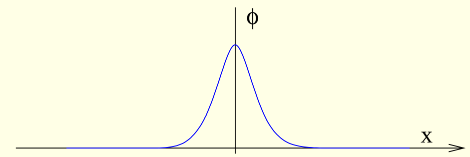
Classical finite energy solutions: sine-Gordon theory







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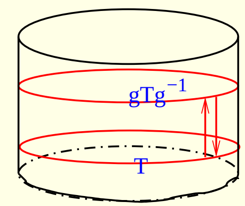

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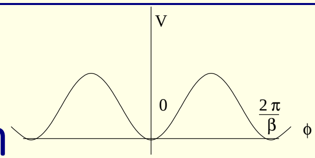
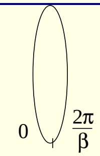
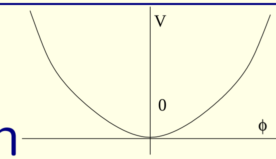
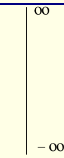
sine-Gordon  target 	$\beta \leftrightarrow ib$	sinh-Gordon  target 
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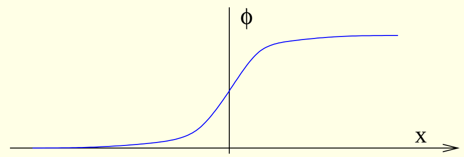

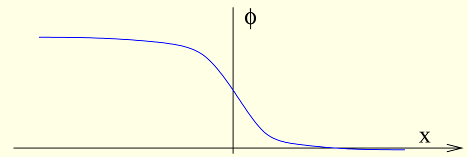

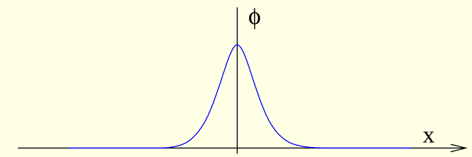

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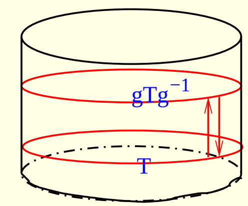

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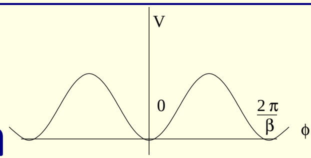
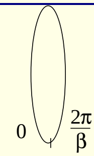
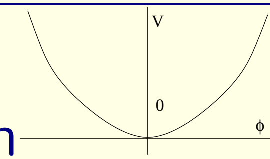
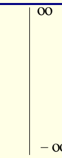
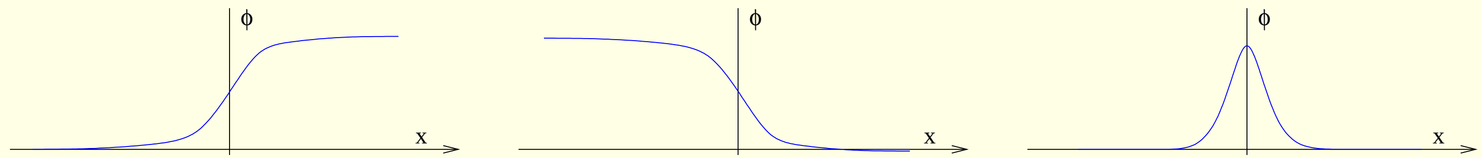
conserved $Q_{\pm 1}[\varphi] = E[\varphi] \pm P[\varphi] = \int \left\{ \frac{1}{2}(\partial_\pm\varphi)^2 + \frac{m^2}{\beta^2}(1-\cos\beta\varphi) \right\} dx$

charges: $Q_{\pm 3}[\varphi] = \int \left\{ \frac{1}{2\beta^2}(\partial_\pm^2\varphi)^2 - \frac{1}{8}(\partial_\pm\varphi)^4 + \frac{m^2}{\beta^2}(\partial_\pm\varphi)^2(1-\cos\beta\varphi) \right\} dx$

Classical integrable models: sin(e/h)-Gordon theory

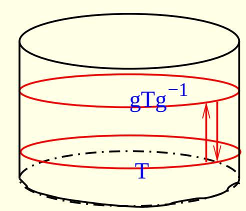
sine-Gordon  target 	$\beta \leftrightarrow ib$	sinh-Gordon  target 
$\mathcal{L} = \frac{1}{2\beta^2}(\partial\beta\varphi)^2 - \frac{m^2}{\beta^2}(1 - \cos\beta\varphi)$		$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 - \frac{m^2}{b^2}(\cosh b\varphi - 1)$

Classical finite energy solutions: sine-Gordon theory



soliton	anti-soliton	breather

Integrability: $\partial_x A_t - \partial_t A_x + [A_x, A_t] = 0 \leftrightarrow T(\lambda) = P\exp\oint A(x)_\nu dx^\nu$



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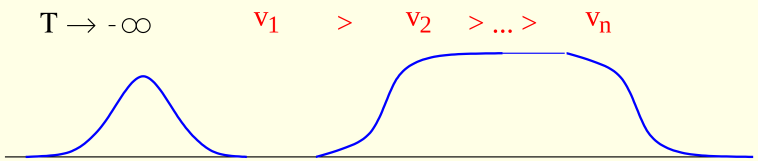
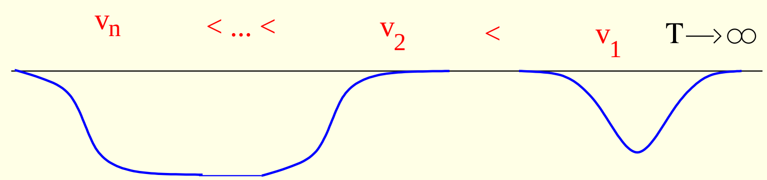
Classical factorized scattering: $\Delta T_1(v_1, v_2, \ldots, v_n) = \sum_i \Delta T_1(v_1, v_i)$





Quantum integrability: sine-Gordon $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{\beta^2}(1 - \cos\beta\phi)$

Perturbed Conformal Field Theory	Lagrangian perturbation theory
$\mathcal{L}_{CFT} + \mu\mathcal{L}_{pert} = \frac{1}{2}(\partial\phi)^2 + \mu(V_\beta + V_{-\beta})$	$\mathcal{L}_0 + V_{pert} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 - \beta^2 U$
$h_\beta = \frac{\beta^2}{4\pi}$ definite scaling $V_\beta =: e^{i\beta\phi}:$	free massive boson+pert.

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$h_\beta=\frac{\beta^2}{4\pi}$ definite scaling $V_\beta=:e^{i\beta\phi}:$	free massive boson+pert.

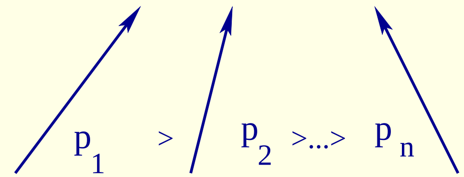
Quantum conservation laws
$\partial_-\Lambda_4=0\to\partial_-\Lambda_4=\lambda\partial_+\Theta_2$
$[\lambda]=2-h_\beta$, $[\Lambda_4]=4$,
counting argument
Nonlocal symmetry: $U_q(\hat{sl_2})$
$[h,J_\pm]=\pm J_\pm$
$[J_+,J_-]=\frac{q^h-q^{-h}}{q-q^{-1}}$
$\Delta(J_\pm)=q^{\frac{h}{2}}\otimes J_\pm+J_\pm\otimes q^{-\frac{h}{2}}$
$[S,\Delta(J_\pm)]=0$
parameter relation: $q=e^{-i\frac{\pi}{p}}$

Correlators=$\sum_{loops} Feynman\, diagrams$
Asymptotic states $E(p)=\sqrt{p^2+m^2}$
S-matrix↔correlators LSZ
$\langle p_1',p_2'\vert\mathcal{O}\vert p_1,p_2\rangle=$
$\bar{\mathcal{D}}_1'\bar{\mathcal{D}}_2'\mathcal{D}_1\mathcal{D}_2\langle 0\vert T(\mathcal{O}\varphi(1)\varphi(2)\varphi(3)\varphi(4))\vert 0\rangle$
$\mathcal{D}_j=i\int d^2x_je^{ip_jx-iE_jt}\left\{\partial_t^2-\partial_x^2+m^2\right\}$
unitarity, crossing symmetry, analyticity
rapidity $p_i=m\cosh\theta_i$
$S(\theta)=S(-\theta)^{-1}=S(i\pi-\theta)=$
$=1-\frac{1}{4}ib^2\mathrm{csch}\theta-\frac{b^4(\mathrm{csch}\theta(\pi\mathrm{csch}\theta-i))}{32\pi}+\ldots$

Bootstrap scheme

Bootstrap program

Asymptotic states $|p_1, p_2, \dots, p_n\rangle_{in/out}$
form a representation of global symmetry:



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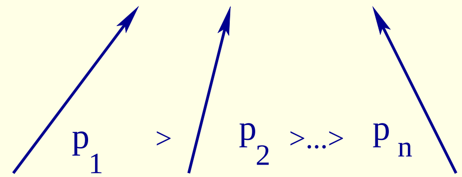

Lorentz: $P = \sum_i p_i \quad E = \sum_i E(p_i)$
dispersion relation $E(p) = \sqrt{m^2 + p^2}$

Bootstrap program

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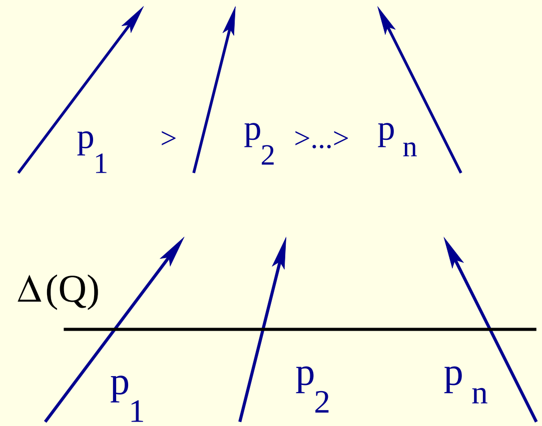

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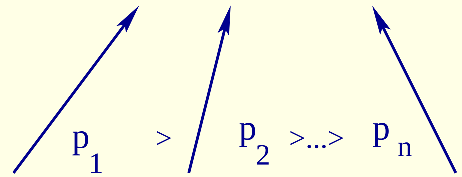

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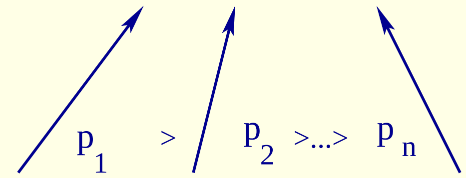

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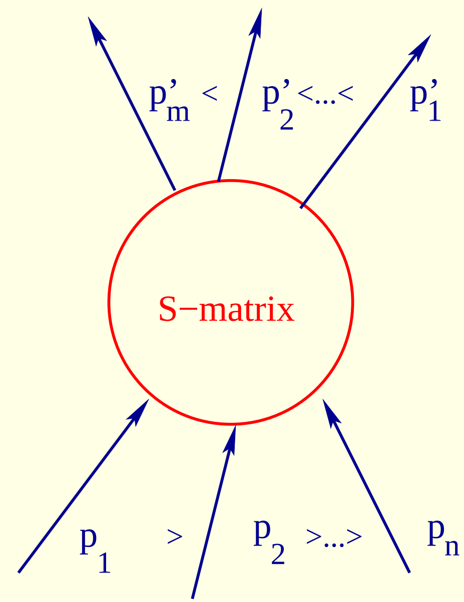

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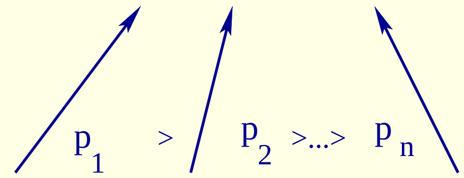

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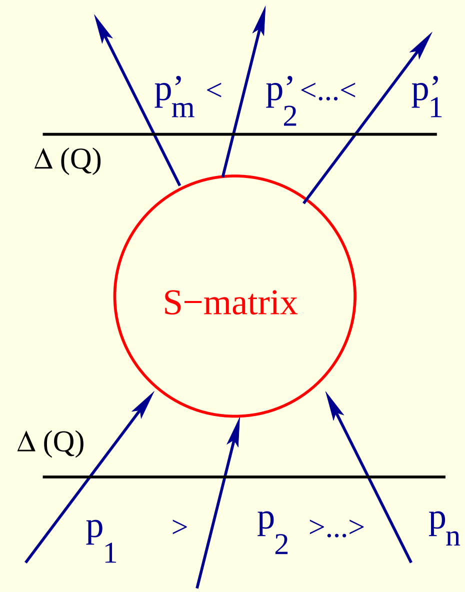

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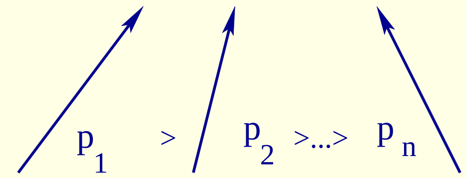

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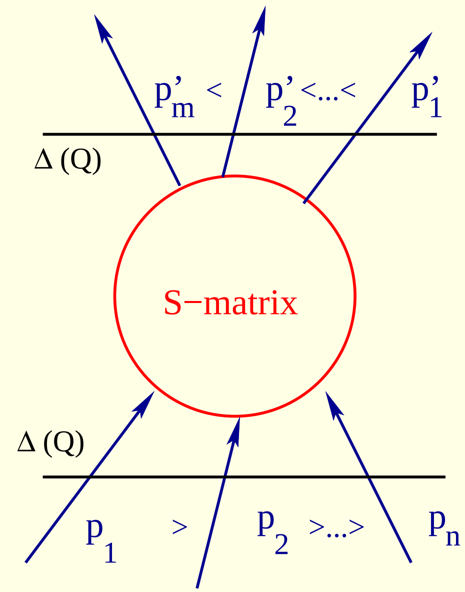

Scattering matrix S: $|out\rangle \to |in\rangle$
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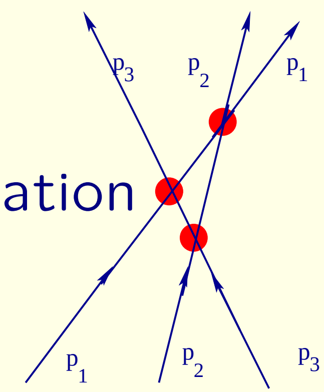
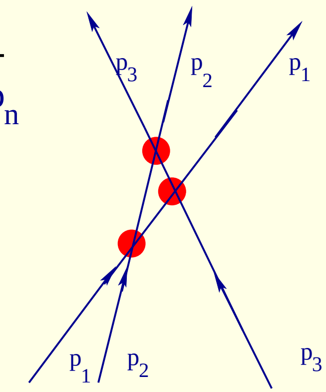
Higher spin concerved charge
factorization + Yang-Baxter equation
$S_{123} = S_{23} S_{13} S_{12} = S_{12} S_{13} S_{23}$

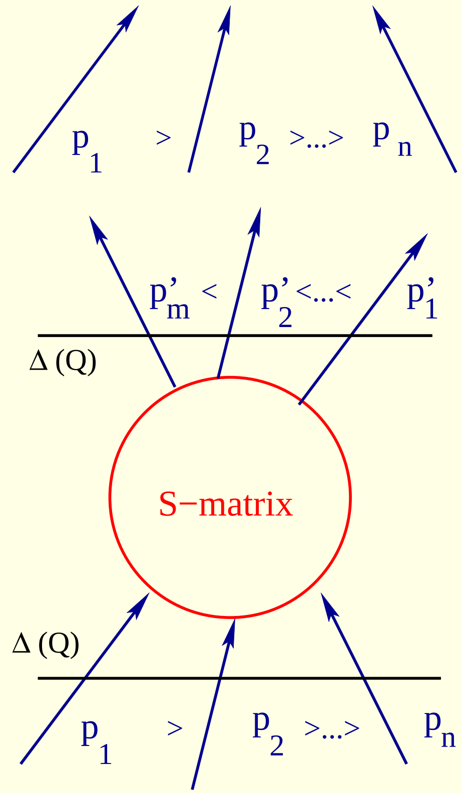




S-matrix = scalar . Matrix

Bootstrap program

Asymptotic states $|p_1, p_2, \ldots, p_n\rangle_{in/out}$
form a representation of global symmetry:



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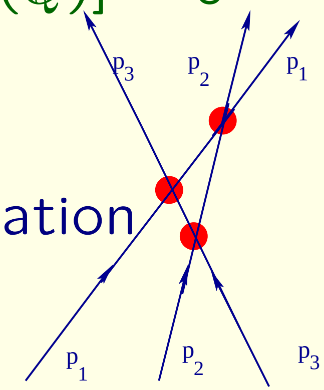

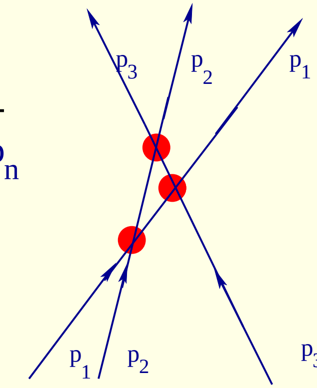

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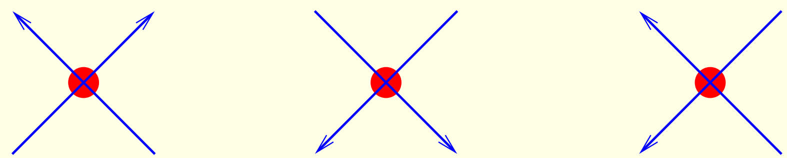
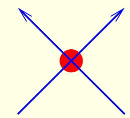


Unitarity $S_{12}S_{21} = Id$
Crossing symmetry $S_{12} = S_{2\bar{1}}$
Maximal analyticity: all poles have physical origin $\rightarrow$ boundstates, anomalous thresholds

Bootstrap program: diagonal

Diagonal: **S-matrix = scalar** $S(p_1, p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh\theta$

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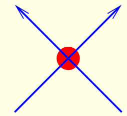
Diagonal: S-matrix = scalar $S(p_1, p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh \theta$

Unitarity $S(\theta)S(-\theta) = 1$

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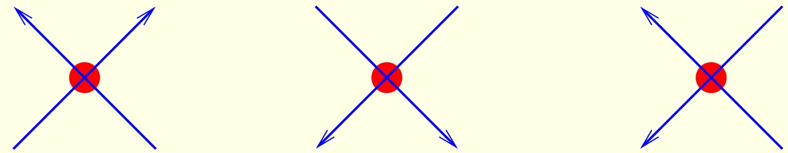

Maximal analyticity: all poles have physical origin

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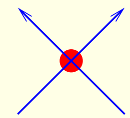
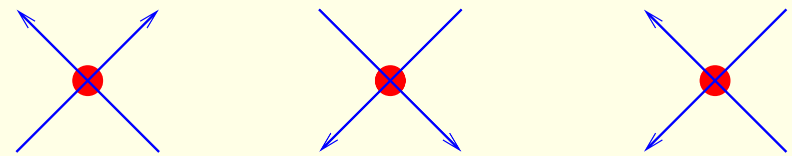
Diagonal: S-matrix = scalar $S(p_1, p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh \theta$



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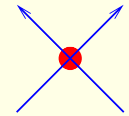
Minimal solution: $S(\theta) = 1$ Free boson

Bootstrap program: diagonal

Diagonal: S-matrix = scalar $S(p_1,p_2)=S(\theta_1-\theta_2)$  $p=m\sinh\theta$

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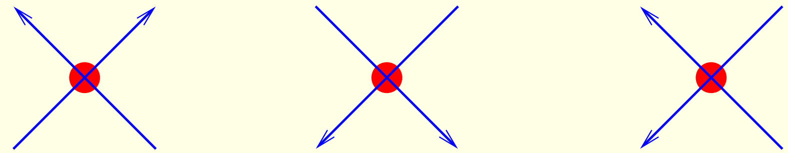
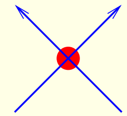


Maximal analyticity: all poles have physical origin

Minimal solution: $S(\theta)=1$ Free boson

CDD factor: $S(\theta)=\frac{\sinh\theta-i\sin p\pi}{\sinh\theta+i\sin p\pi}$

Bootstrap program: diagonal

Diagonal: S-matrix = scalar $S(p_1,p_2)=S(\theta_1-\theta_2)$  $p=m\sinh\theta$

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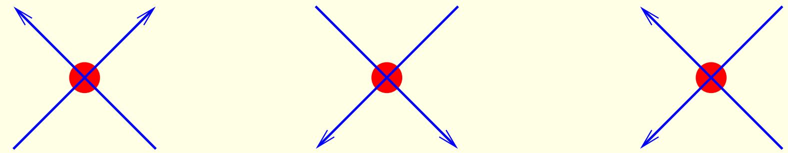


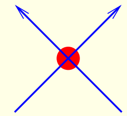
Maximal analyticity: all poles have physical origin

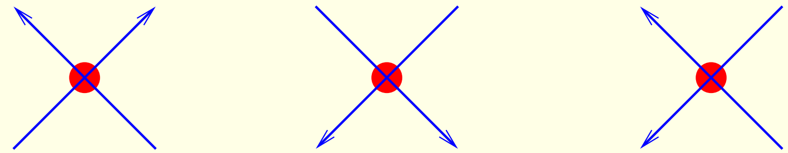
Minimal solution: $S(\theta)=1$ Free boson

$V\sim\cosh b\phi \leftrightarrow p^{-1}=1+\frac{b^2}{8\pi}>0$

CDD factor: $S(\theta)=\frac{\sinh\theta-i\sin p\pi}{\sinh\theta+i\sin p\pi}$ Sinh-Gordon

Bootstrap program: diagonal

Diagonal: S-matrix = scalar $S(p_1,p_2)=S(\theta_1-\theta_2)$  $p=m\sinh\theta$



Unitarity $S(\theta)S(-\theta)=1$

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Maximal analyticity: all poles have physical origin

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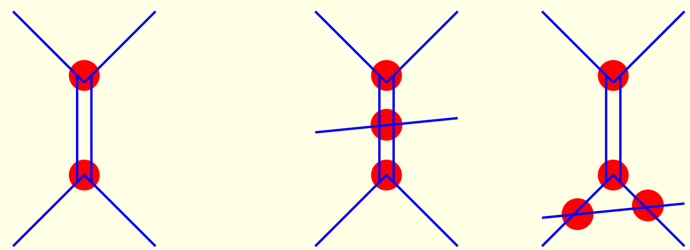


Maximal analyticity: $S(\theta)=\frac{\sinh\theta+i\sin p\pi}{\sinh\theta-i\sin p\pi}$

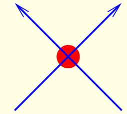
pole at $\theta=ip\pi$ $\to$boundstate B^2

bootstrap: $S_{12}(\theta)=S_{11}(\theta-\frac{i\pi p}{2})S_{11}(\theta+\frac{i\pi p}{2})$

new particle if $p\neq\frac{2}{3}$ otherwise Lee-Yang

Bootstrap program: diagonal

Diagonal: S-matrix = scalar $S(p_1,p_2)=S(\theta_1-\theta_2)$ $p=m\sinh\theta$



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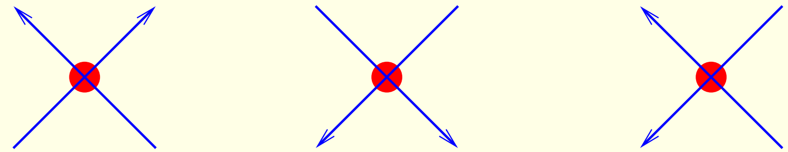
Maximal analyticity: all poles have physical origin



Minimal solution: $S(\theta)=1$ Free boson $V\sim\cosh b\phi\leftrightarrow p^{-1}=1+\frac{b^2}{8\pi}>0$

CDD factor: $S(\theta)=\frac{\sinh\theta-i\sin p\pi}{\sinh\theta+i\sin p\pi}$ Sinh-Gordon

Maximal analyticity: $S(\theta)=\frac{\sinh\theta+i\sin p\pi}{\sinh\theta-i\sin p\pi}$

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bootstrap: $S_{12}(\theta)=S_{11}(\theta-\frac{i\pi p}{2})S_{11}(\theta+\frac{i\pi p}{2})$

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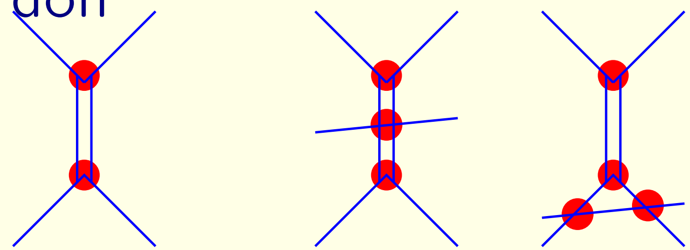
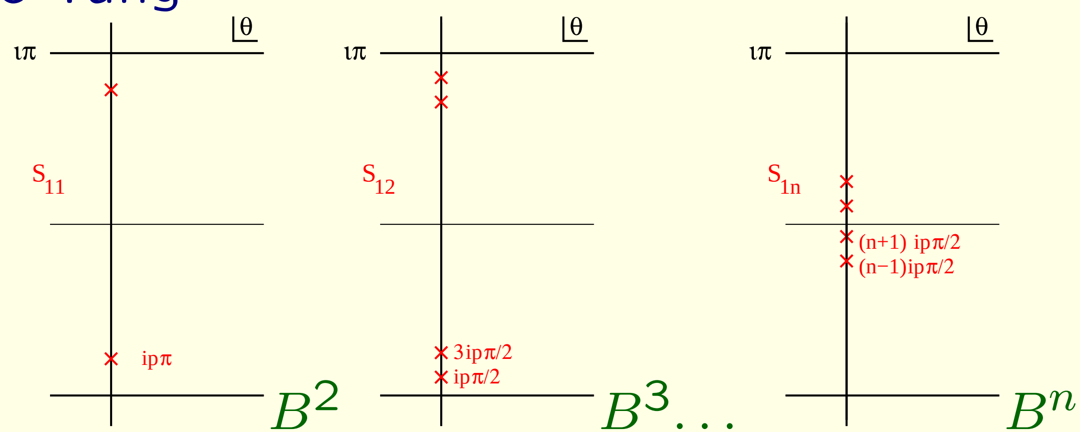


Maximal analyticity:

all poles have physical origin

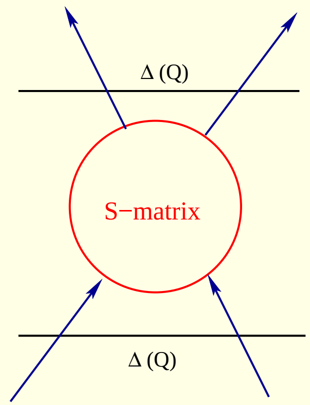
$\rightarrow$sine-Gordon solitons



Bootstrap program: sine-Gordon

Nondiagonal scattering: S-matrix = scalar . Matrix soliton doublet $\begin{pmatrix} s \\ \bar{s} \end{pmatrix}$

Matrix:
global symmetry $U_q(\hat{sl}_2)$
2d evaluation reps
$[S,\Delta(Q)]=0$



$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{-\sin\alpha\pi}{\sin\alpha(\pi+i\theta)} & \frac{\sin i\theta\alpha}{\sin\alpha(\pi+i\theta)} & 0 \\ 0 & \frac{\sin i\alpha\theta}{\sin\alpha(\pi+i\theta)} & \frac{-\sin\alpha\pi}{\sin\alpha(\pi+i\theta)} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Bootstrap program: sine-Gordon

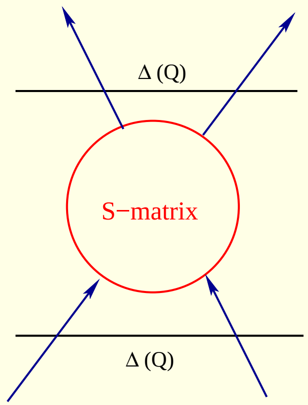
Nondiagonal scattering: $\boxed{\text{S-matrix} = \text{scalar . Matrix}}$ soliton doublet $\begin{pmatrix} s \\ \bar{s} \end{pmatrix}$

Matrix:

global symmetry $U_q(s\hat{l}_2)$

2d evaluation reps

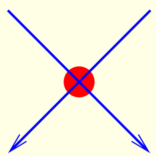
$[S, \Delta(Q)] = 0$



$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{-\sin \alpha\pi}{\sin \alpha(\pi + i\theta)} & \frac{\sin i\theta\alpha}{\sin \alpha(\pi + i\theta)} & 0 \\ 0 & \frac{\sin i\alpha\theta}{\sin \alpha(\pi + i\theta)} & \frac{-\sin \alpha\pi}{\sin \alpha(\pi + i\theta)} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Unitarity

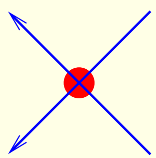
$\mathcal{S}(\theta)\mathcal{S}(-\theta) = 1$



$$\prod_{l=1}^{\infty} \left[\frac{\Gamma(2(l-1)\alpha + \frac{\alpha i\theta}{\pi})\Gamma(2l\alpha + 1 + \frac{\alpha i\theta}{\pi})}{\Gamma((2l-1)\alpha + \frac{\alpha i\theta}{\pi})\Gamma((2l-1)\alpha + 1 + \frac{\alpha i\theta}{\pi})} / (\theta \to -\theta) \right]$$

Crossing symmetry

$\mathcal{S}(\theta) = \mathcal{S}^{c_1}(i\pi - \theta)$



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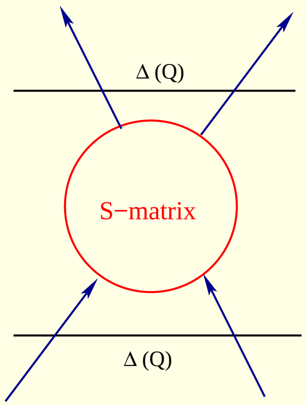

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{-\sin\alpha\pi}{\sin\alpha(\pi+i\theta)} & \frac{\sin i\theta\alpha}{\sin\alpha(\pi+i\theta)} & 0 \\ 0 & \frac{\sin i\alpha\theta}{\sin\alpha(\pi+i\theta)} & \frac{-\sin\alpha\pi}{\sin\alpha(\pi+i\theta)} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

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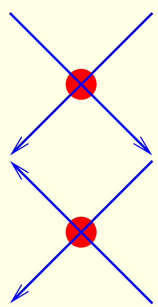
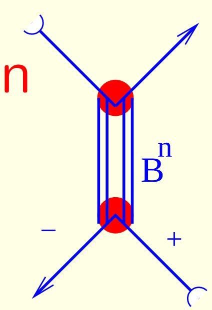
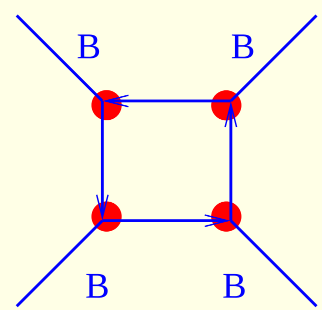


$$\prod_{l=1}^{\infty} \left[\frac{\Gamma(2(l-1)\alpha + \frac{\alpha i\theta}{\pi})\Gamma(2l\alpha + 1 + \frac{\alpha i\theta}{\pi})}{\Gamma((2l-1)\alpha + \frac{\alpha i\theta}{\pi})\Gamma((2l-1)\alpha + 1 + \frac{\alpha i\theta}{\pi})} / (\theta \to -\theta) \right]$$

Maximal analyticity:
all poles have physical origin
either boundstates or
anomalous thresholds
$p = \alpha^{-1}$
[Zamolodchikov^2]





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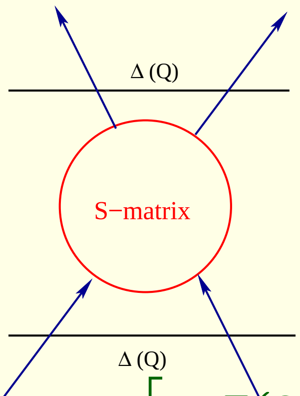

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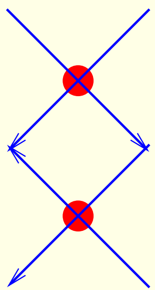

$$\prod_{l=1}^{\infty}\left[\frac{\Gamma(2(l-1)\alpha+\frac{\alpha i\theta}{\pi})\Gamma(2l\alpha+1+\frac{\alpha i\theta}{\pi})}{\Gamma((2l-1)\alpha+\frac{\alpha i\theta}{\pi})\Gamma((2l-1)\alpha+1+\frac{\alpha i\theta}{\pi})}/(\theta\to-\theta)\right]$$

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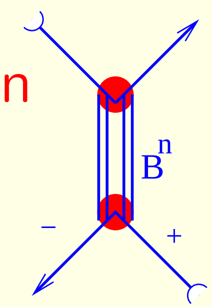
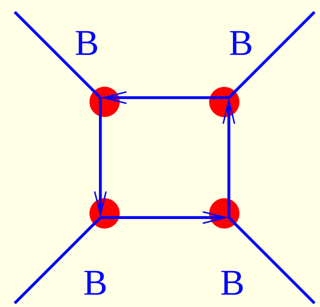
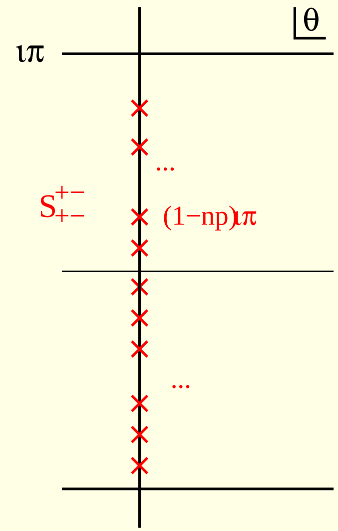
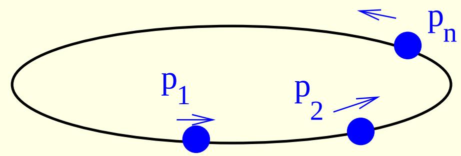






QFTs in finite volume

Finite volume spectrum



QFTs in finite volume

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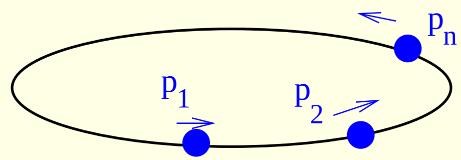

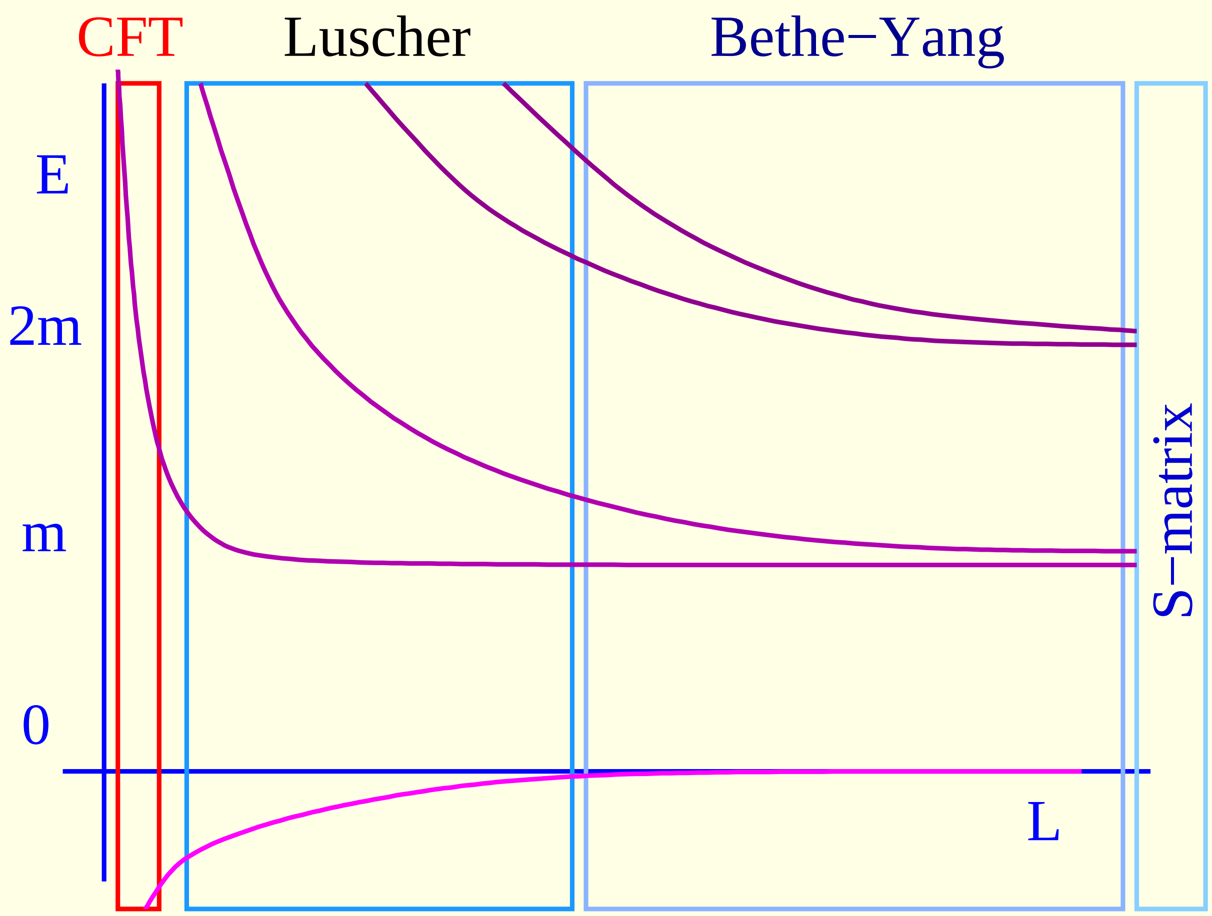

QFTs in finite volume

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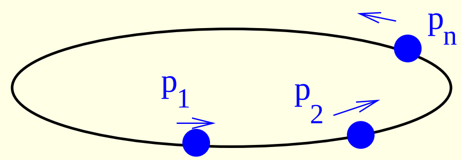

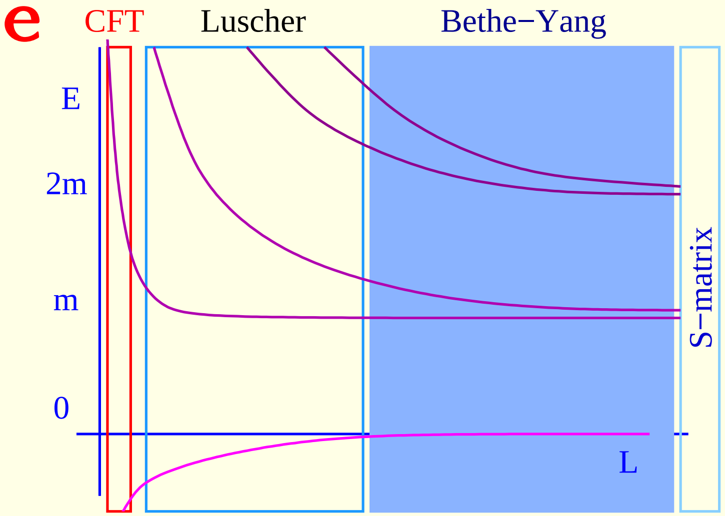

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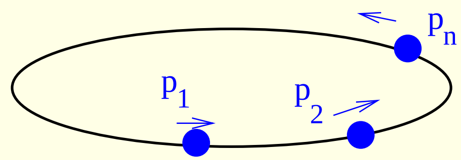
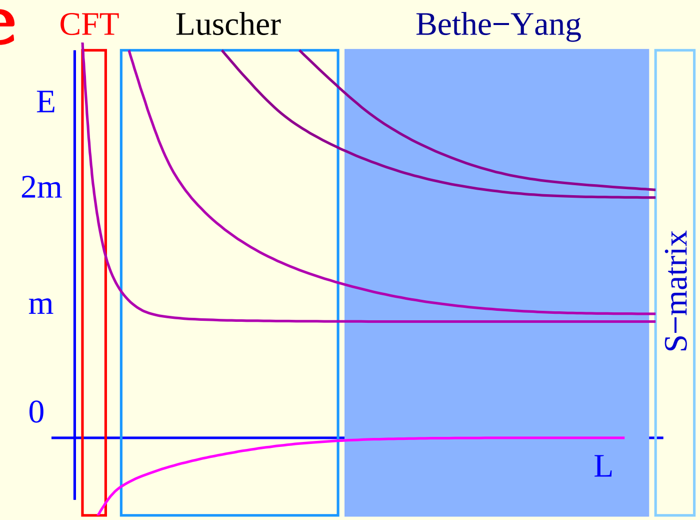
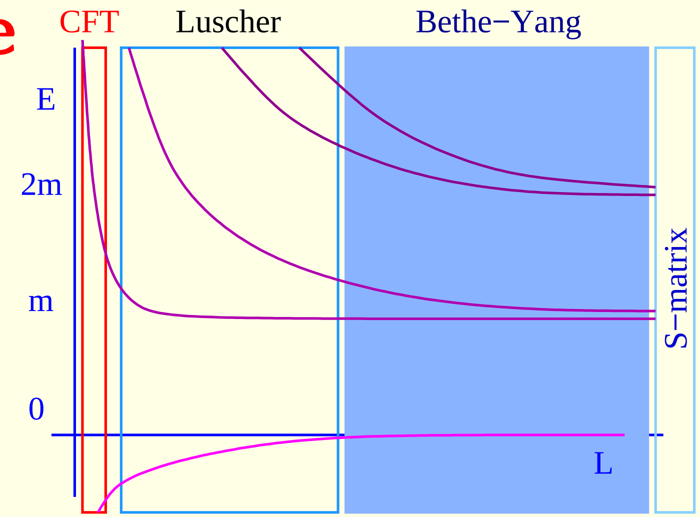


Infinite volume spectrum:

$E(p_1, \ldots, p_n) = \sum_i E(p_i)$ $\qquad p_i \in R$



QFTs in finite volume



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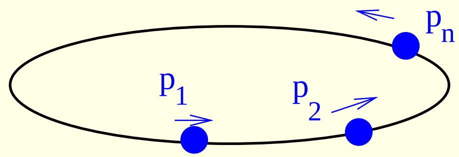

Infinite volume spectrum:

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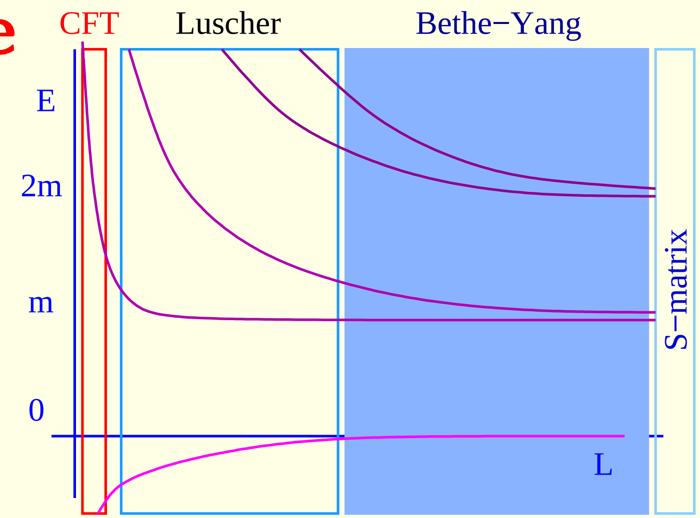
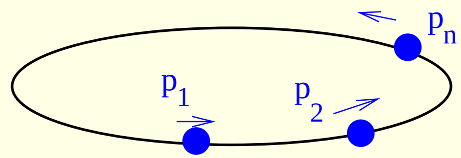
Polynomial volume corrections:

Bethe-Yang; p_i quantized. Diagonal

QFTs in finite volume



Finite volume spectrum



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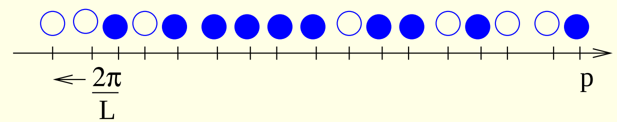
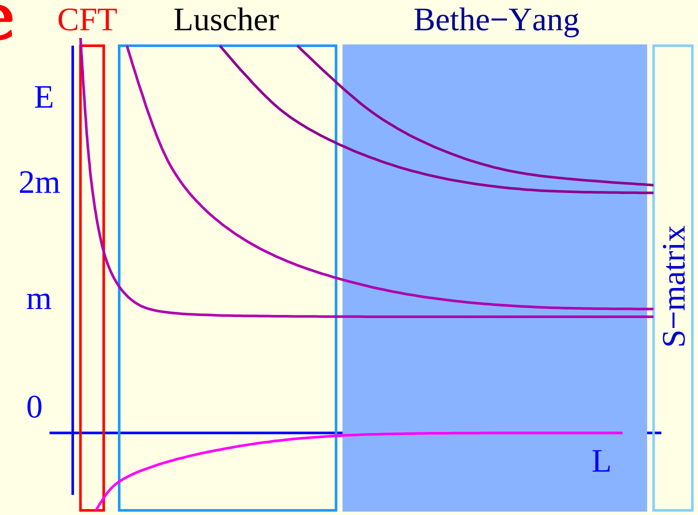
$e^{ip_jL} S(p_j,p_1)\ldots S(p_j,p_n) = -1 \quad ; \quad S(0) = -1$



$p_jL + \sum_k \frac{1}{i} \log S(p_j,p_k) = (2n+1)i\pi$

QFTs in finite volume



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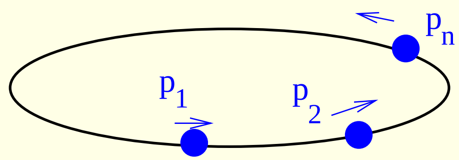

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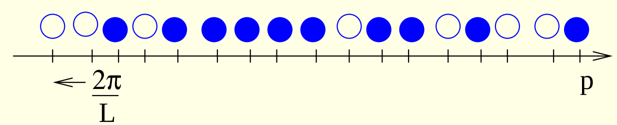

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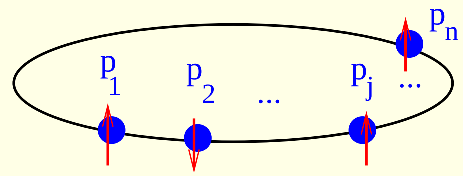
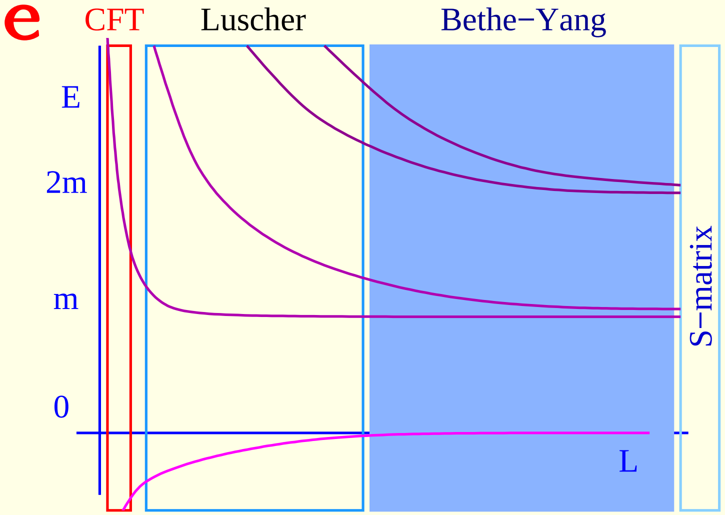
Non-diagonal, sine-Gordon



$e^{ip_jL}\mathcal{S}(p_j,p_1)\dots\mathcal{S}(p_j,p_n)\Psi=-\Psi \quad S(0)=-P$

QFTs in finite volume



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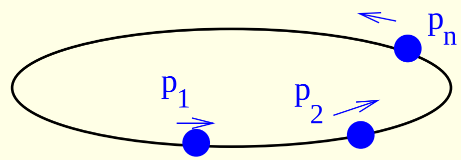

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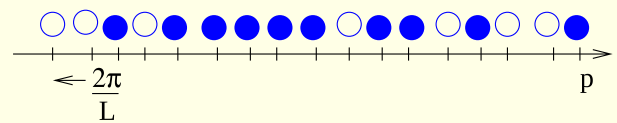

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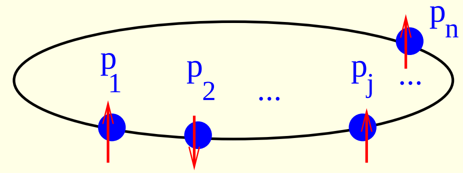
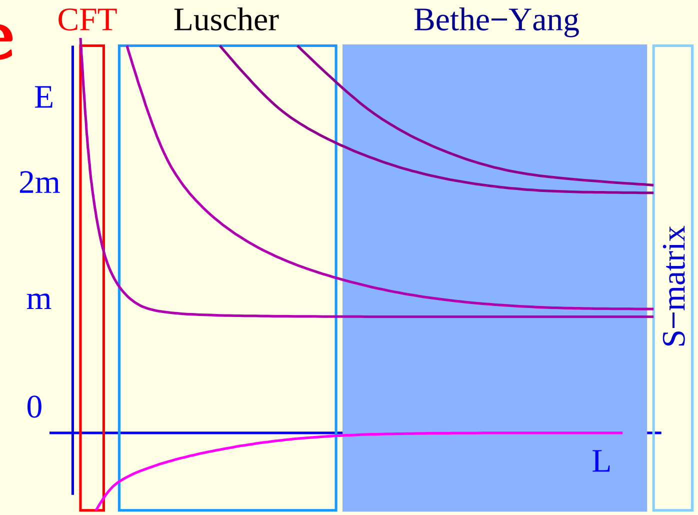
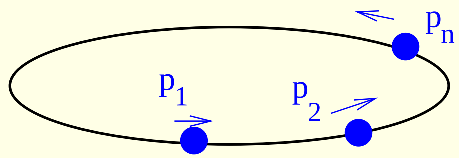


Inhomogenous XXZ spin-chain spectral problem $e^{iL \sinh \theta_j} T(\theta_j) S_0(\theta_j) = -1$

QFTs in finite volume



Finite volume spectrum



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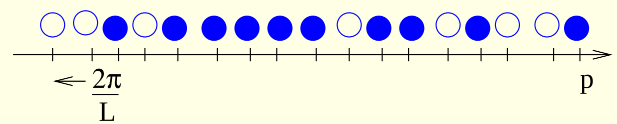

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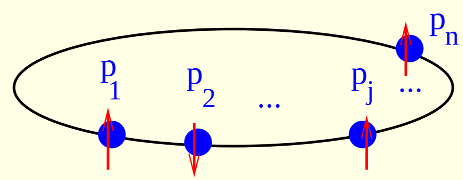

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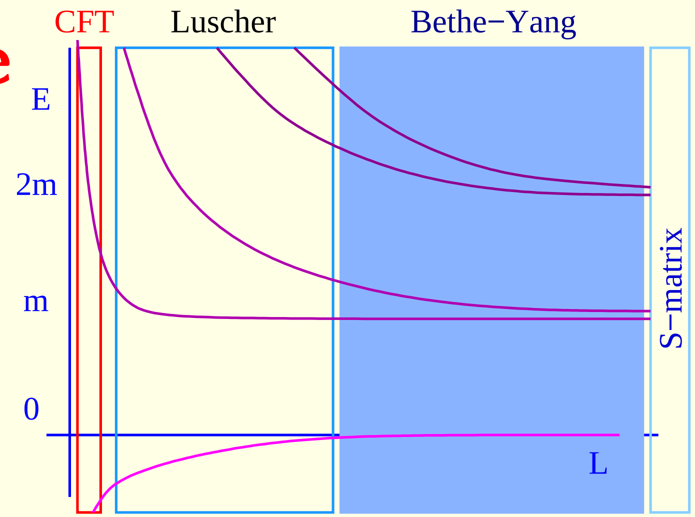
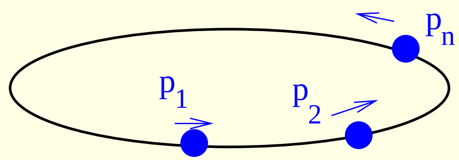
$T(\theta)Q(\theta) = Q(\theta+i\pi)T_0(\theta-\frac{i\pi}{2}) + Q(\theta-i\pi)T_0(\theta+\frac{i\pi}{2}) = Q^{++}T^- + Q^{--}T^+$

QFTs in finite volume



Finite volume spectrum



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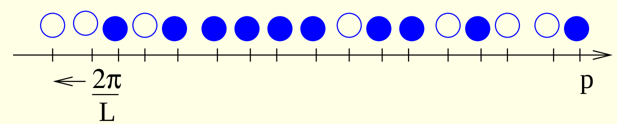

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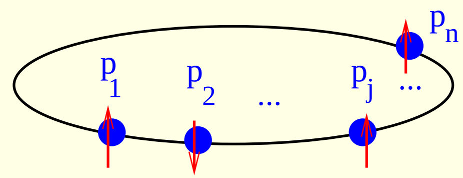

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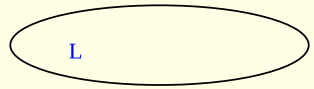
$$T(\theta)Q(\theta) = Q(\theta + i\pi) T_0(\theta - \tfrac{i\pi}{2}) + Q(\theta - i\pi) T_0(\theta + \tfrac{i\pi}{2}) = Q^{++}T^- + Q^{--}T^+$$

$Q(\theta) = \prod_\beta \sinh(\lambda(\theta - w_\beta))$

$T_0(\theta) = \prod_j \sinh(\lambda(\theta - \theta_j))$

Bethe Ansatz: $\dfrac{T_0(w_\alpha - \frac{i\pi}{2}) Q(w_\alpha + i\pi)}{T_0(w_\alpha + \frac{i\pi}{2}) Q(w_\alpha - i\pi)} = \dfrac{T_0^- Q^{++}}{T_0^+ Q^{--}}\Big|_\alpha = -1$

Thermodynamic Bethe Ansatz: diagonal

Ground-state energy exactly 

[Zamolodchikov]

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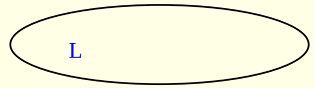
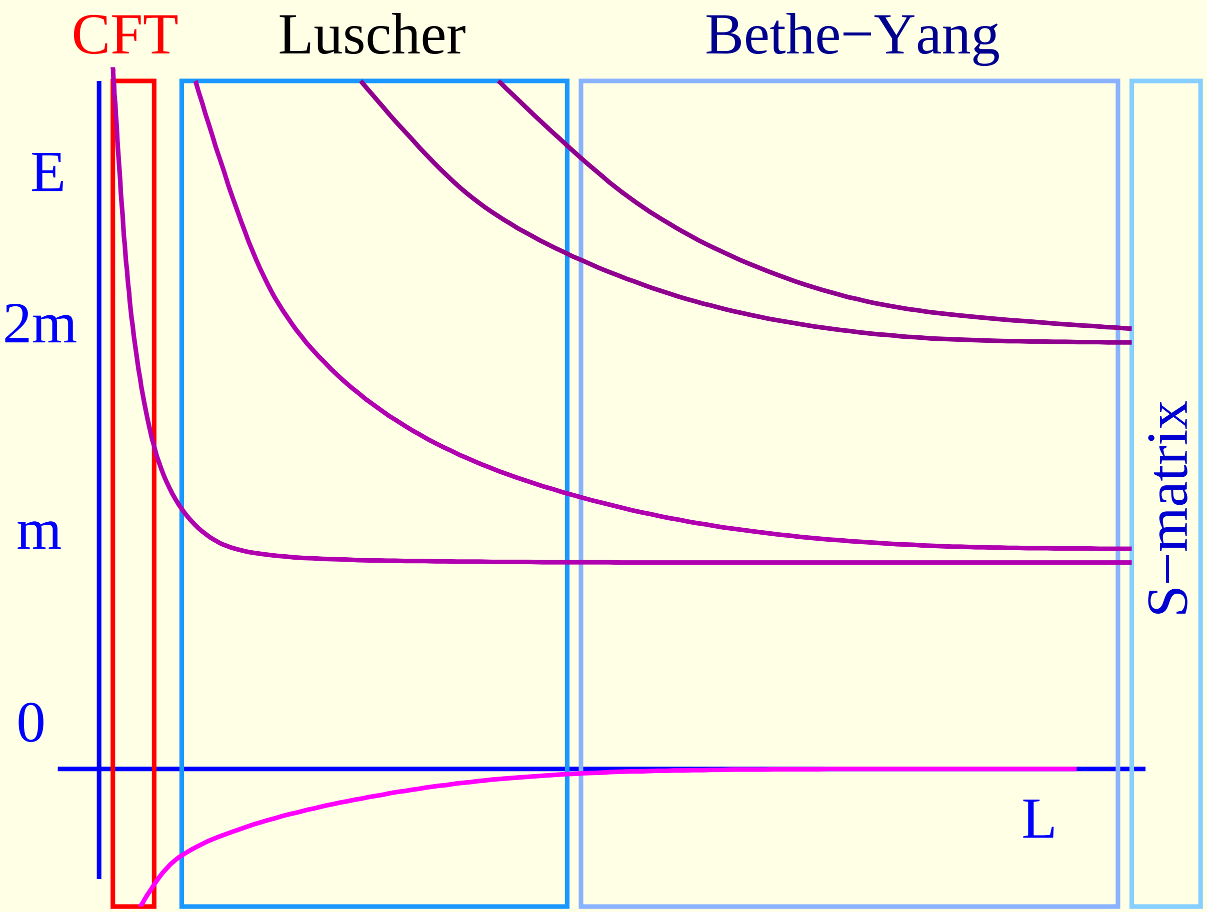
Ground-state energy exactly 

[Zamolodchikov]



Thermodynamic Bethe Ansatz: diagonal

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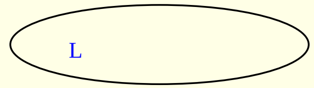
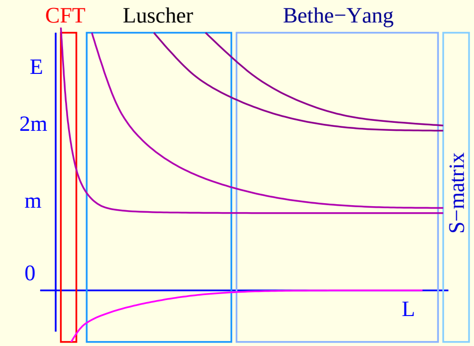


[Zamolodchikov]



Thermodynamic Bethe Ansatz: diagonal

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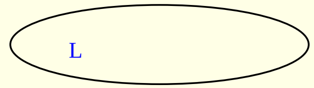

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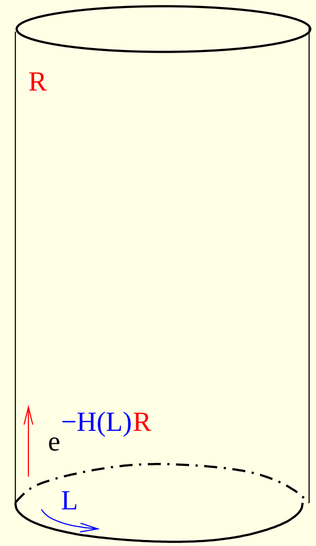
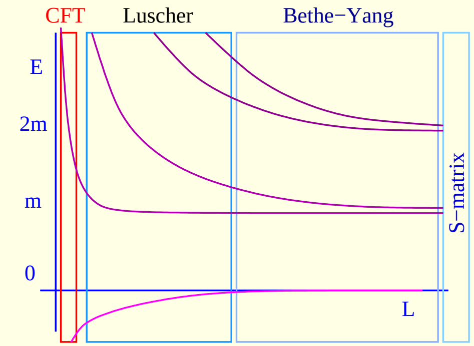


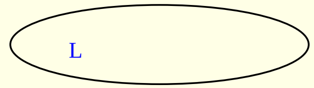


Euclidian partition function:

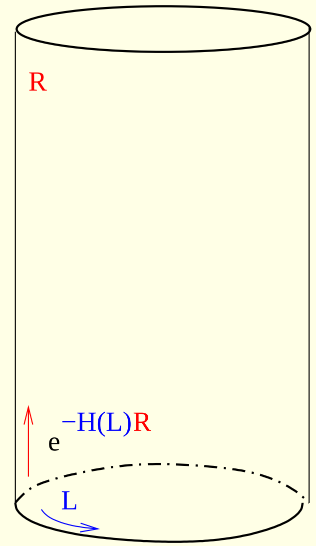
$$Z(L,R) =_{R\to\infty} \mathrm{Tr}(e^{-H(L)R})$$
$$Z(L,R) =_{R\to\infty} e^{-E_0(L)R}(1 + e^{-\Delta E\, R})$$

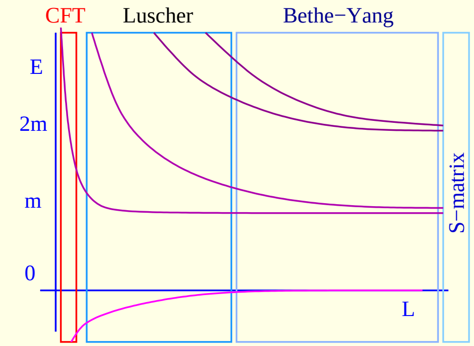
Thermodynamic Bethe Ansatz: diagonal

Ground-state energy exactly



[Zamolodchikov]





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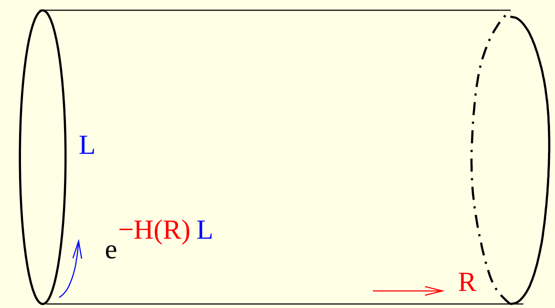

Exchange space and Euclidian time

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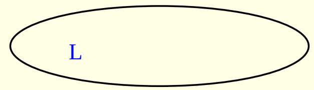

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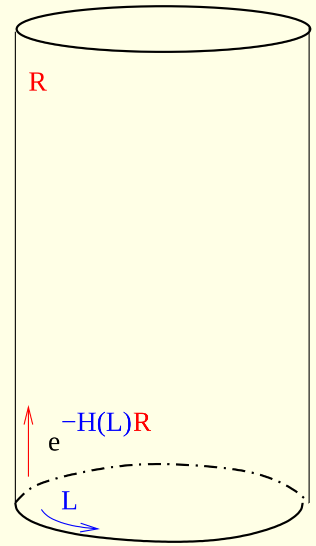

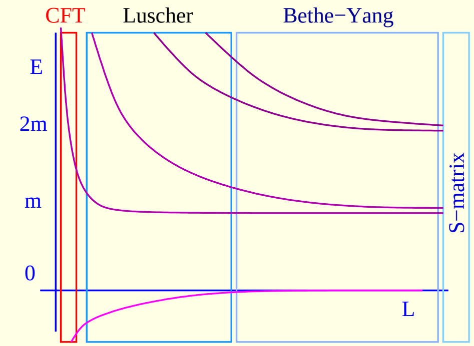

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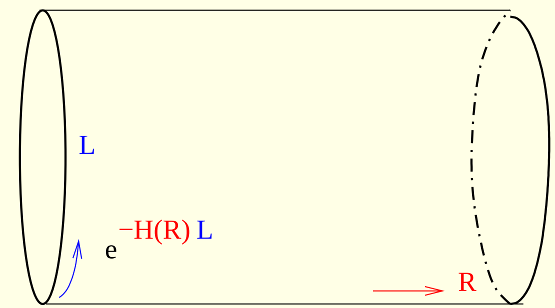

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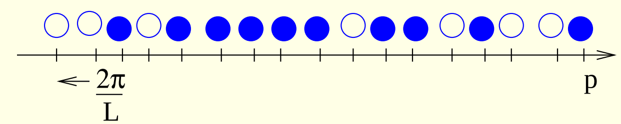

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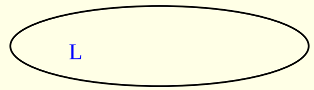
Ground-state energy exactly



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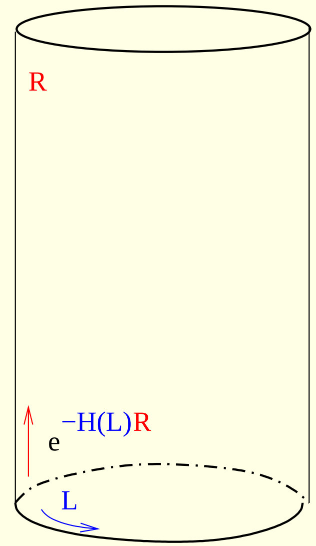

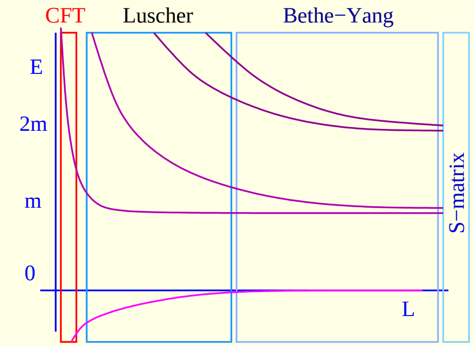

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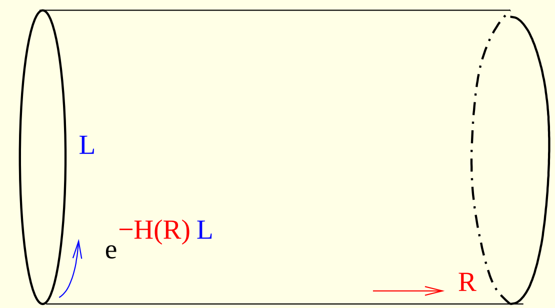

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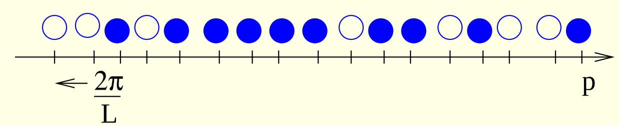

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Thermodynamic Bethe Ansatz: diagonal

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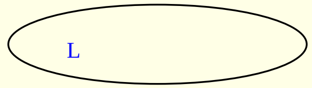

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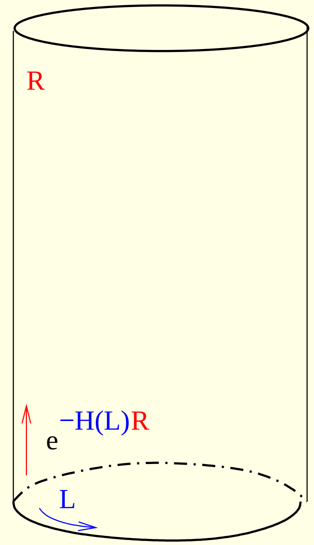

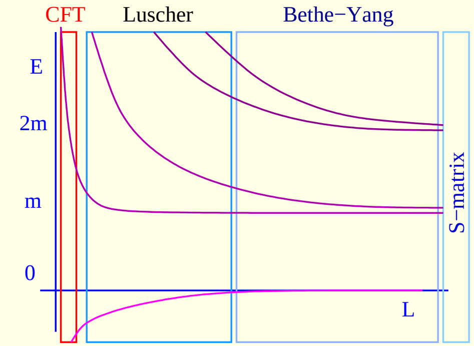

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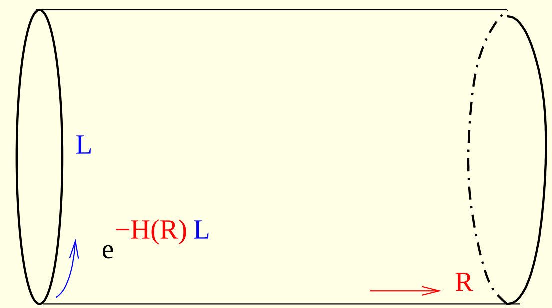

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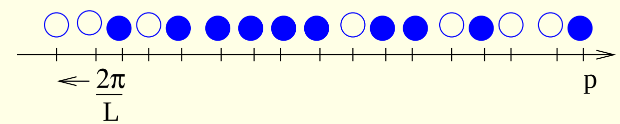

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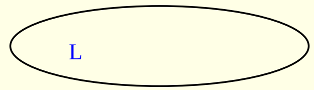
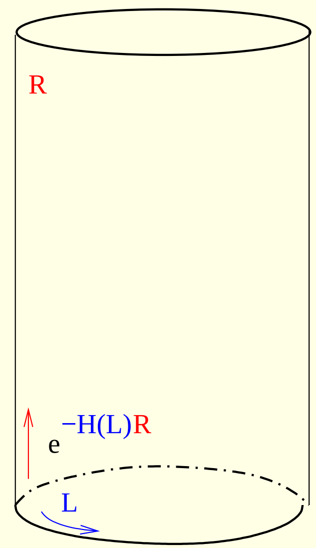
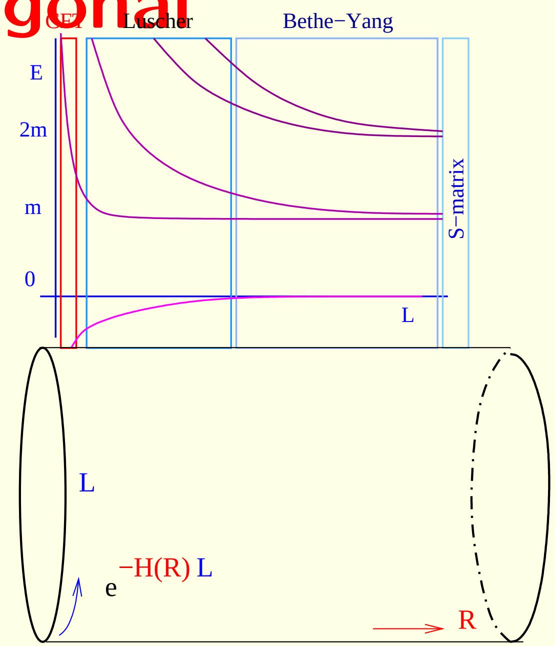
Thermodynamic Bethe Ansatz: diagonal

Ground-state energy exactly

[Zamolodchikov]







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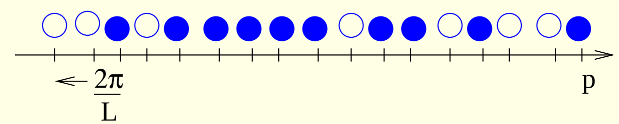

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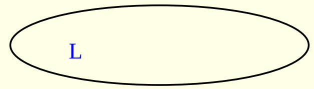
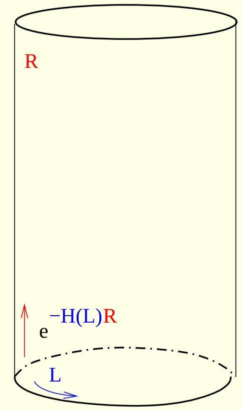
Saddle point : $\epsilon(p) = \ln\frac{\rho_h(p)}{\rho(p)}$ $\boxed{\epsilon(p) = E(p)L + \int \frac{dp}{2\pi} i d_p \log S(p',p)\log(1+e^{-\epsilon(p^{'})})}$

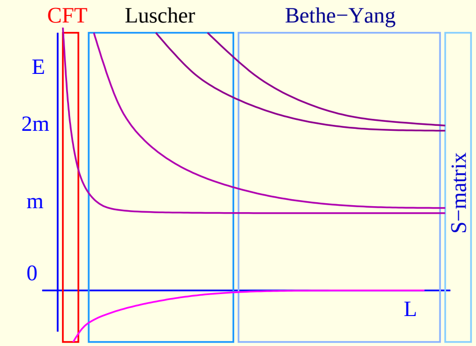
Ground state energy exactly: $\boxed{E_0(L) = -\int\frac{dp}{2\pi}\log(1+e^{-\epsilon(p)})}$ Lee-Yang, sh-G

Thermodynamic Bethe Ansatz: non-diagonal

Ground-state energy exactly







[Tateo]

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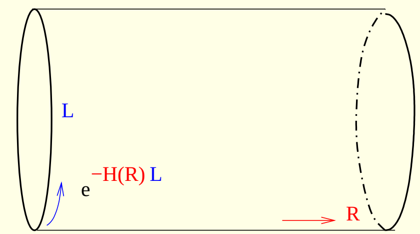

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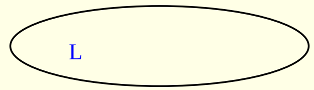

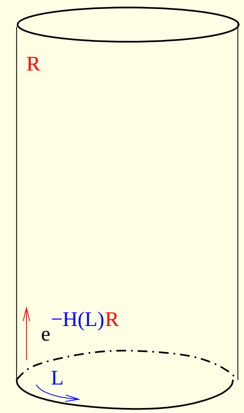

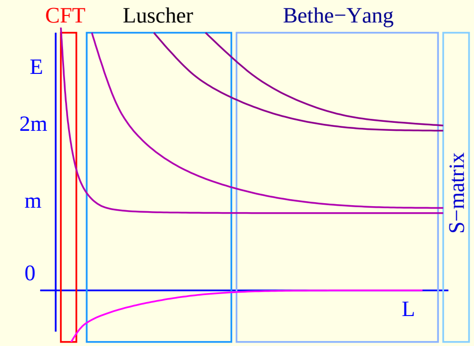

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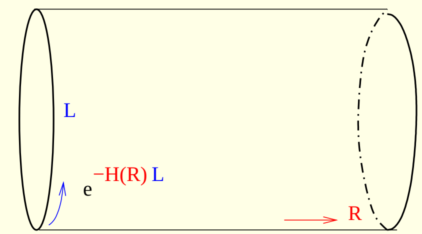

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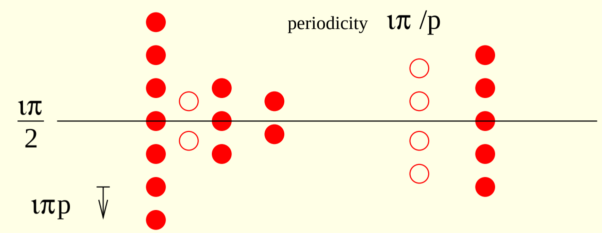
Finite particle/hole + Bethe root density $\rho^0,\rho^0_h,\rho^i,\rho^i_h$:



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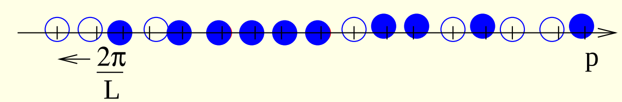

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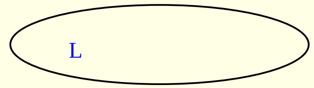

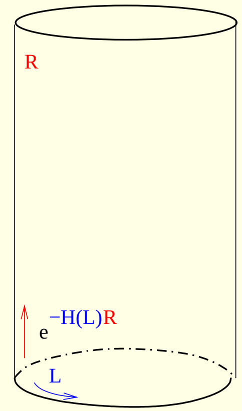

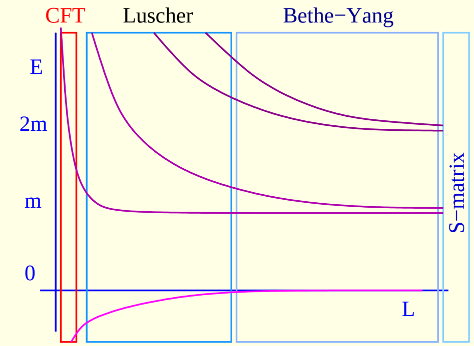

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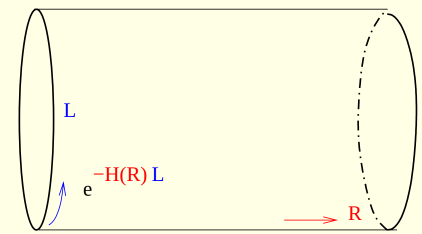

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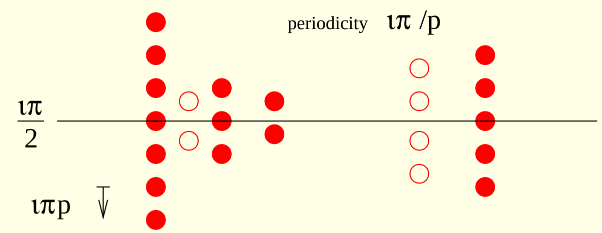

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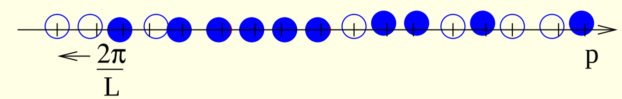

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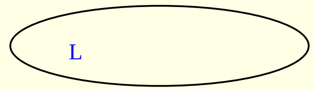

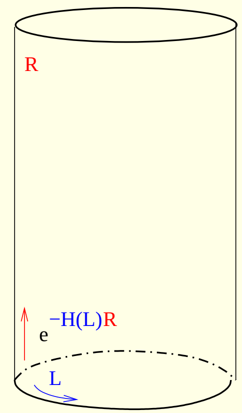

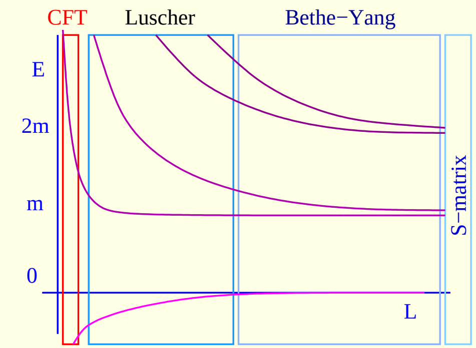

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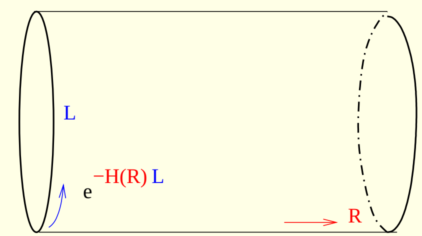

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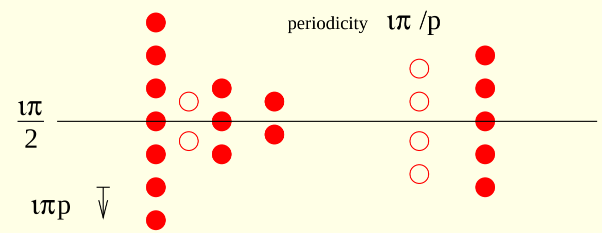

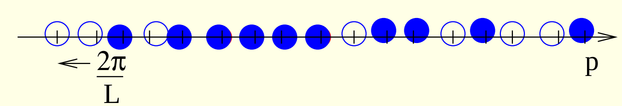

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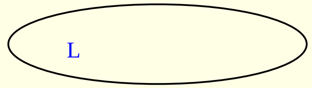

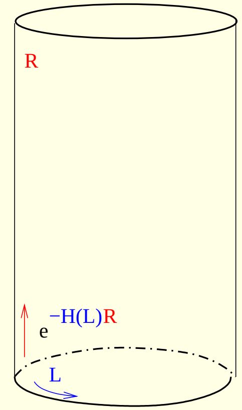

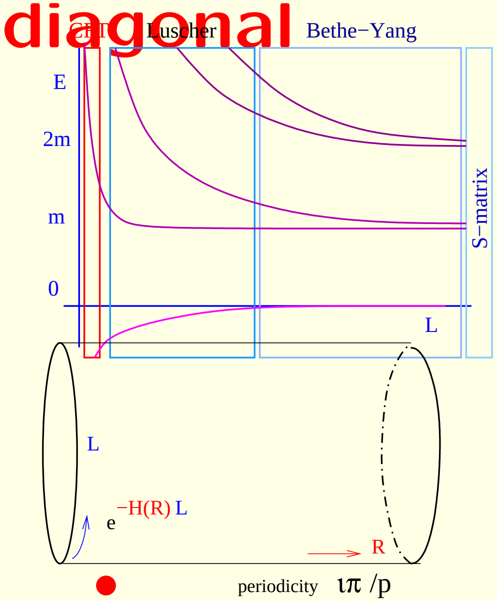

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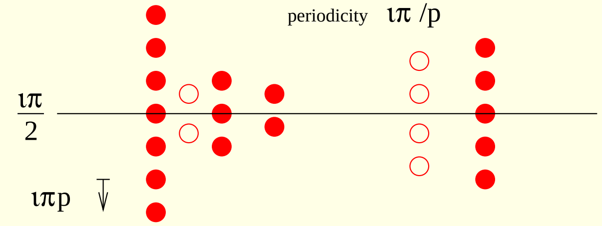

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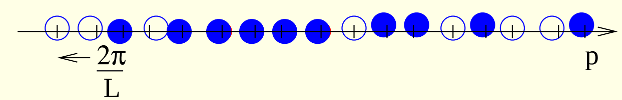

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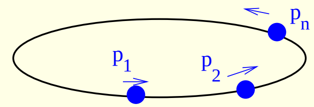
Saddle point : $\epsilon^i(\theta) = -\ln\frac{\rho^i(p)}{\rho^i_h(p)}$ $\boxed{\epsilon^j(\theta) = \delta_0^j E(p)L - \int K_i^j(p',p)\log(1+e^{-\epsilon^i(p')})dp'}$

Ground state energy exactly: $\boxed{E_0(L) = -\int\frac{dp}{2\pi}\log(1+e^{-\epsilon_0(\theta)})d\theta}$

Thermodynamic Bethe Ansatz: excited diagonal

Excited state energy exactly



[an idea]

Thermodynamic Bethe Ansatz: excited diagonal

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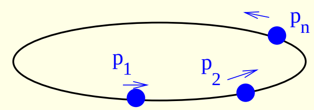

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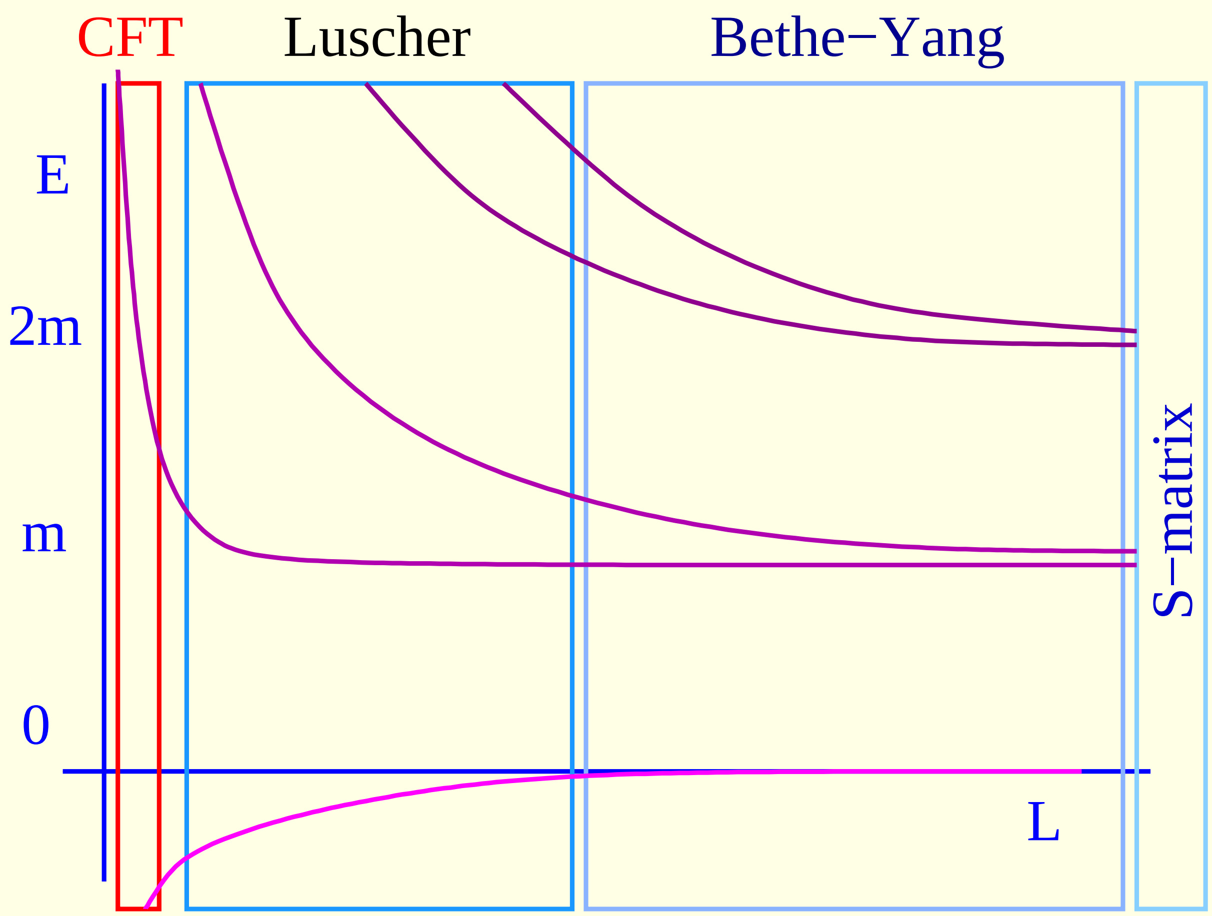

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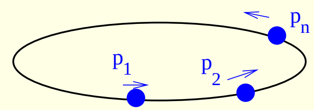

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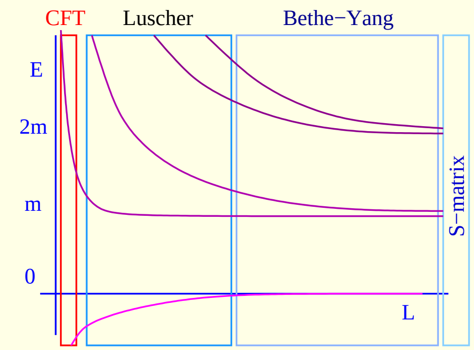

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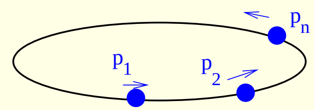

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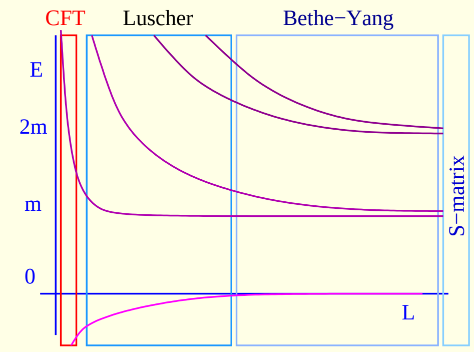
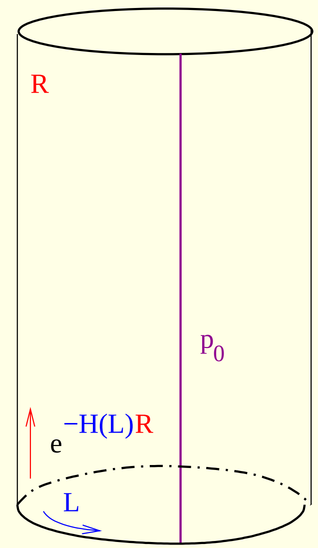


Particles are like defects:



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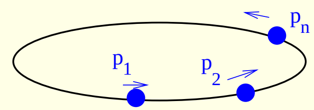

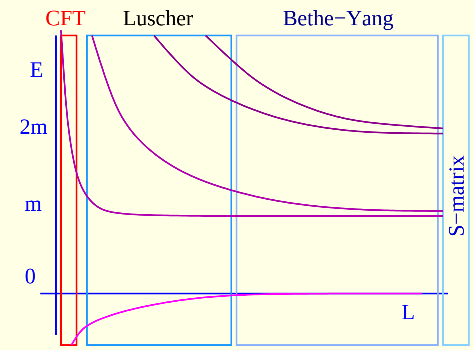

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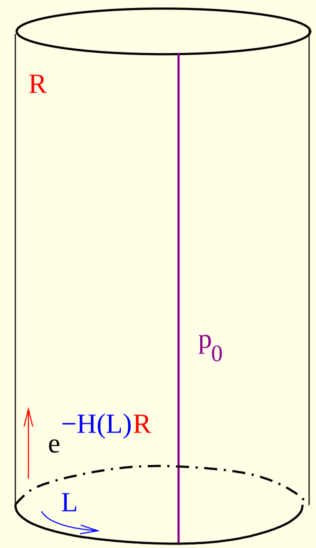

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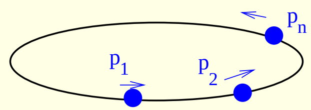
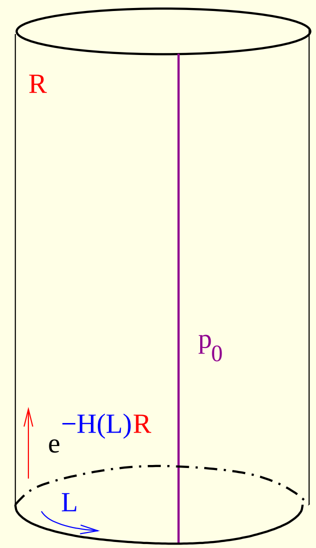
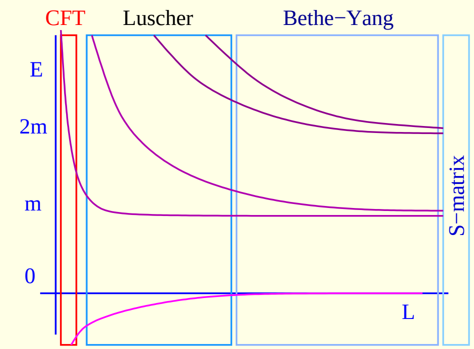
Thermodynamic Bethe Ansatz: excited diagonal

Excited state energy exactly

[an idea]







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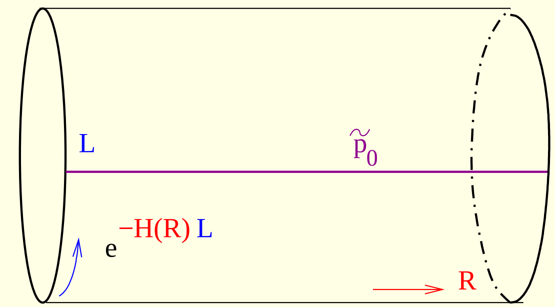

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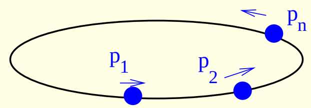

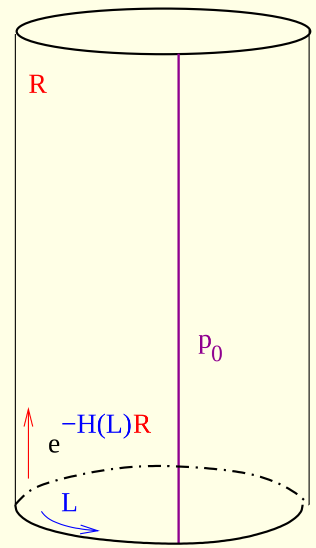

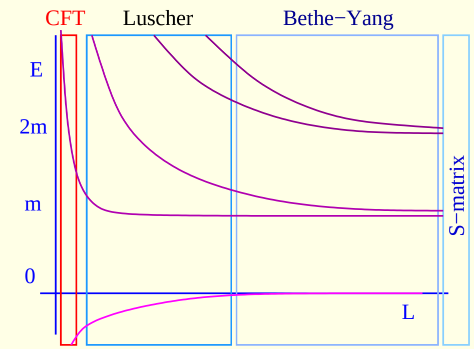

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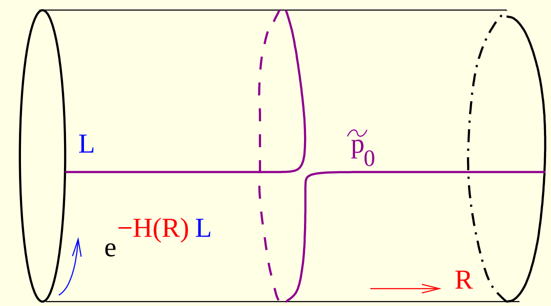

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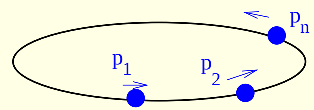

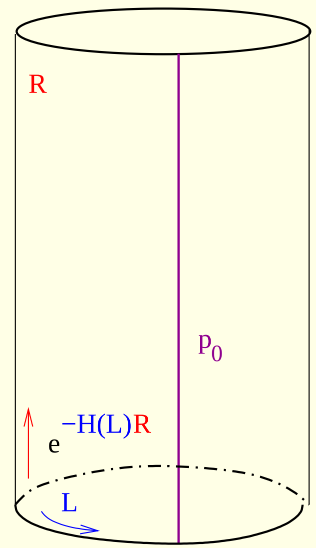

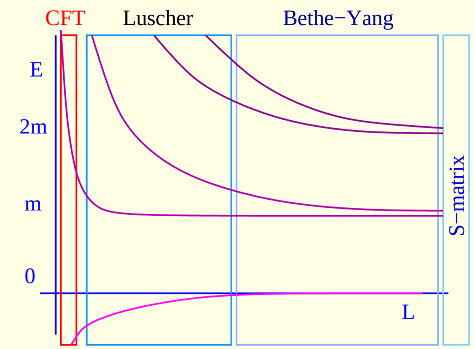

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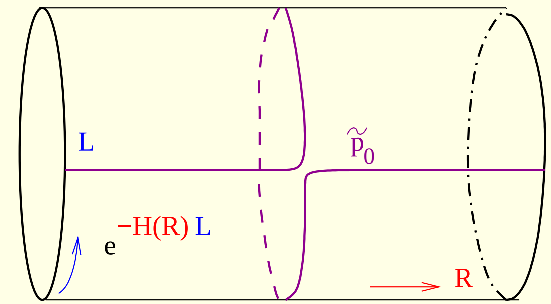

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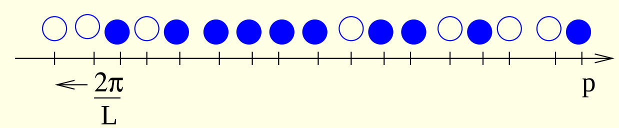

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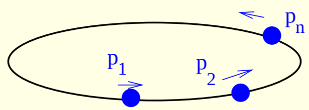
Thermodynamic Bethe Ansatz: excited diagonal

Excited state energy exactly



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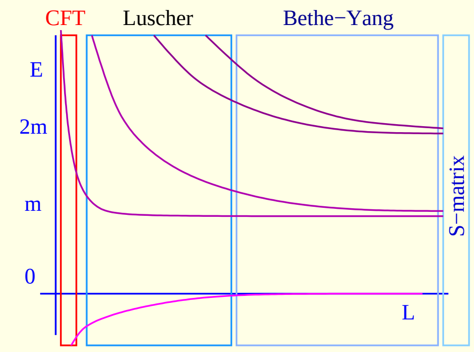

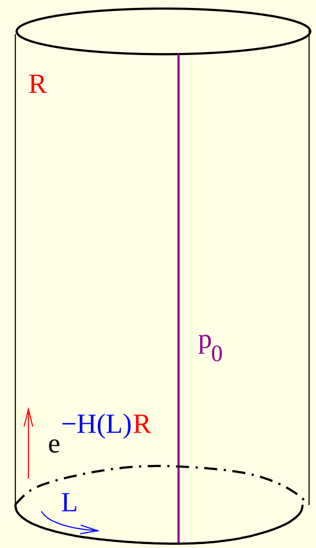

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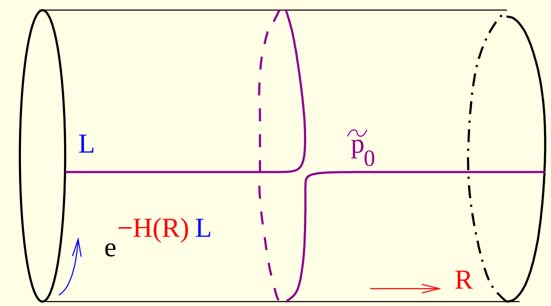

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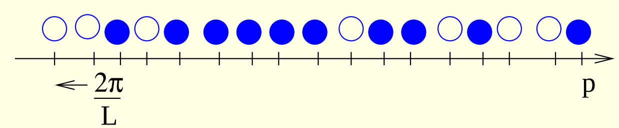

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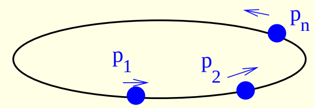
$$p_j R + \sum_k \frac{1}{i}\log S(p_j,p_k) = (2n+1)i\pi \quad \longrightarrow \quad R + \int(-id_p \log S(p,p'))\rho(p^{'})dp^{'} = 2\pi(\rho+\rho_h)$$

Thermodynamic Bethe Ansatz: excited diagonal

Excited state energy exactly



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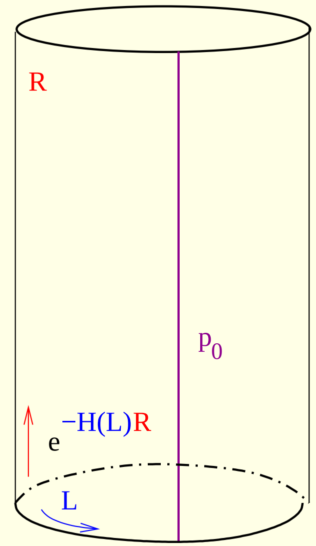

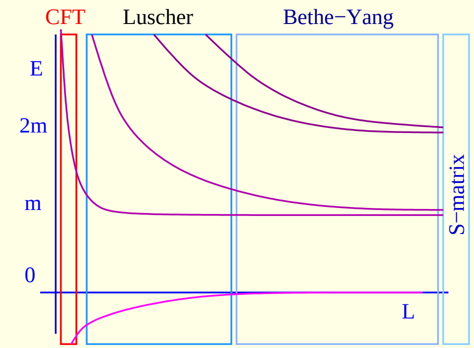

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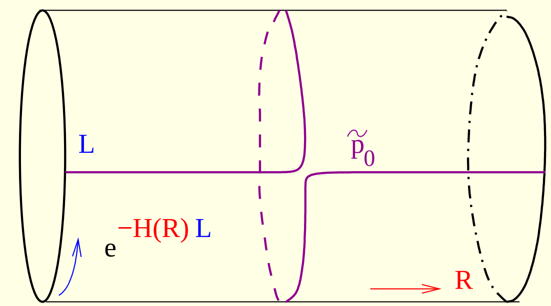

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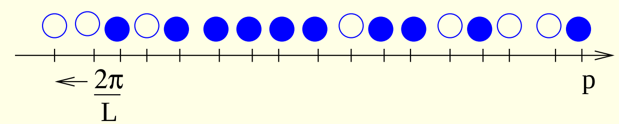

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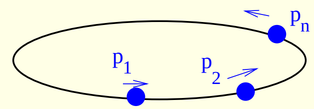
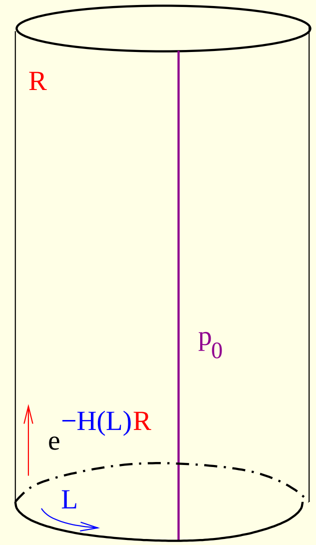
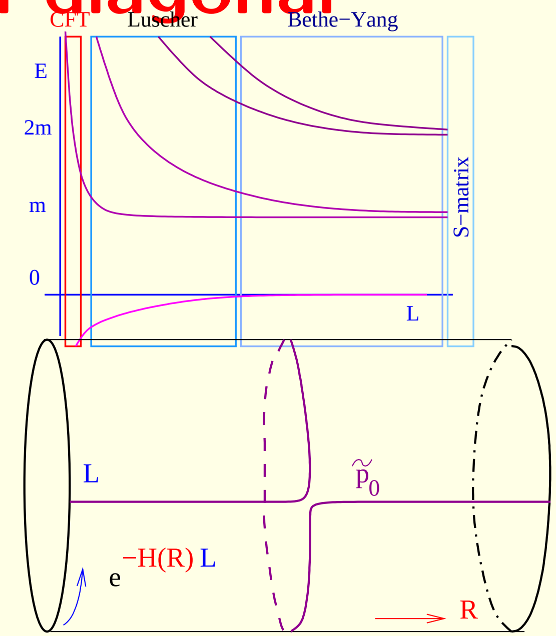
Thermodynamic Bethe Ansatz: excited diagonal

Excited state energy exactly

[an idea]







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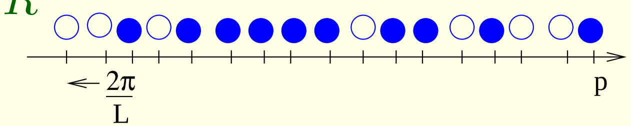

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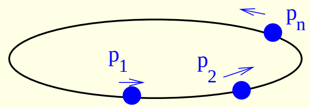
Excited state energy exactly: $E_d(L) = E(\tilde{p}_0) - \int \frac{dp}{2\pi}\log(1+e^{-\epsilon(p)})$ sh-G

Excited states TBA, Y-system: diagonal

Excited states exactly 

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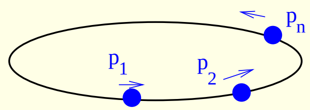

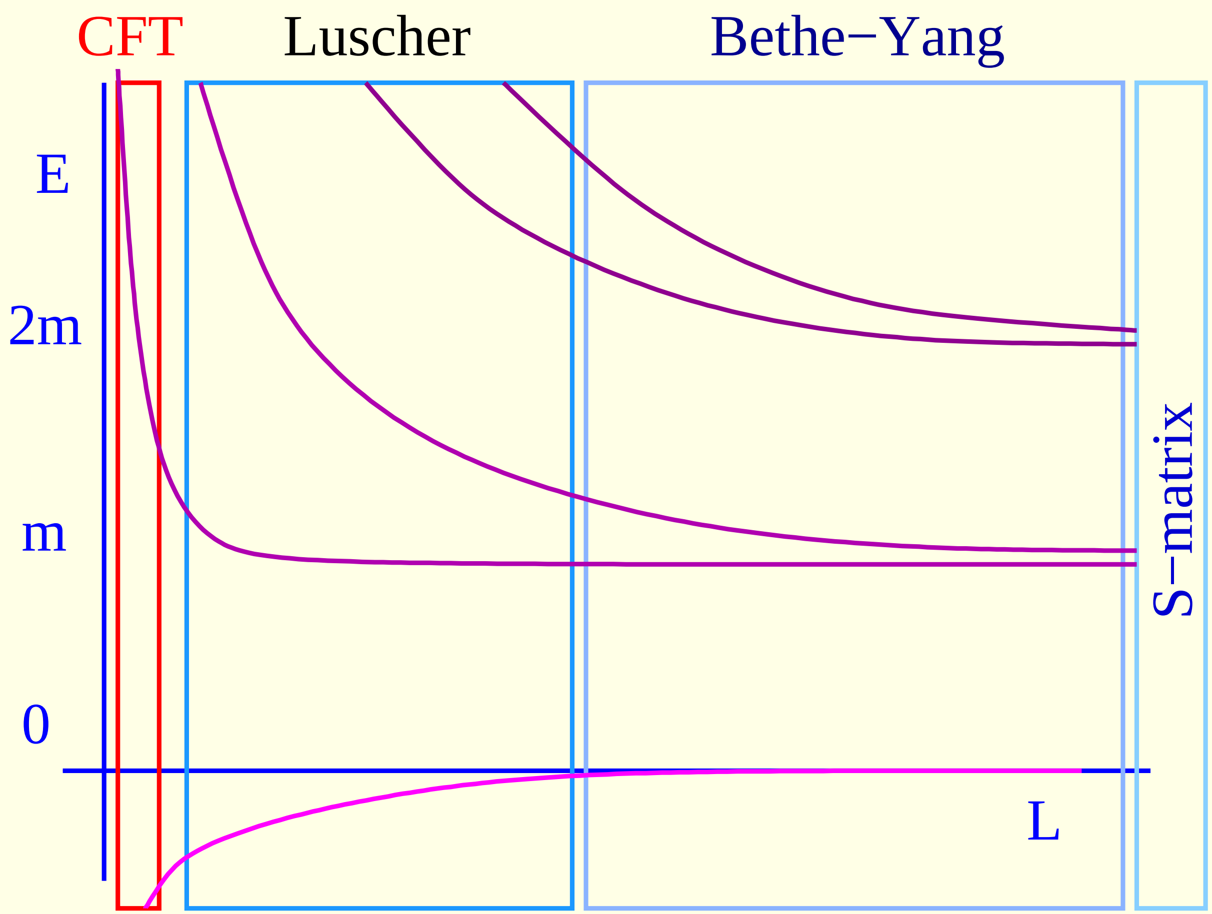

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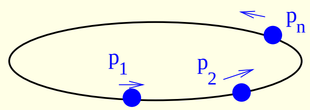

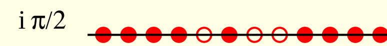

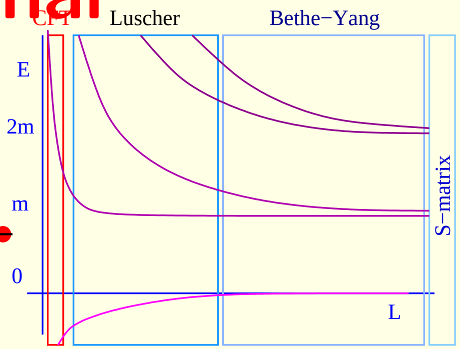

By lattice regularization: sinh-Gordon [Teschner]

$$\epsilon(\theta) = mL\cosh\theta \qquad + \int \frac{d\theta'}{2\pi} i d_\theta \log S(\theta-\theta') \log(1+e^{-\epsilon(\theta')})$$

$$E_{\{n_j\}}(L) = \qquad -m\int \frac{d\theta}{2\pi}\cosh\theta \log(1+e^{-\epsilon(\theta)})$$

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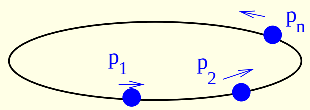

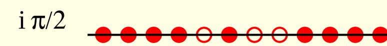

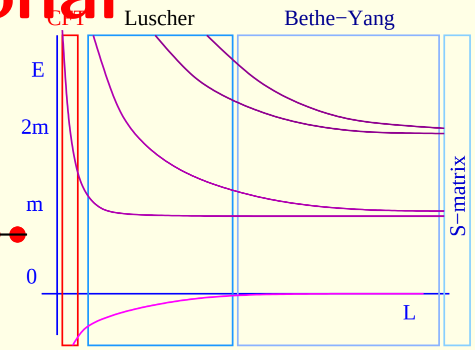

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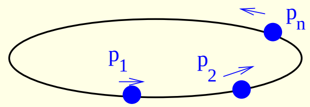

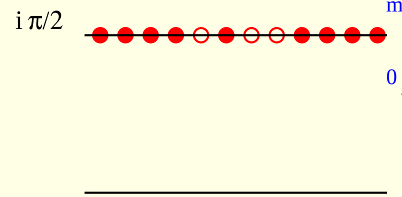

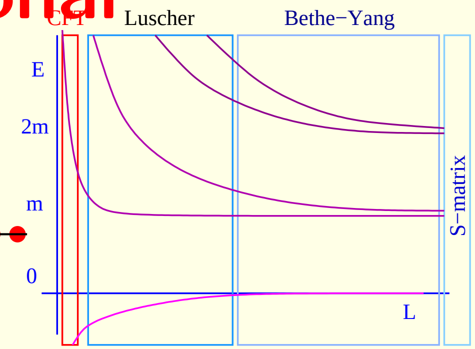

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$$E_{\{n_j\}}(L) = \frac{m}{i}\sum \sinh\theta_j - m\int \frac{d\theta}{2\pi}\cosh\theta\,\log(1+e^{-\epsilon(\theta)})$$

Excited states TBA, Y-system: diagonal

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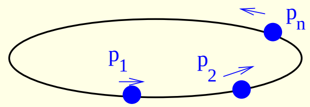

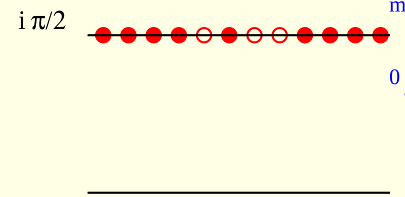

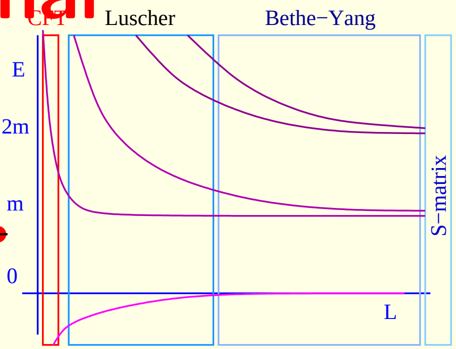

By lattice regularization: sinh-Gordon [Teschner]

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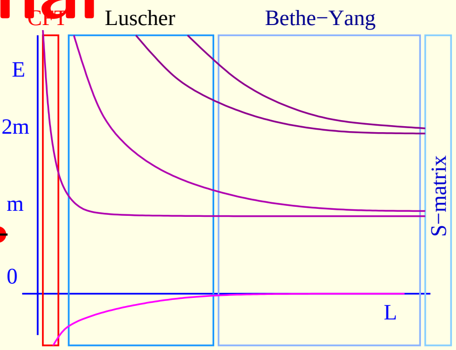
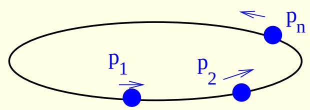
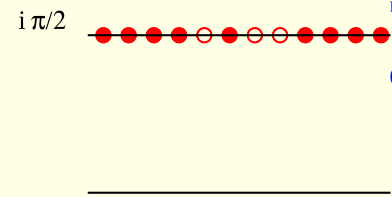
$$E_{\{n_j\}}(L) = \frac{m}{i}\sum \sinh\theta_j - m\int\frac{d\theta}{2\pi}\cosh\theta\,\log(1+e^{-\epsilon(\theta)})\,;\; Y(\theta_j) = e^{\epsilon(\theta_j)} = -1$$

Excited states TBA, Y-system: diagonal



Excited states exactly





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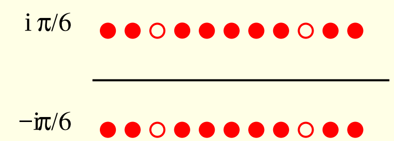

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$$\epsilon(\theta) = mL\cosh\theta \qquad + \int_{-\infty}^{\infty} \frac{d\theta'}{2\pi} i d_\theta \log S(\theta-\theta') \log(1+e^{-\epsilon(\theta')})$$

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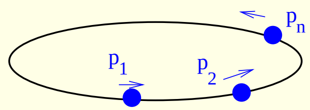
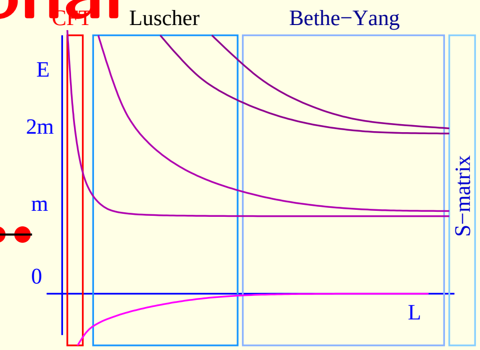
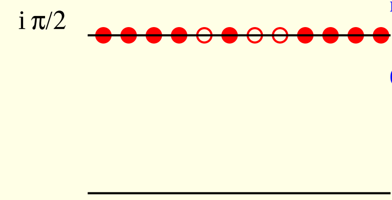
Lüscher corrections: differ by μ term

Excited states TBA, Y-system: diagonal

Excited states exactly







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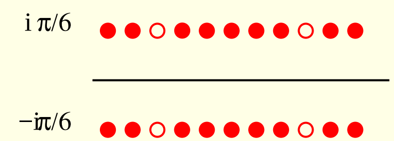

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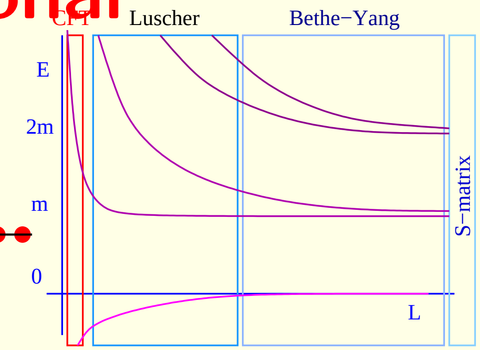
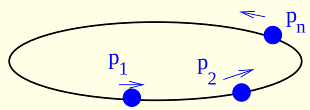
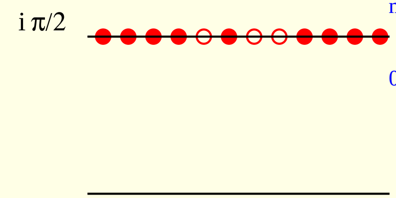


Excited states exactly





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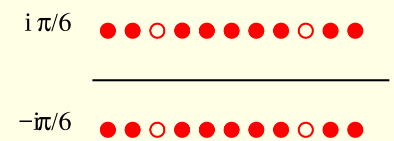

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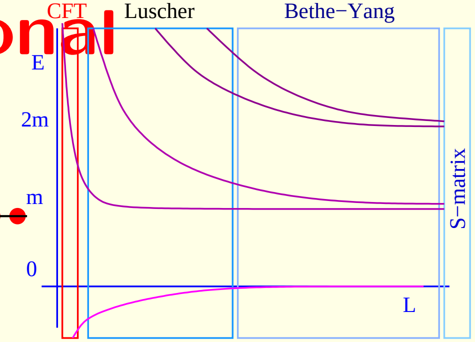

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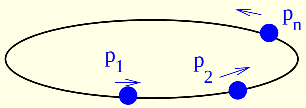

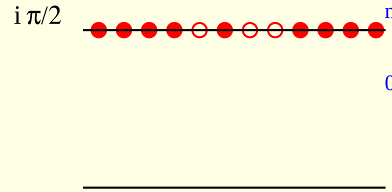

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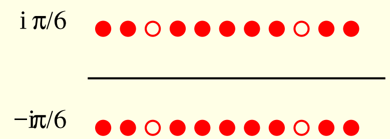

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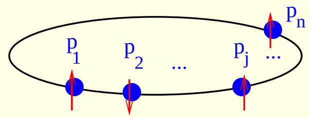
Lüscher corrections: differ by μ term

$$S(\theta-\tfrac{i\pi}{3})S(\theta+\tfrac{i\pi}{3}) = S(\theta) \to Y(\theta+\tfrac{i\pi}{3})Y(\theta-\tfrac{i\pi}{3}) = 1+Y(\theta)$$

Y-system+analyticity=TBA $\leftrightarrow$ scalar . Matrix [Bazhanov,Lukyanov,Zamolodchikov]

Excited states TBA, Y-system: Non-diagonal

Excited states exactly 

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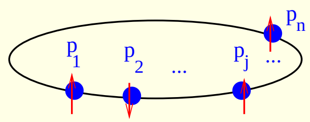

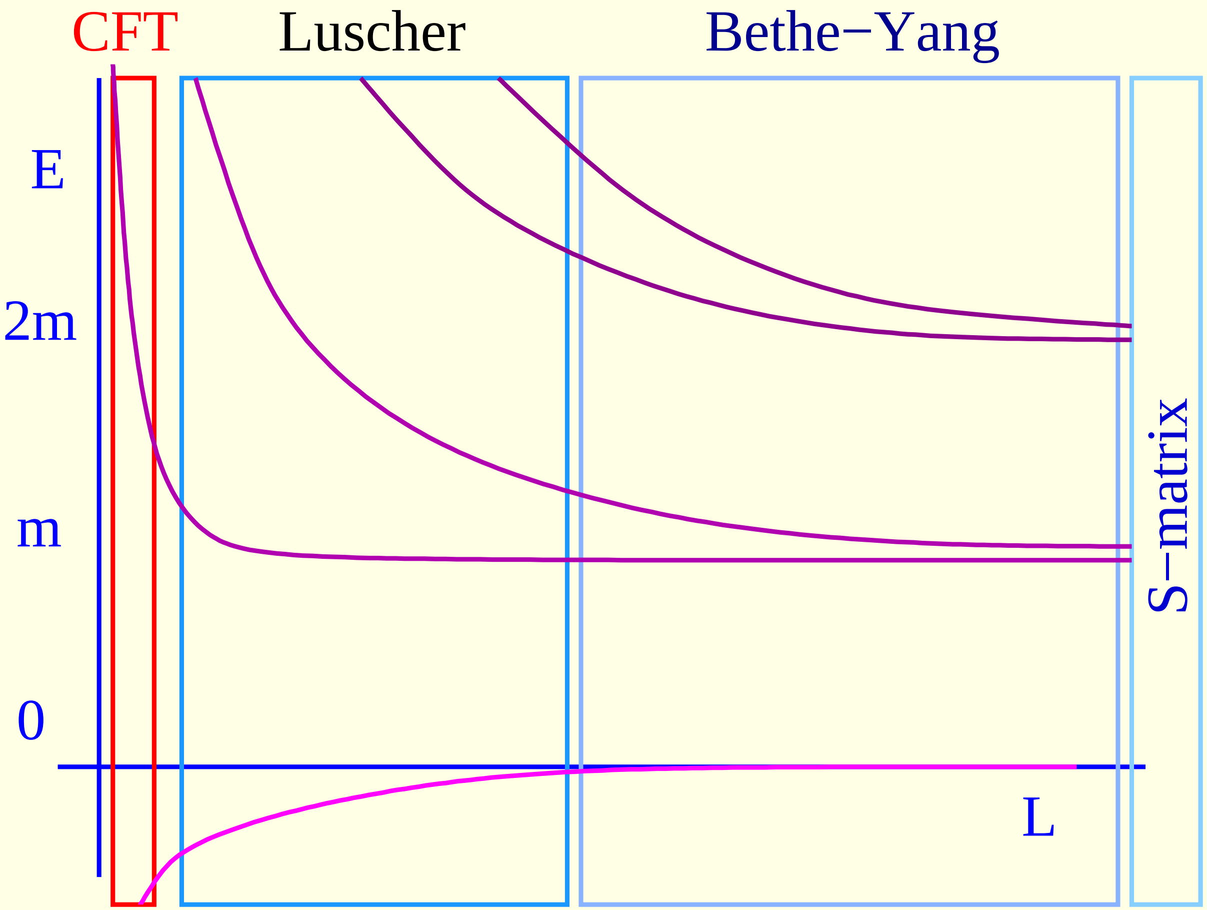

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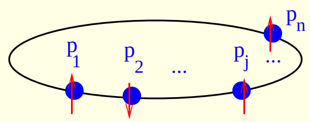

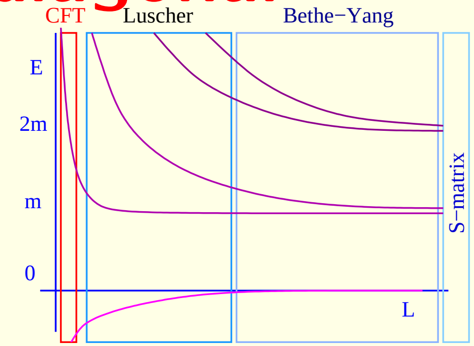

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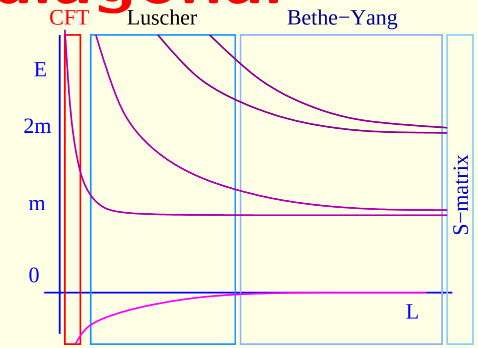

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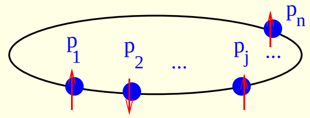
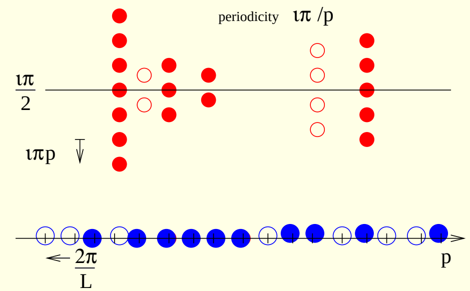
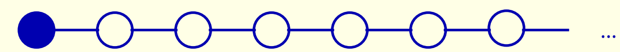




Y-system: sine-Gordon

$$Y_s(\theta+\frac{i\pi p}{2})Y_s(\theta-\frac{i\pi p}{2})=(1+Y_{s-1})(1+Y_{s+1})$$



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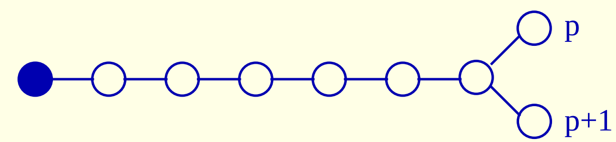

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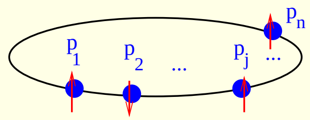

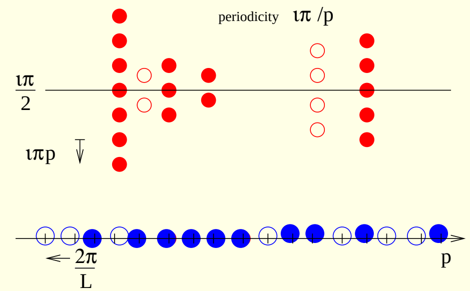

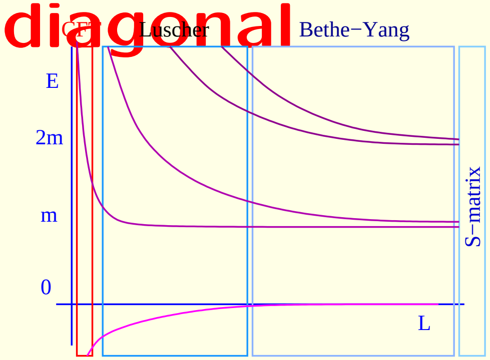

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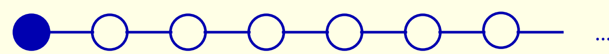

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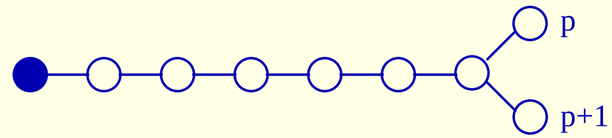

Excited states: analyticity from Lüscher [Balog, Hegedus]

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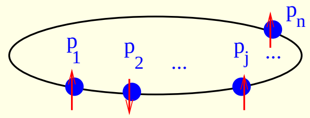

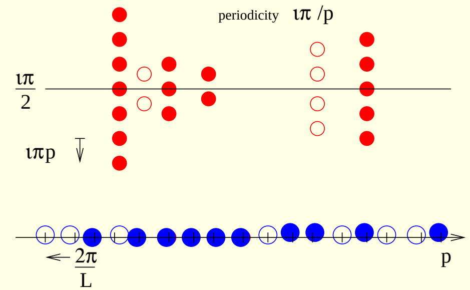

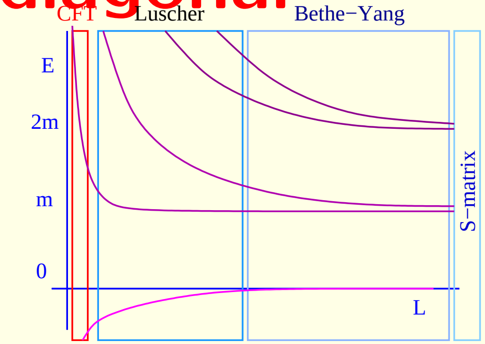

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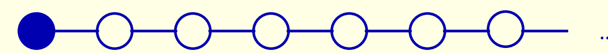

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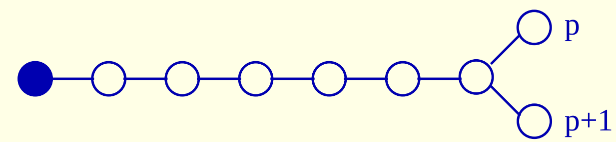

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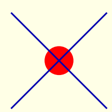
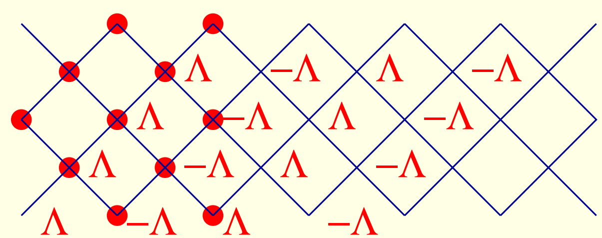
Lattice regularization:

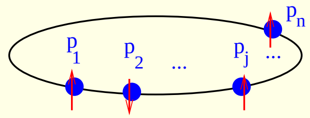
[Destri,de Vega,Ravanini,...]

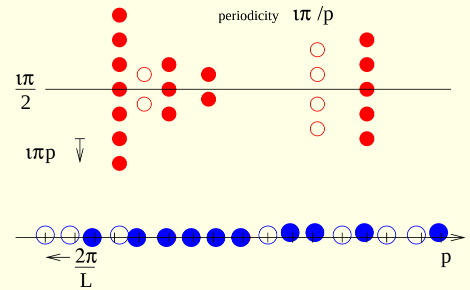


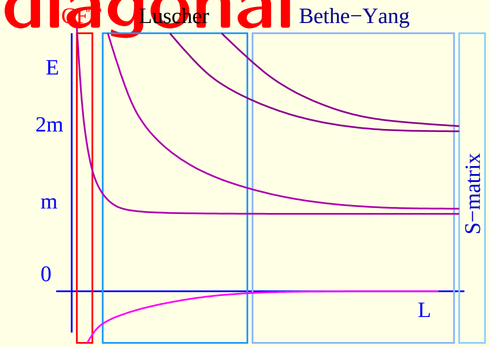


Excited states TBA, Y-system: Non-diagonal

Excited states exactly







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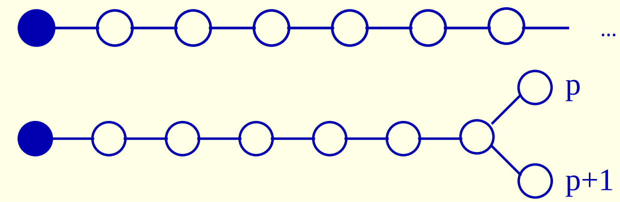

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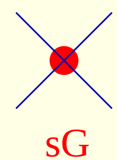
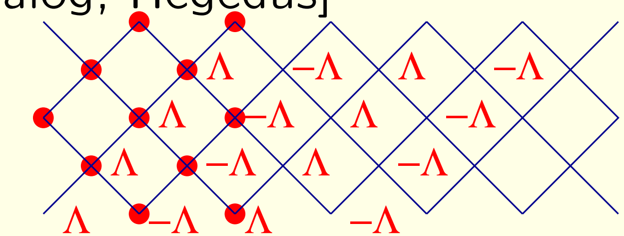




$Z(\theta)=ML\sinh\theta+\text{source}(\theta|\{\theta_k\})+2\Im m\int dxG(\theta-x-i\epsilon)\log\left[1-e^{iZ(x+i\epsilon)}\right]$

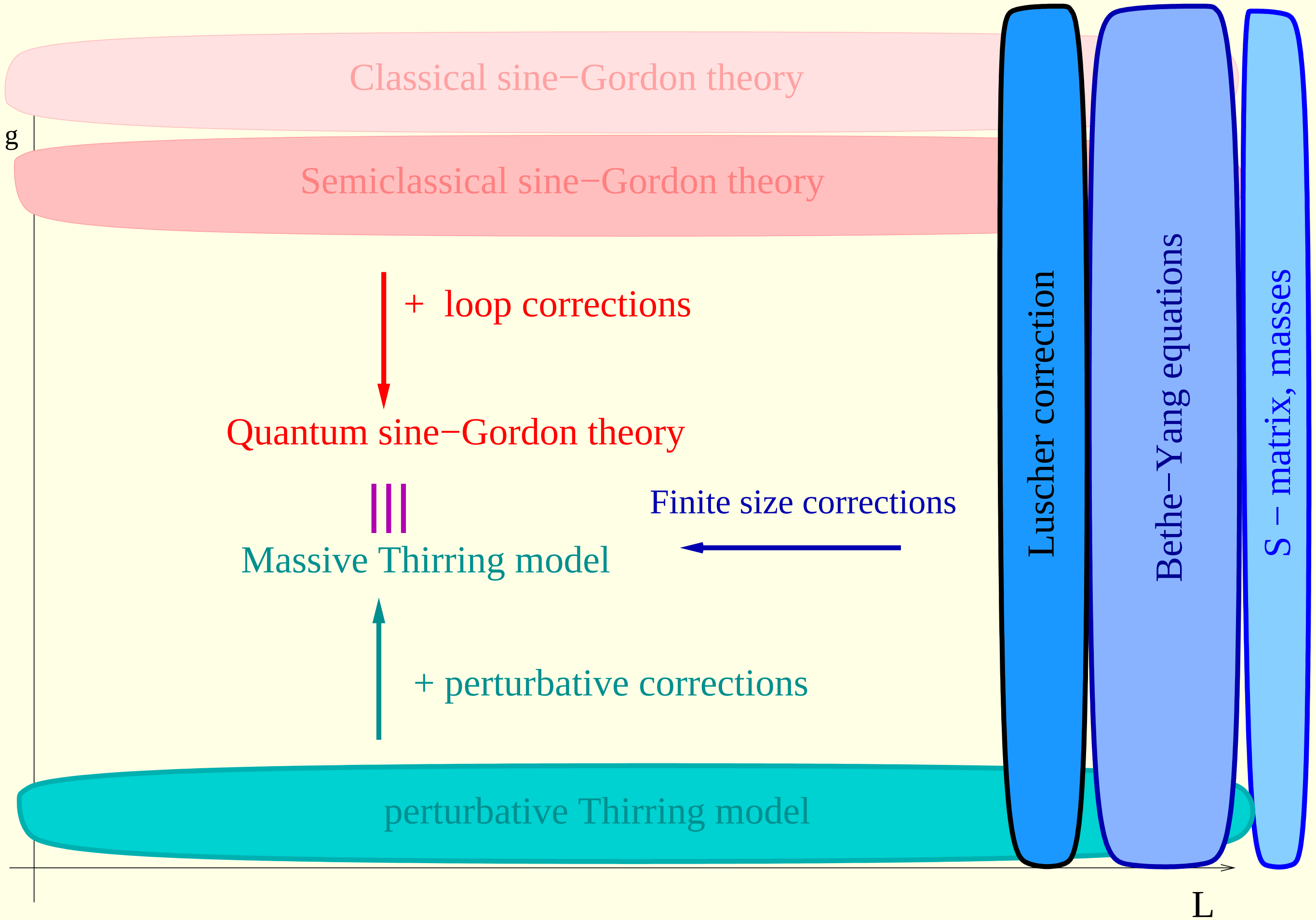
$\text{source}(\theta|\{\theta_k\})=-i\sum_k\log S_{++}^{++}(\theta-\theta_k)$ kernel: $G(\theta)=-i\partial_\theta\log S_{++}^{++}(\theta)$

Energy: $E=M\sum_k\cosh\theta_k-2M\Im m\int dxG(\theta+i\epsilon)\log\left[1-e^{iZ(x+i\epsilon)}\right]$

Bethe-Yang $e^{iZ(\theta_k)}=-1$

Sine-Gordon/massive Thirring duality

$$\mathcal{L}_{SG} = \frac{1}{2}\partial_\nu\Phi\partial^\nu\Phi + \frac{m^2}{\beta^2} : \cos(\beta\Phi) : \qquad 0 < \beta^2 < 8\pi,$$

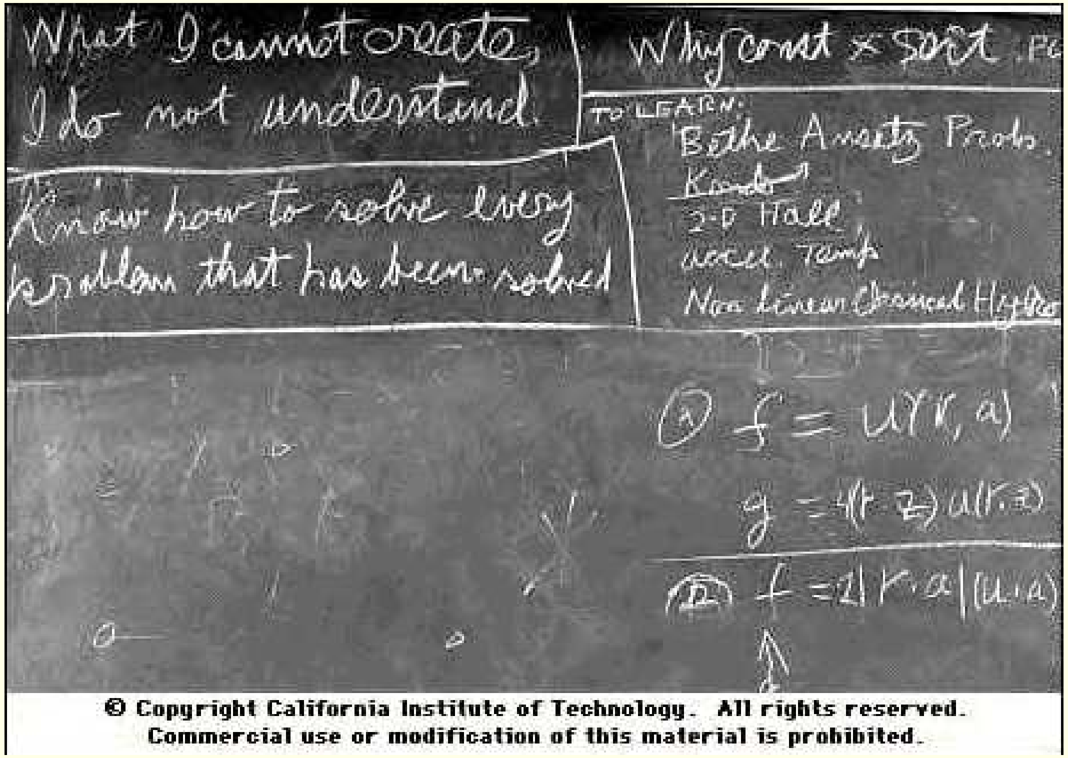


strong-weak duality:

$$1 + \frac{g}{4\pi} = \frac{4\pi}{\beta^2} = \frac{p+1}{2p}$$

$$\mathcal{L}_{MT} = \bar{\Psi}(i\gamma_\nu\partial^\nu + m_0)\Psi - \frac{g}{2}\bar{\Psi}\gamma^\nu\Psi\bar{\Psi}\gamma_\nu\Psi$$

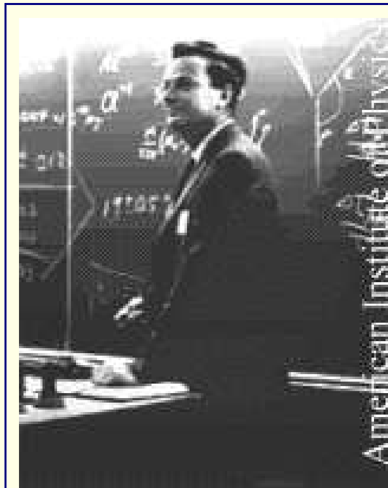
AdS/CFT duality: t' Hooft $\longleftrightarrow$ Integrability in QCD: Feynman



What I cannot create I do not understand.

To learn: Bethe Ansatz Probs., $\leftarrow$ Kondo, 2D

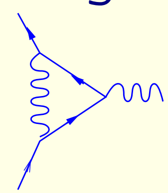
Hall, accel temp, Non linear classical Hydro



Richard Feynman
(1918–1988)

 1965

QED:
Feynman graphs



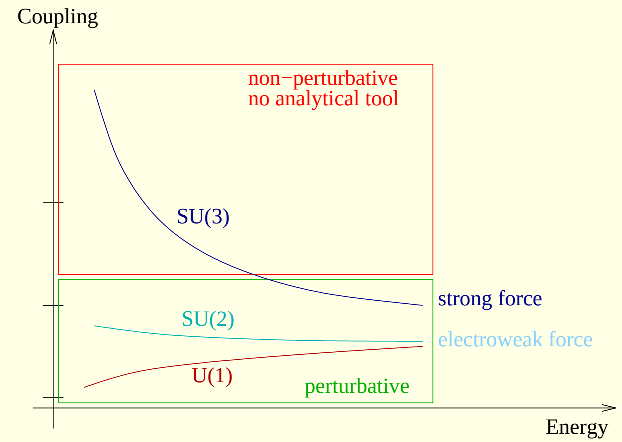
Strong interaction?

CFT: maximally supersymmetric gauge theory

Feynman: *If you want to learn about nature, to appreciate nature, it is necessary to understand the language she speaks in.*

Fundamental interactions: gauge theory

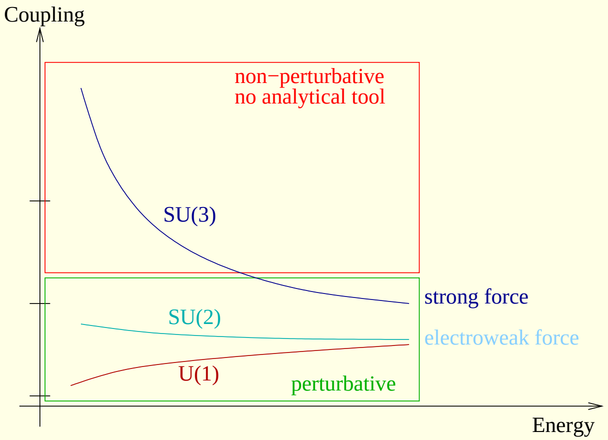
interaction	particles	gauge theory
electromagnetic	photon+electron	$U(1)$
electroweak	$W^{\pm}, Z$ μ,ν+Higgs	$SU(2) \times U(1)$
strong	gluon+quarks	$SU(3)$



only analytical tool: perturbation theory

CFT: maximally supersymmetric gauge theory

Feynman: *If you want to learn about nature, to appreciate nature, it is necessary to understand the language she speaks in.*



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only analytical tool: perturbation theory

maximally supersymmetric gauge theory (harmonic oscillator)

interaction	particles	gauge theory
$\mathcal{N} = 4$ supersymm.	gluon+quarks+scalars	$SU(N)$

all fields $N^2 - 1$ component matrix

$\Psi_{1,2,3,4}$

$A_\mu \nearrow \Psi_{1,2,3,4} \searrow \Phi_{1,2,3,4,5,6}$

$A_\mu \searrow \overline{\Psi}_{1,2,3,4} \nearrow \Phi_{1,2,3,4,5,6}$

$\overline{\Psi}_{1,2,3,4}$

$$\mathcal{L} = \frac{2}{g_{YM}^2} \int d^4x \mathrm{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \overline{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \overline{\Psi} [\Phi, \Psi]$$

no running $\beta = 0 \rightarrow$ CFT
no confinement
supersymmetric

heavy ion collision:
finite T $\rightarrow$ SUSY is broken
quark-gluon plasma is not confined

CFT: Observables

maximally supersymmetric gauge theory

A $\quad \Psi_{1,2,3,4}$ $\quad \overline{\Psi}_{1,2,3,4}$ $\quad \Phi_{1,2,3,4,5,6}$ fields $SU(N)$ matrices

$$\mathcal{S} = \frac{2}{g_{YM}^2}\int d^4x \,\mathrm{Tr}\left[-\tfrac{1}{4}F^2 - \tfrac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$$

$$V(\Phi,\Psi) = \tfrac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$$

observables

parameters: g_{YM}, N

observables: partition function

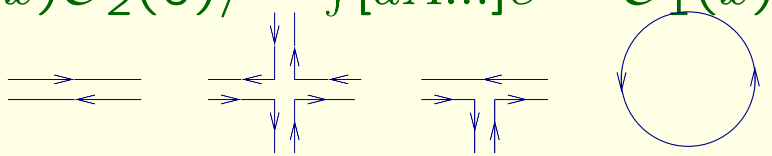
gauge-invariant operators

$\mathcal{O}(x) = \mathrm{Tr}(A^{L_1}\Psi^{L_2}\Phi^{L_3}..)$,det()

correlators: $\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle$

correlators: $\langle \mathcal{O}_1(x)\mathcal{O}_2(0)\rangle = \int [dA...]e^{-i\mathcal{S}}\mathcal{O}_1(x)\mathcal{O}_2(0) = \langle \mathcal{O}_1(x)\mathcal{O}_2(0)e^{-iV}\rangle_0$

perturbation:



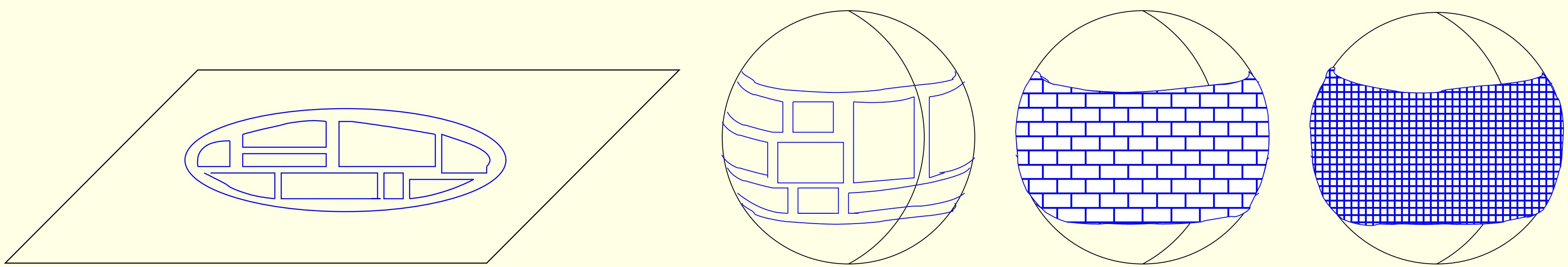
g_{YM}^2 $\quad g_{YM}^{-2}$ $\quad g_{YM}^{-2}$ $\quad N$

genus exp.

$g_{YM}^2 N^3 = N^2\lambda$, $\lambda = g_{YM}^2 N$

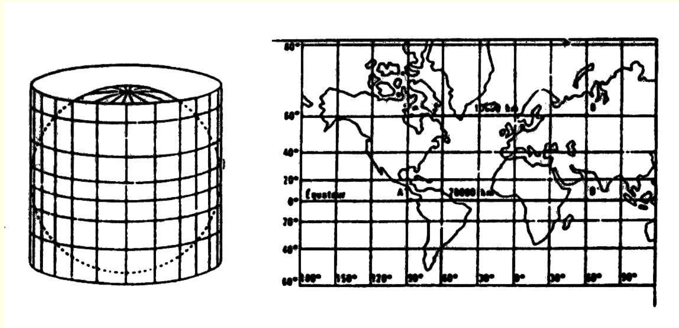
partition func. $\quad Z(\lambda,\frac{1}{N}) = N^2\sum_g(\frac{1}{N})^{2g}\sum_n \alpha(g,n)\lambda^n$ $\quad$ string interactions?

(t' Hooft)



AdS: string theory on Anti de Sitter

positively curved space



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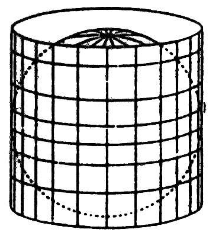
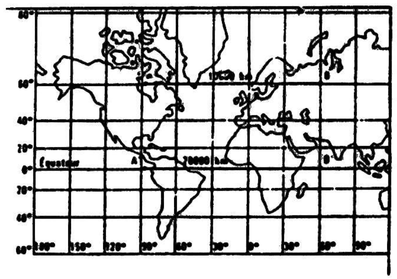
positively curved space Anti de Sitter: negatively curved space





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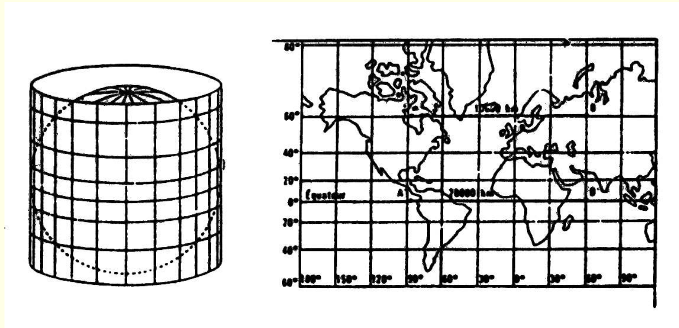

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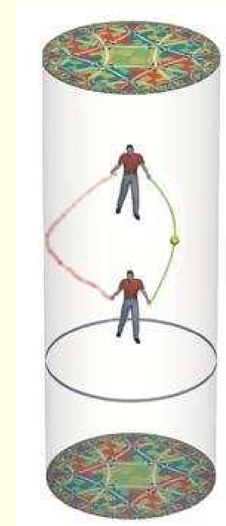

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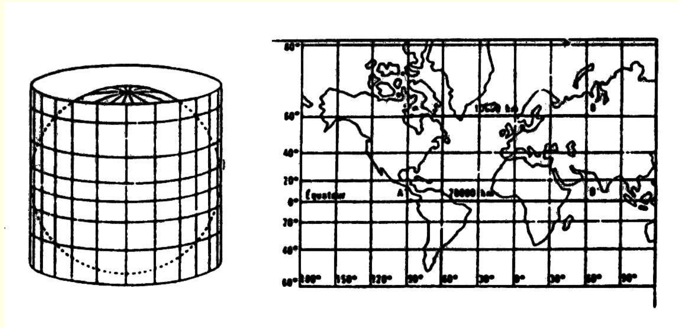

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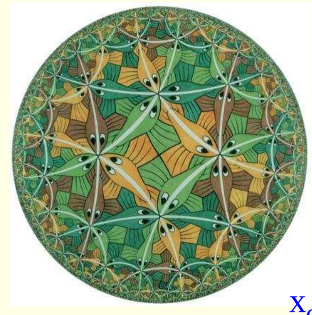
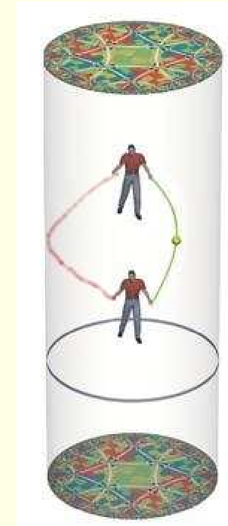
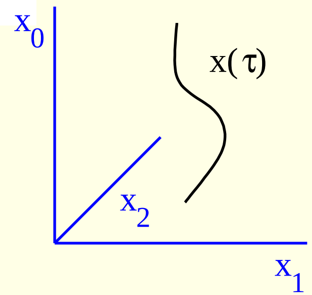




relativistic point particle: $ds^2 = -dx_0^2 + dx_1^2 + \ldots$

$S \propto$ worldline $\propto \int ds = \int \sqrt{\dot{x} \cdot \dot{x}} d\tau$



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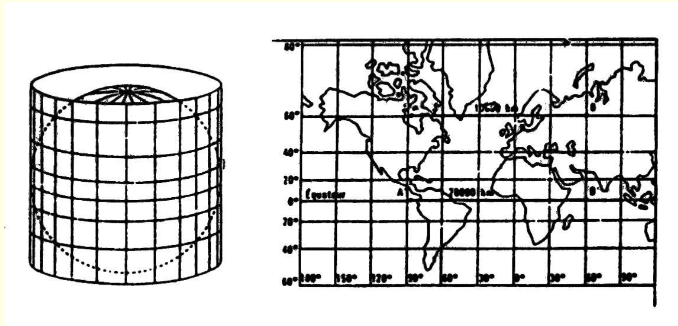

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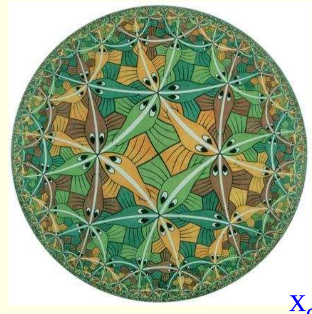

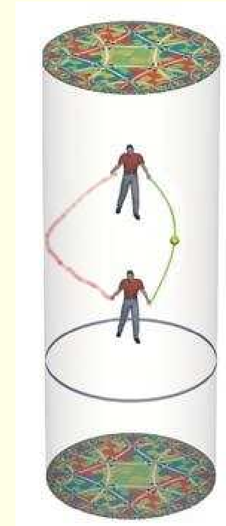

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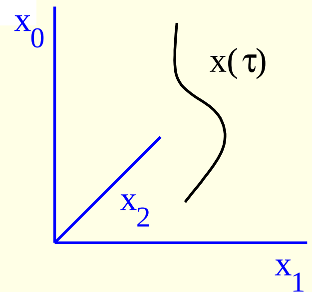

relativistic string: $ds^2 = -dx_0^2 + dx_1^2 + \ldots$

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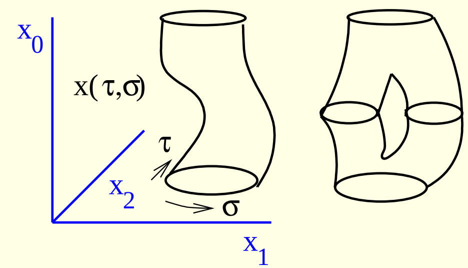

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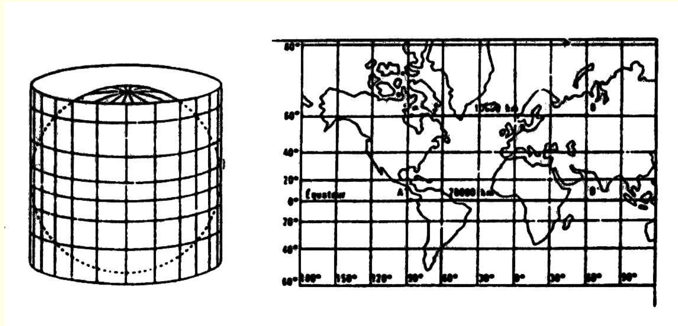

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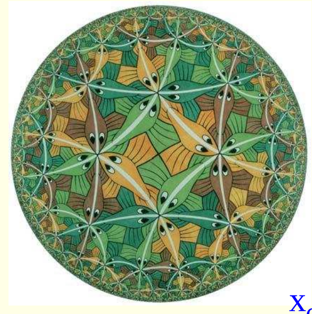

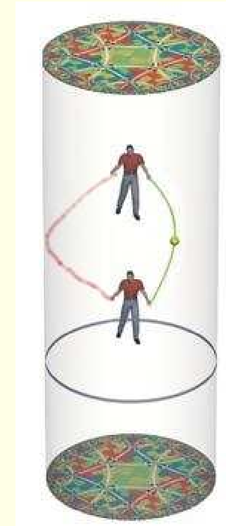

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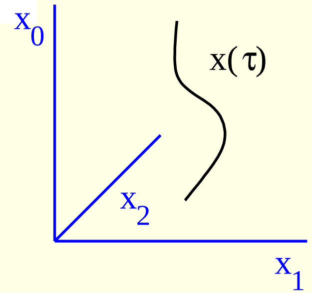

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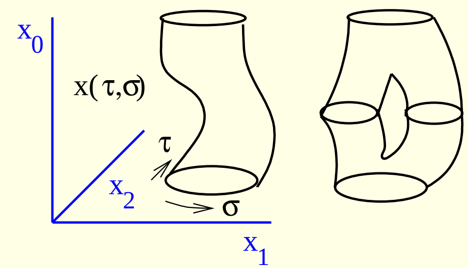
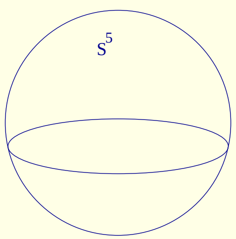
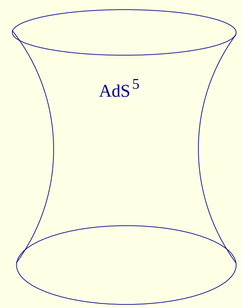




$S^5 : Y_0^2 + Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 + Y_5^2 = R^2$

$AdS_5 : -X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 - X_5^2 = -R^2$

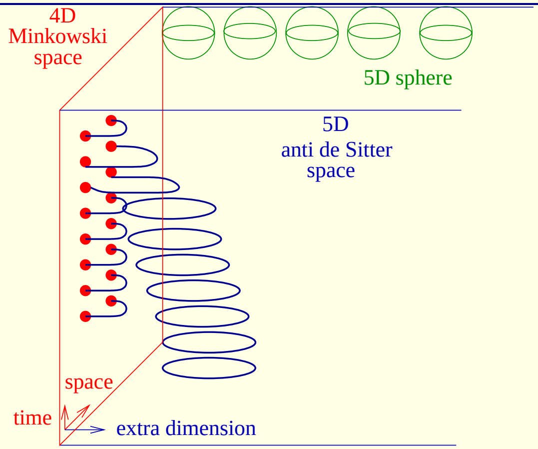


$$S = \frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M \right) + fermions$$

supercoset $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

AdS/CFT correspondence (Maldacena 1998)

II_B superstring on $AdS_5 \times S^5$



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$\equiv$

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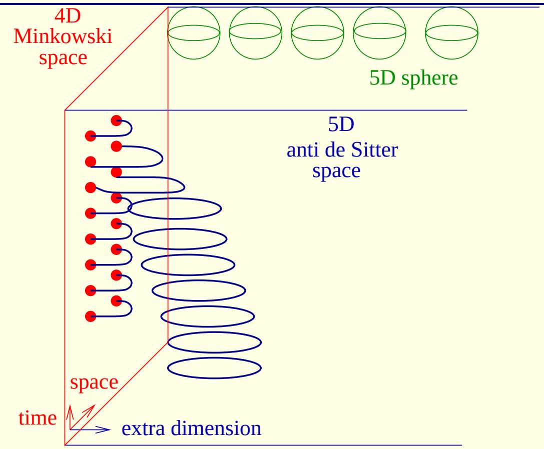

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Coupl.: $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, $g_s = \frac{\lambda}{N} \to 0$

2D QFT

String energy levels: $E(\lambda)$

$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$$

strong$\leftrightarrow$weak

$\Downarrow$

$\lambda = g_{YM}^2 N$, $N \to \infty$ planar

$$\langle \mathcal{O}_n(x) \mathcal{O}_m(0) \rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$$

Anomalous dim $\Delta(\lambda)$

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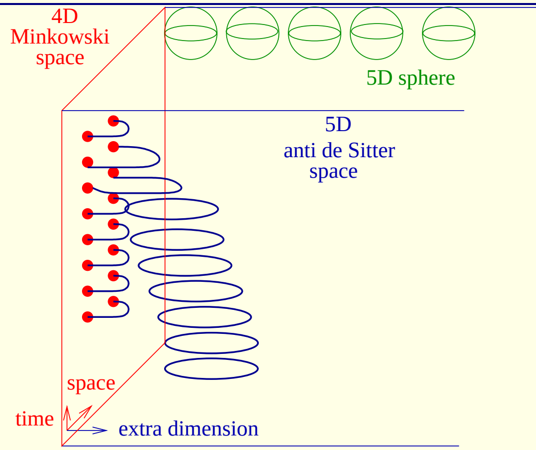

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2D integrable QFT

spectrum: $Q = 1, 2, \ldots, \infty$ dispersion: $\epsilon_Q(p) = \sqrt{Q^2 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}}$

Exact scattering matrix: $S_{Q_1 Q_2}(p_1, p_2, \lambda)$

Motivation: AdS/CFT

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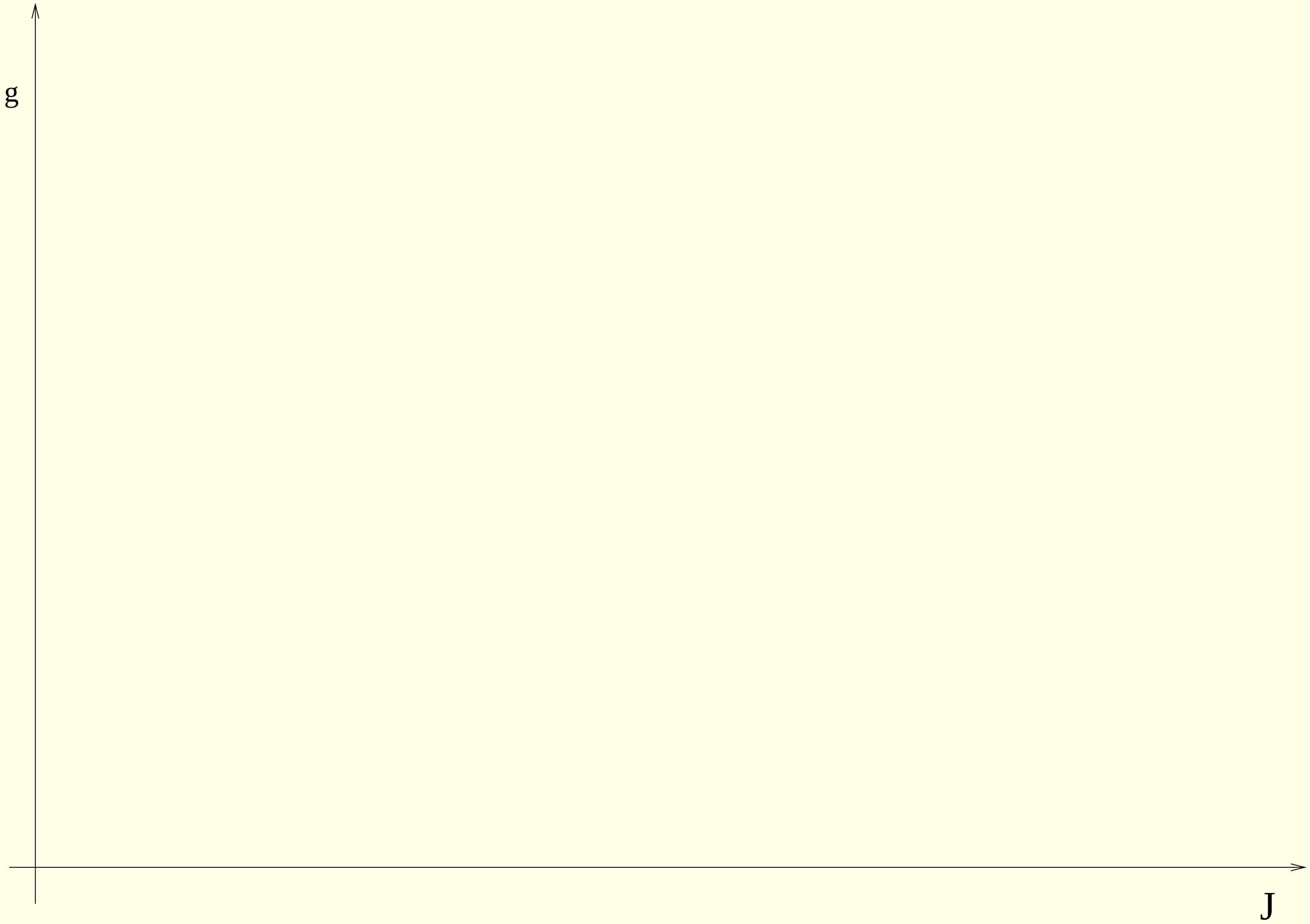

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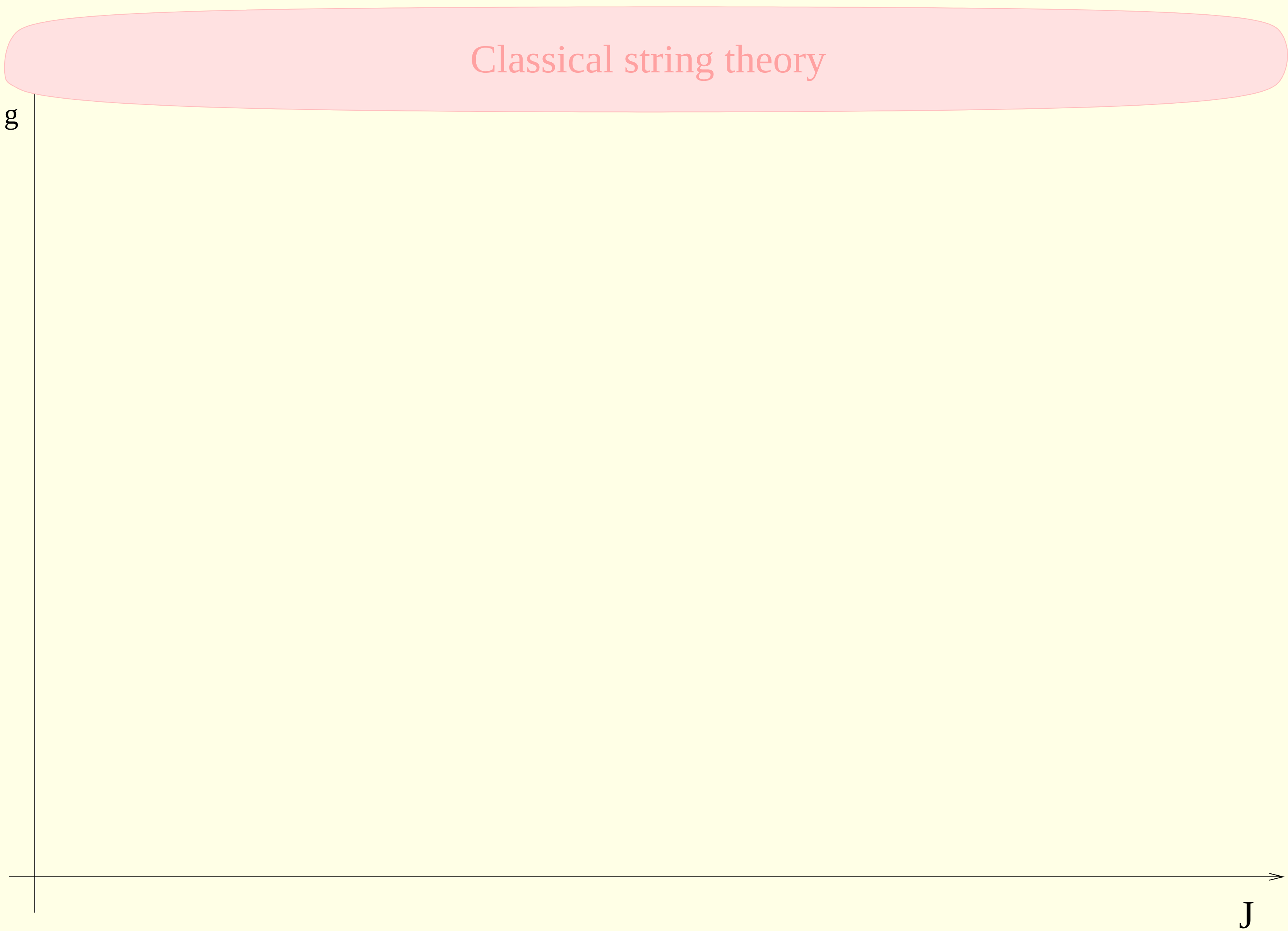

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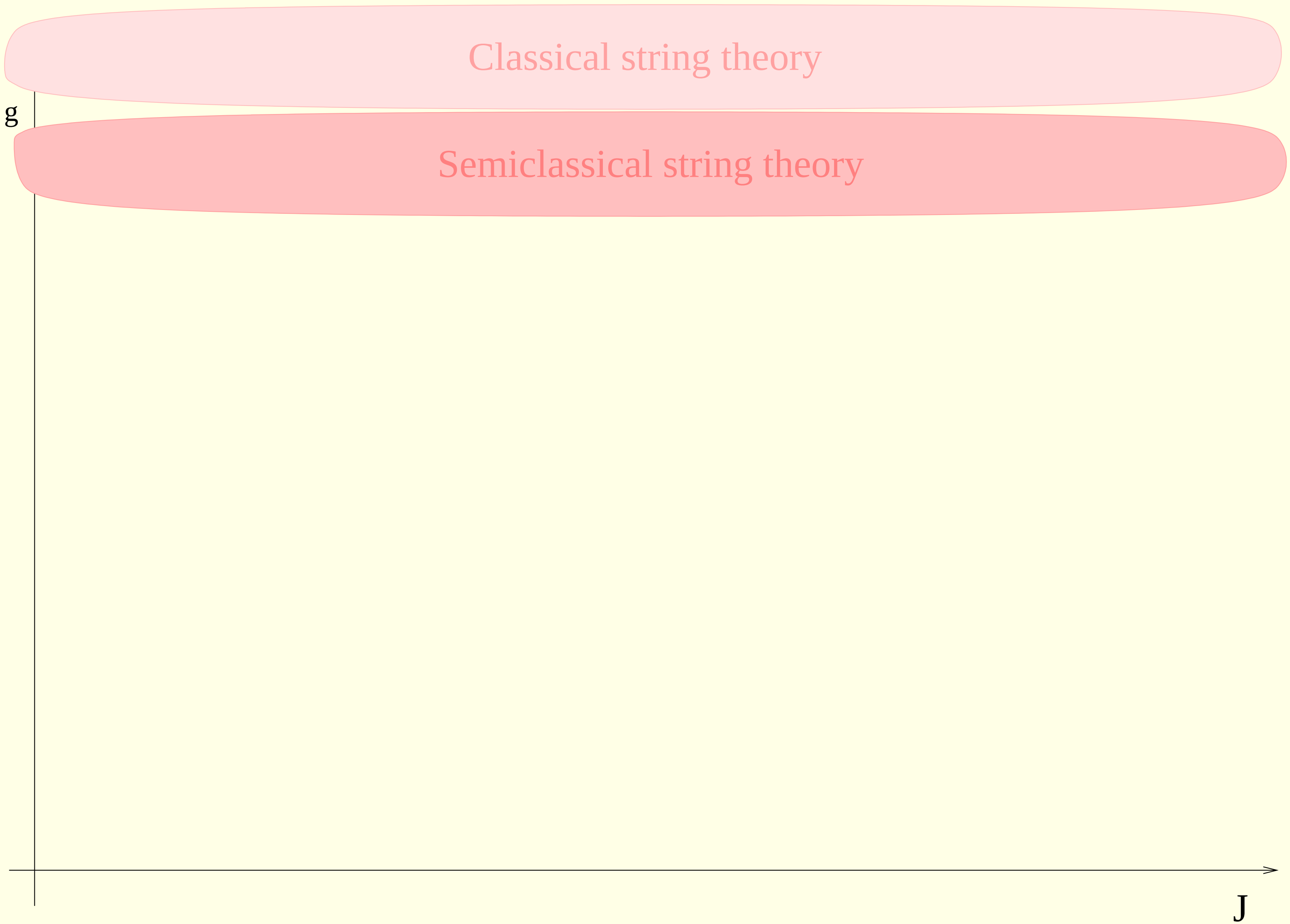

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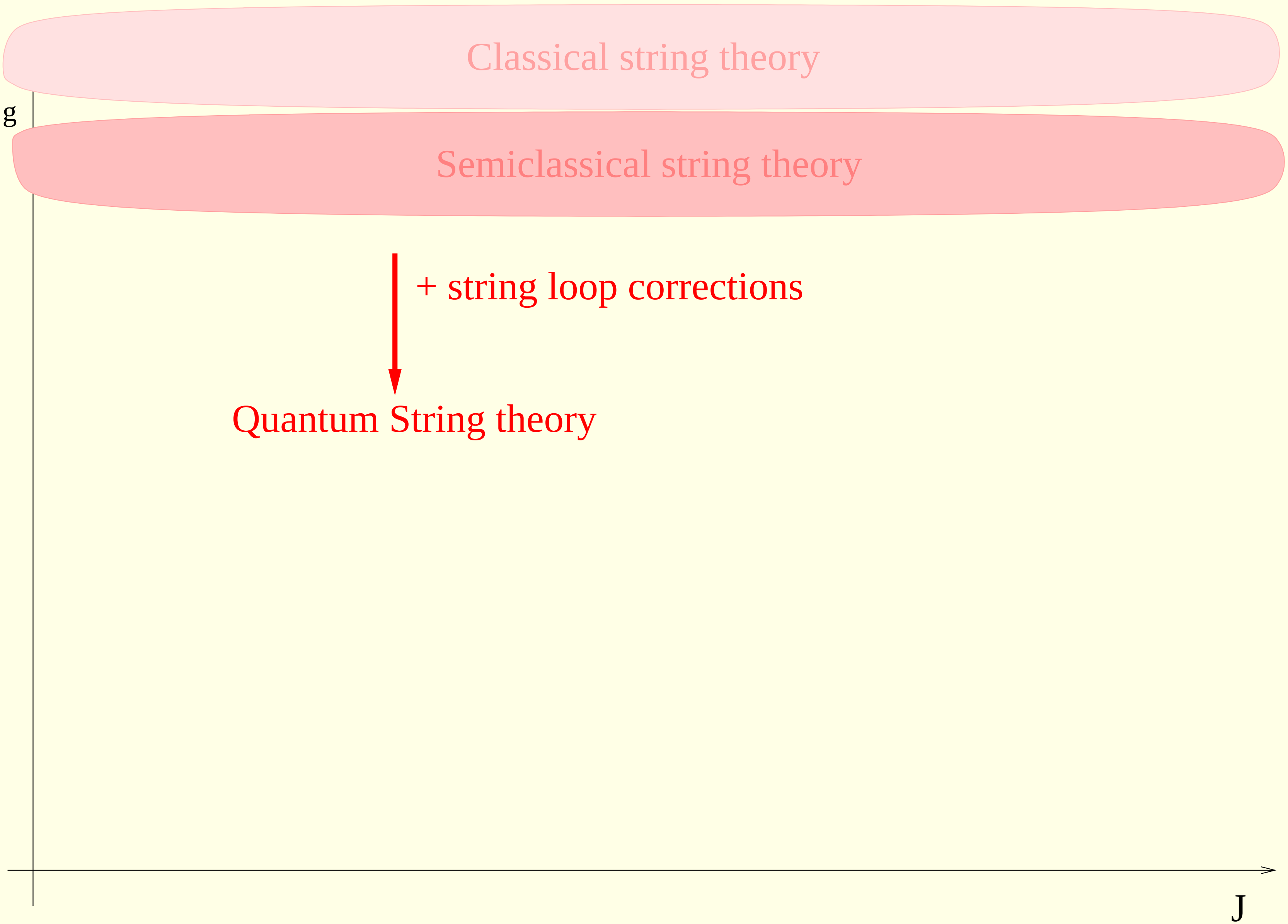

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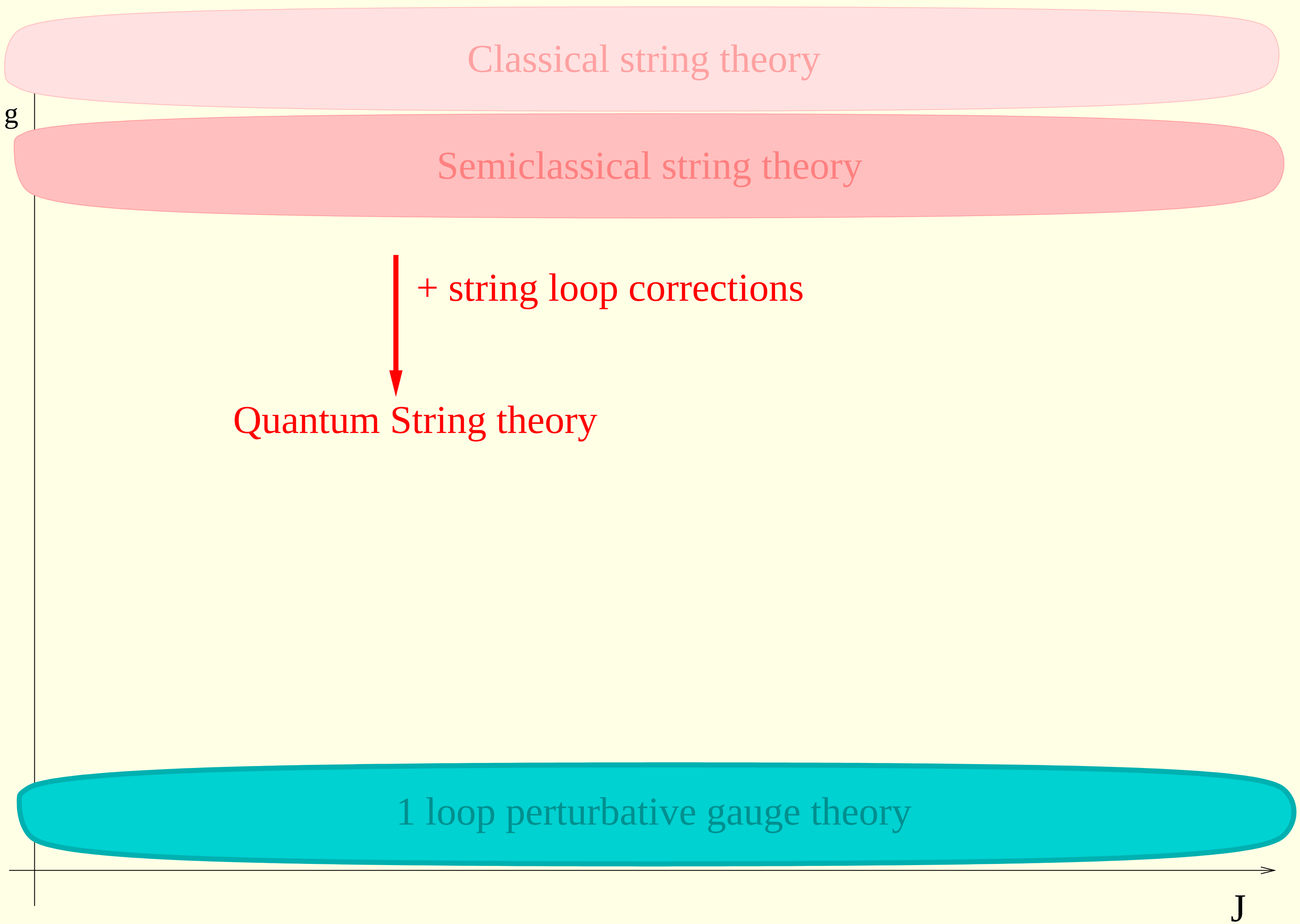

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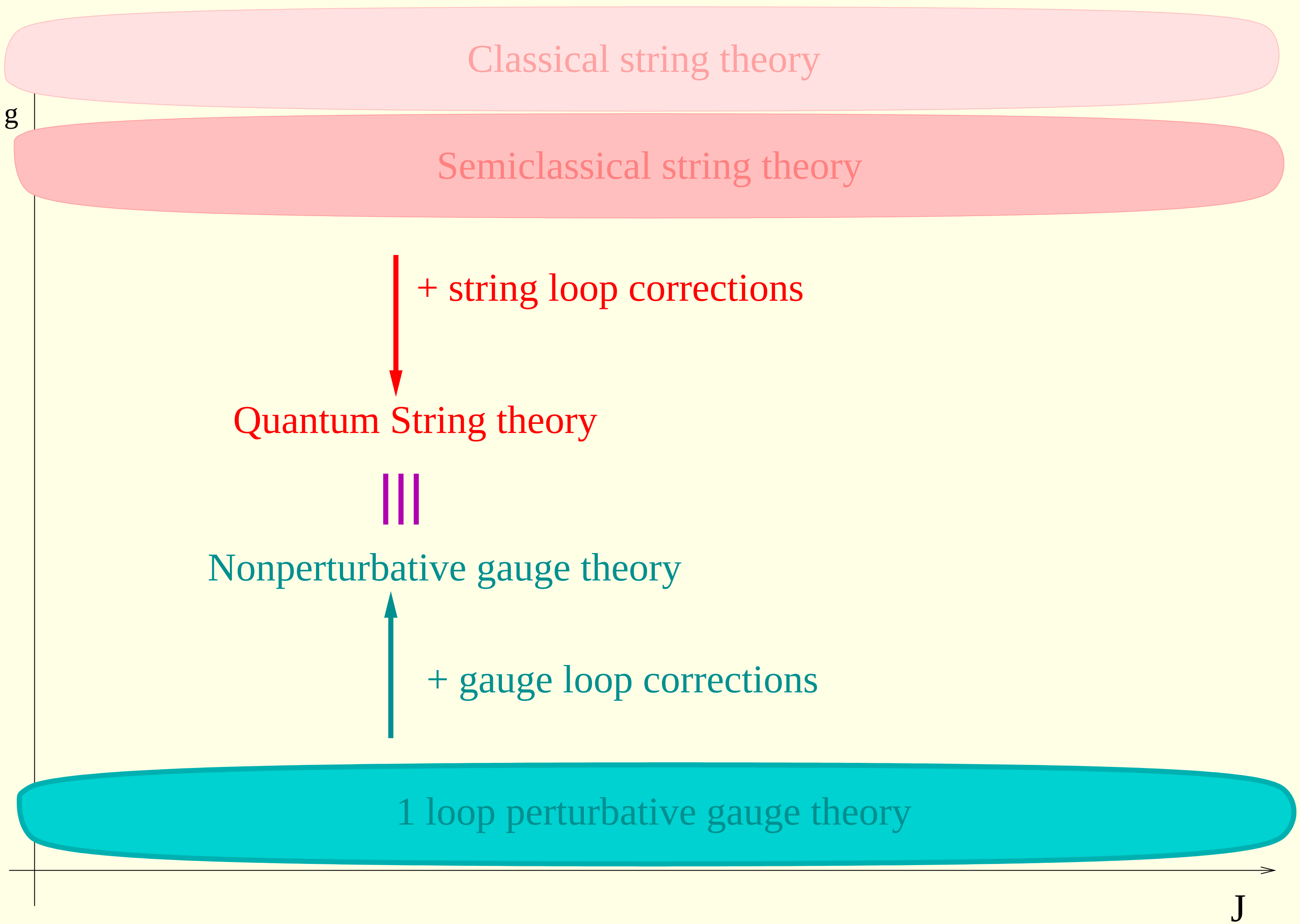

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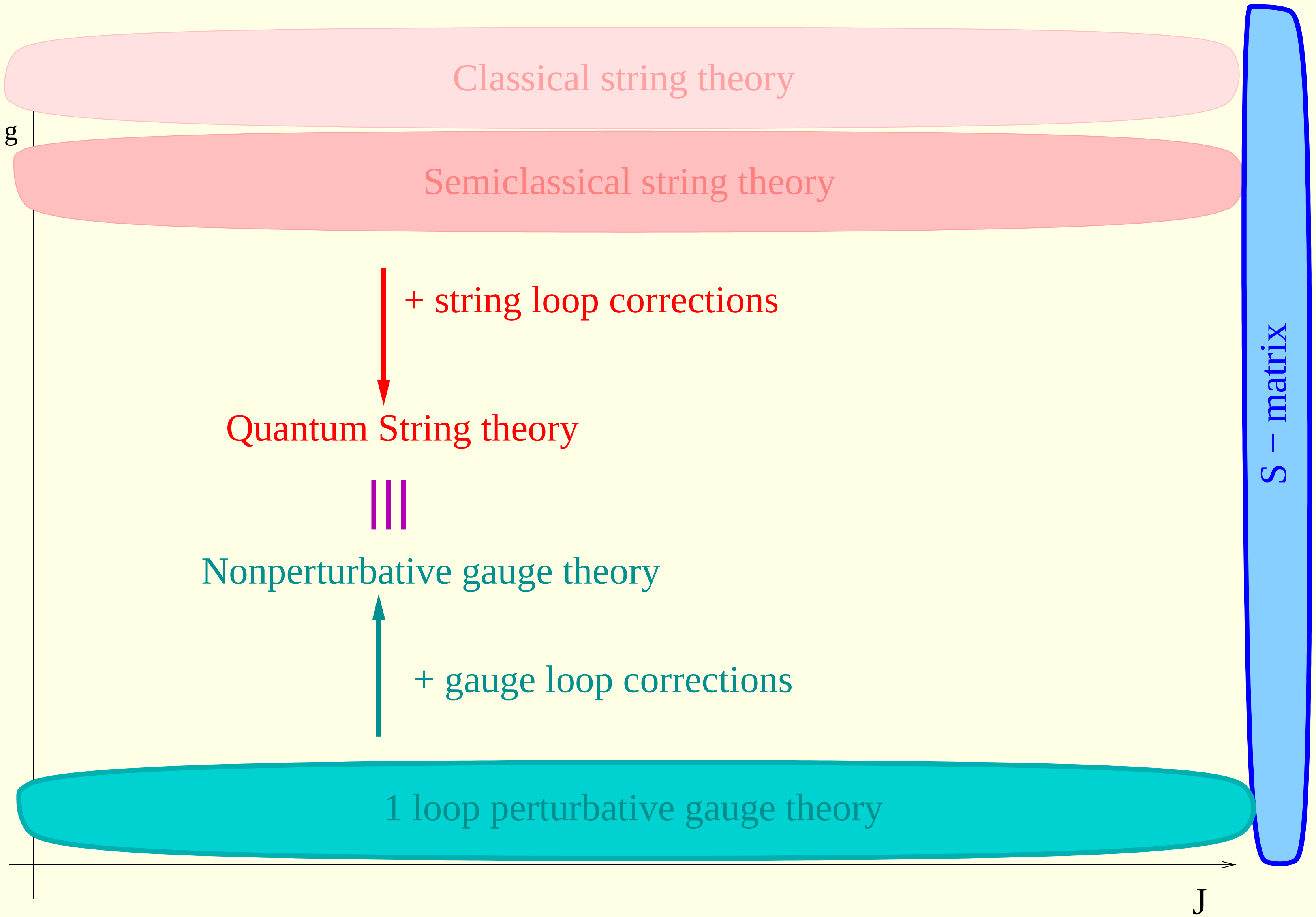

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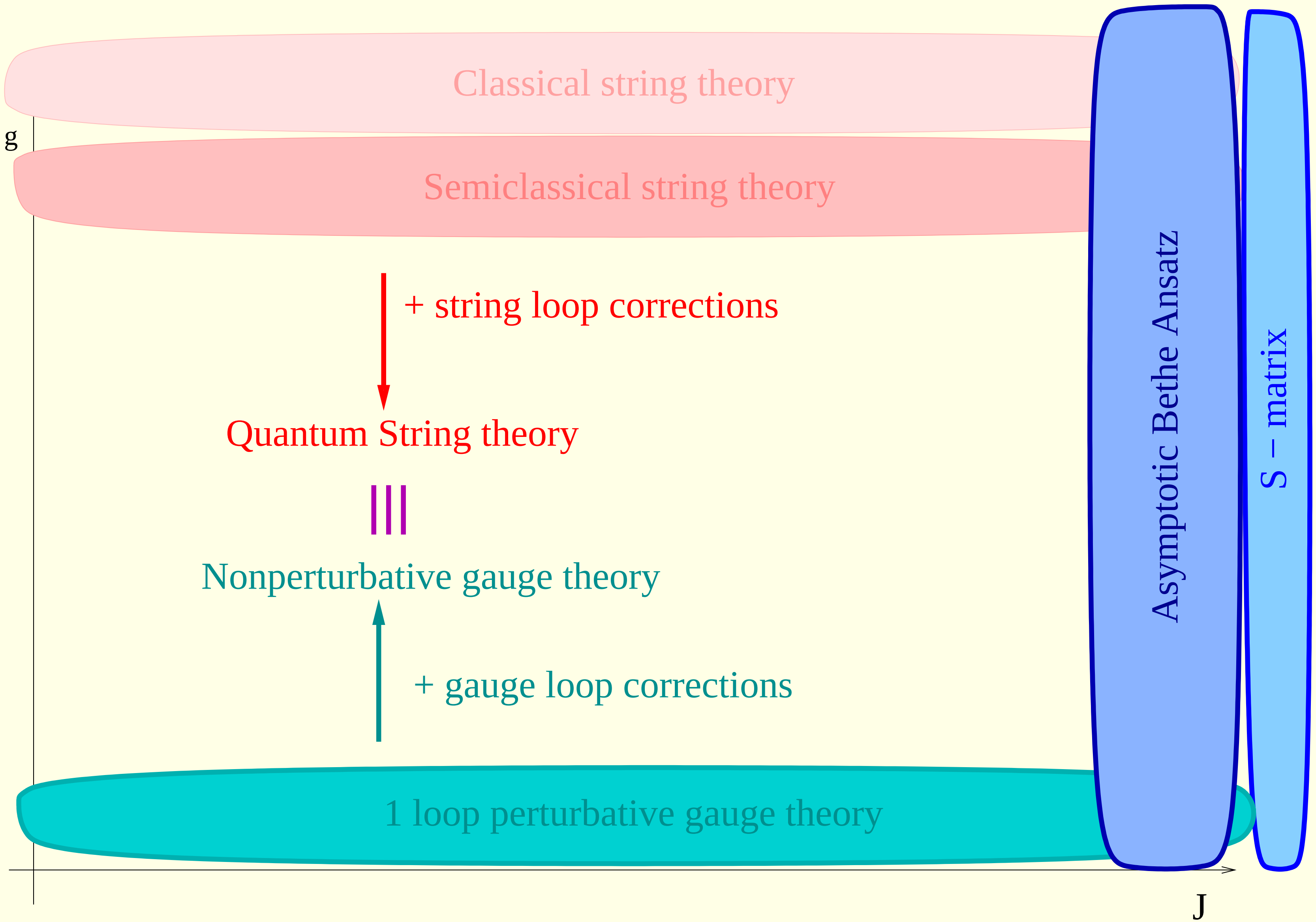

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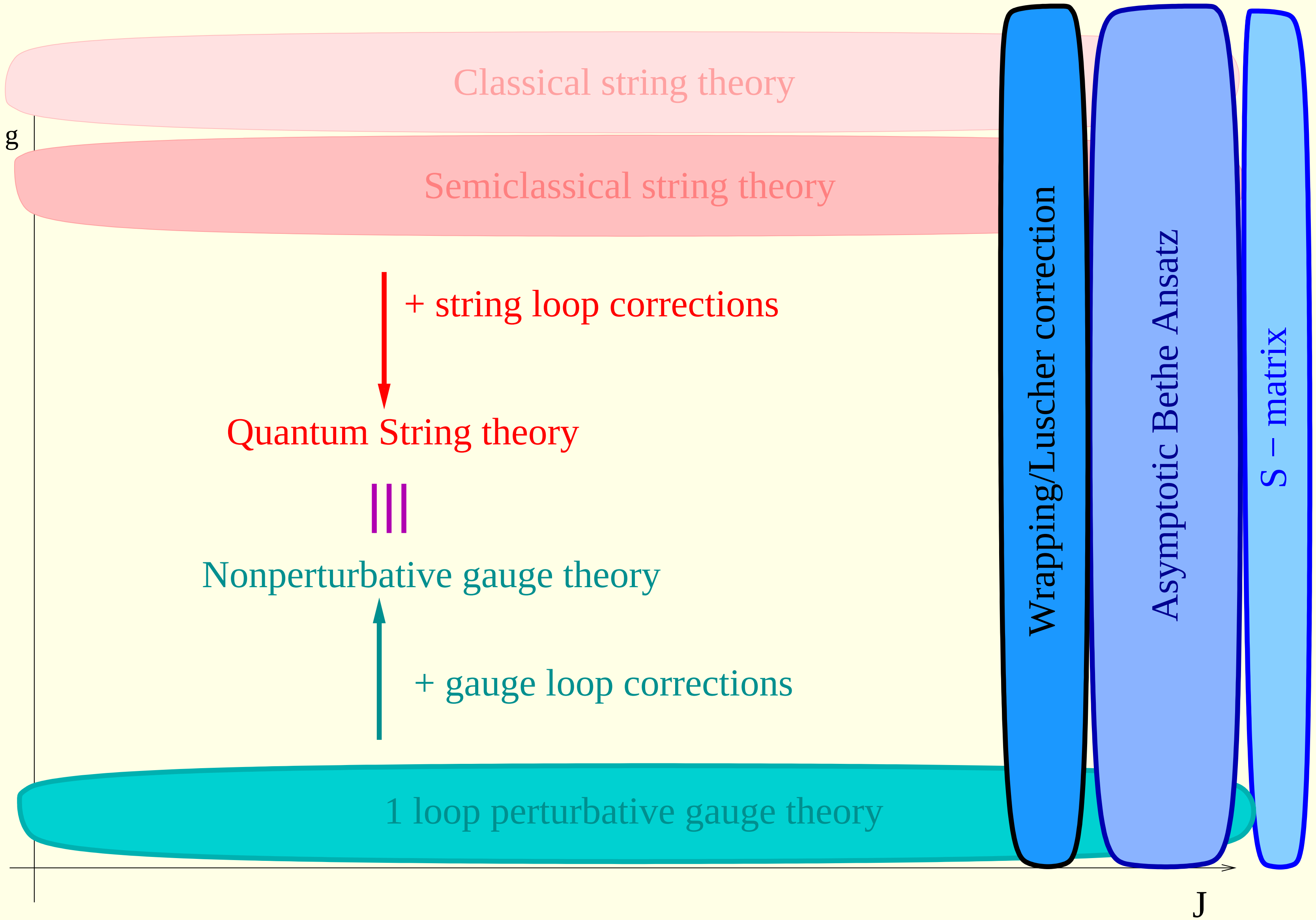
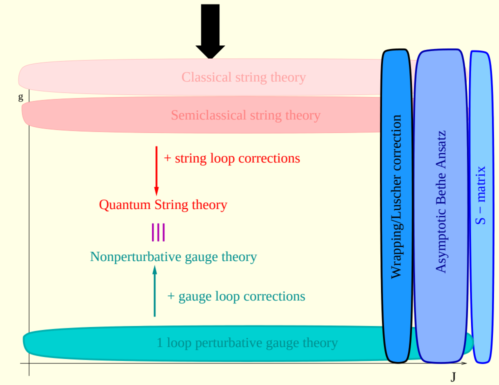
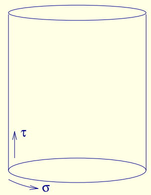
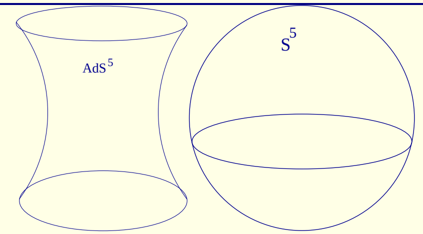


Need finite J (volume) solution of the spectral problem

Classical integrability: AdS



AdS σ model 

target 

$$\mathcal{L} = \frac{R^2}{\alpha'}\left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M\right) + fermions$$

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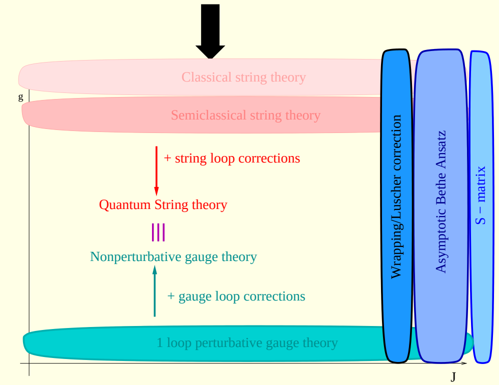
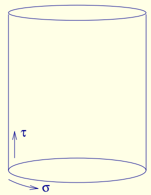
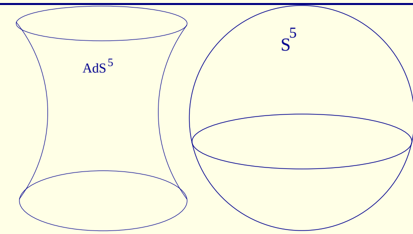


AdS σ model  target 

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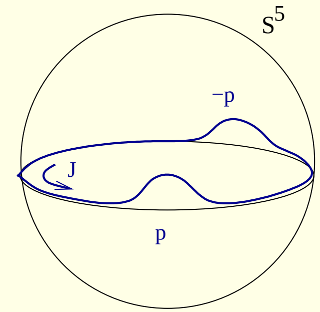
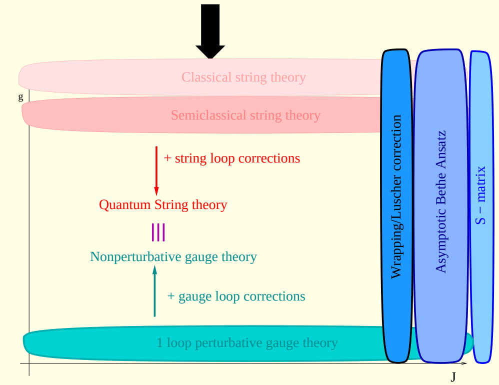
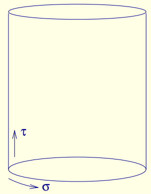
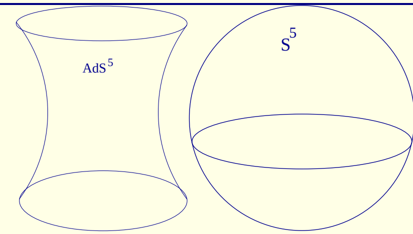
Classical solutions are found, for example magnon:



Classical integrability: AdS

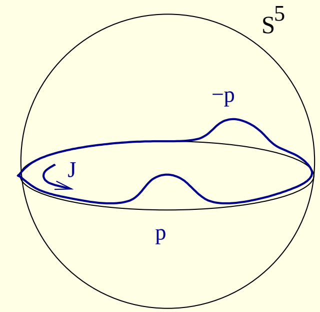


AdS σ model 

target 

$$\mathcal{L} = \frac{R^2}{\alpha'}\left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M\right) + fermions$$

Classical solutions are found, for example magnon:



Coset NLσ model: $g \in \frac{PSU(2,2|4)}{SO(4,1)\times SO(5)}$ $\quad J = g^{-1}dg = J_{||} + J_{\perp}$

Z_4 graded structure: [Metsaev, Tseytlin 03]

$J_{\perp} \rightarrow, J_1, J_2, J_3$ $\quad \mathcal{L} \propto \mathrm{STr}(J_2 \wedge *J_2) - \mathrm{STr}(J_1 \wedge J_3)$

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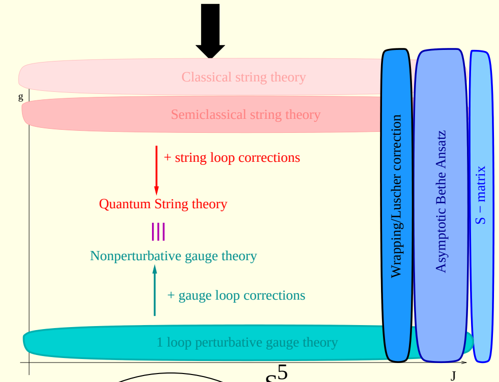
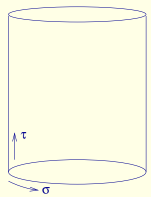
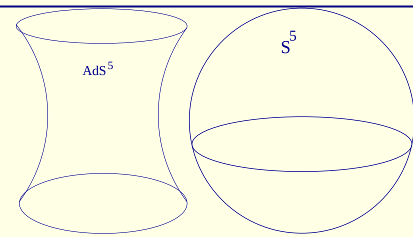


AdS σ model 

target 

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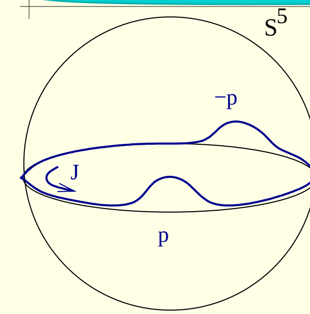

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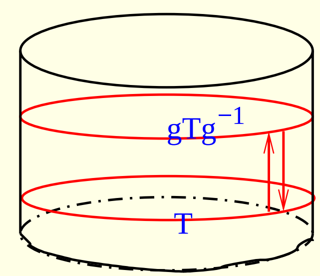
Integrability from flat connection: $dA - A \wedge A = 0$

$$A(\mu) = J_0 + \mu^{-1} J_1 + (\mu^2 + \mu^{-2}) J_2/2 + (\mu^2 + \mu^{-2}) J_2/2 + \mu J_3$$

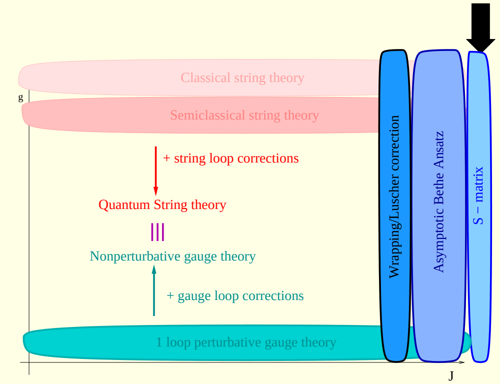
Conserved charges from the trace of the monodromy matrix



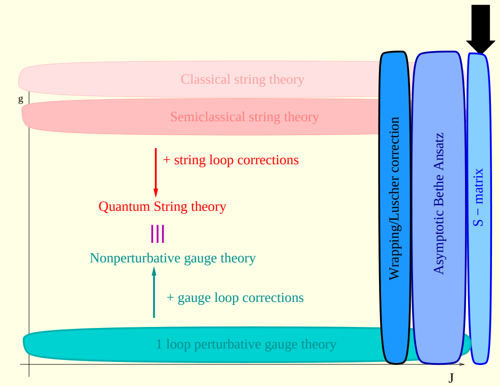
$$T(\mu) = \mathcal{P} \exp \oint A(x)_\mu dx^\mu$$

No proof of quantum integrability: let us assume it!

Bootstrap program: AdS



Nondiagonal scattering: $\boxed{\text{S-matrix} = \text{scalar} \;.\; \text{Matrix}}$



Bootstrap program: AdS

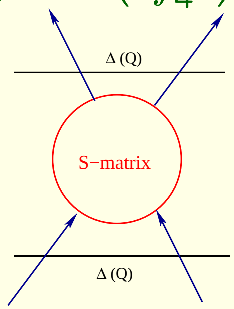
Nondiagonal scattering: S-matrix = scalar . Matrix

Perturbative spectrum: 8 boson + 8 fermion

global symmetry $PSU(2|2)^2$

$$Q = 1 \text{ reps } \begin{pmatrix} b_1 \\ b_2 \\ f_3 \\ f_4 \end{pmatrix} \otimes \begin{pmatrix} b_{\dot{1}} \\ b_{\dot{2}} \\ f_{\dot{3}} \\ f_{\dot{4}} \end{pmatrix}$$

$$[S, \Delta(Q)] = 0$$



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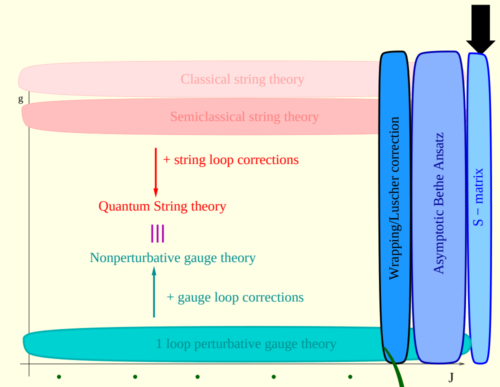

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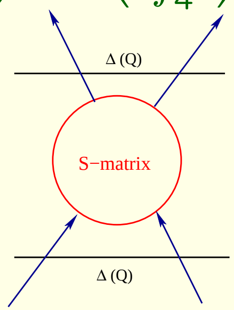
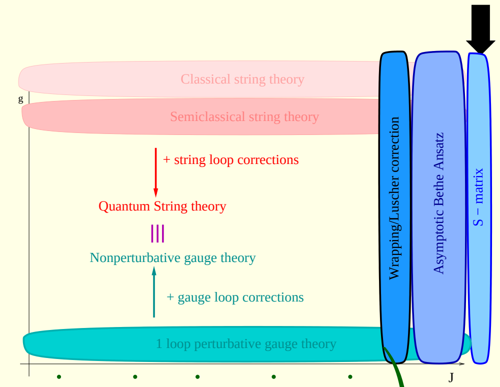
$$\begin{pmatrix}
1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & A & . & . & -b & . & . & . & . & . & . & -c & . & . & c & . \\
. & . & d & . & . & . & . & . & e & . & . & . & . & . & . & . \\
. & -b & . & d & . & . & . & . & . & . & . & . & e & . & . & . \\
. & . & . & . & A & . & . & . & . & . & . & c & . & . & -c & . \\
. & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & d & . & e & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & d & . & . & . & . & . & e & . & . \\
. & . & f & . & . & . & . & . & g & . & . & . & . & . & . & . \\
. & . & . & . & . & . & f & . & . & g & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & a & . & . & . & . & . \\
. & -h & . & . & h & . & . & . & . & . & . & B & . & . & -i & . \\
. & . & . & f & . & . & . & . & . & . & . & . & g & . & . & . \\
. & . & . & . & . & . & . & f & . & . & . & . & . & g & . & . \\
. & h & . & . & -h & . & . & . & . & . & . & -i & . & . & B & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & a
\end{pmatrix}$$

$[S, \Delta(Q)] = 0$





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$$\begin{pmatrix}
1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & A & \cdot & \cdot & -b & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -c & \cdot & \cdot & c & \cdot \\
\cdot & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -b & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & A & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & c & \cdot & \cdot & -c & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d & \cdot & e & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot \\
\cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & f & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -h & \cdot & \cdot & h & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & B & \cdot & \cdot & -i & \cdot \\
\cdot & \cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot \\
\cdot & h & \cdot & \cdot & -h & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -i & \cdot & \cdot & B & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a
\end{pmatrix}$$

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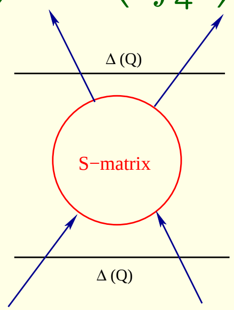


Unitarity

$$\mathcal{S}(z_1, z_2)\mathcal{S}(z_2, z_1) = 1 \qquad S_{11}^{11} = \frac{u_1 - u_2 - i}{u_1 - u_2 + i} e^{i2\theta(z_1, z_2)}$$

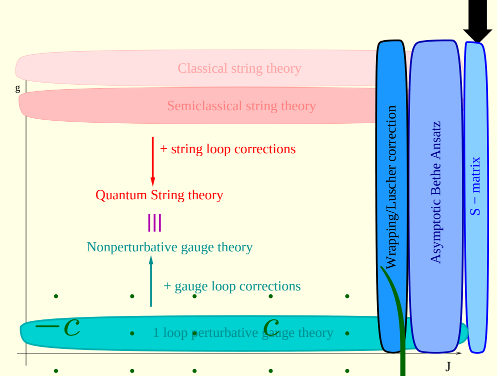
[Beisert,Eden,Staudacher]

Crossing symmetry [Janik] [Volin]

$$u = \tfrac{1}{2} \cot \tfrac{p}{2} E(p) \qquad p = 2\,\mathrm{am}(z)$$

$$\mathcal{S}(z_1, z_2) = \mathcal{S}^{c_1}(z_2, z_1 + \omega_2)$$

Bootstrap program: AdS



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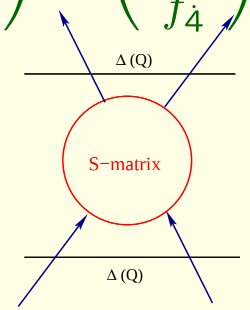

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$$\left(\begin{array}{cccccccccccccccc}
1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & A & \cdot & \cdot & -b & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -c & \cdot & \cdot & c & \cdot \\
\cdot & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -b & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & A & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & c & \cdot & \cdot & -c & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d & \cdot & e & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot \\
\cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & f & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -h & \cdot & \cdot & h & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & B & \cdot & \cdot & -i & \cdot \\
\cdot & \cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot \\
\cdot & h & \cdot & \cdot & -h & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -i & \cdot & \cdot & B & \cdot \\
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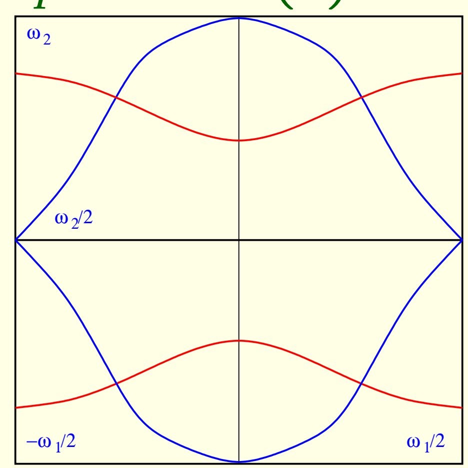
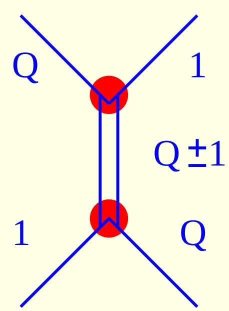


Maximal analyticity:

boundstates atyp symrep: $Q \in \mathbf{N}$

anomalous thresholds

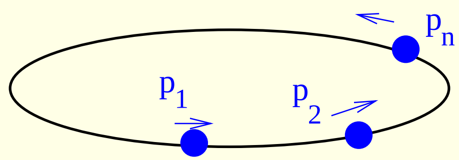
[N.Dorey,Maldacena,Hofman,Okamura]

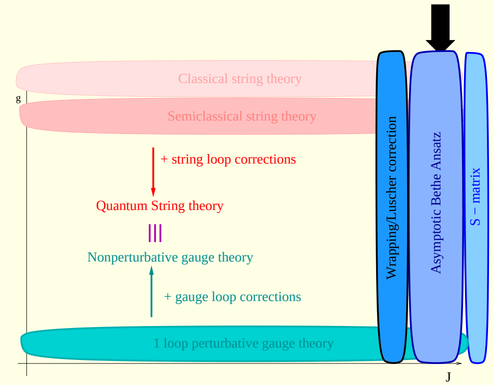


Physical domain

Bethe-Yang=Asymptotic Bethe Ansatz

Finite volume spectrum





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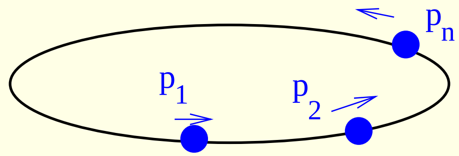

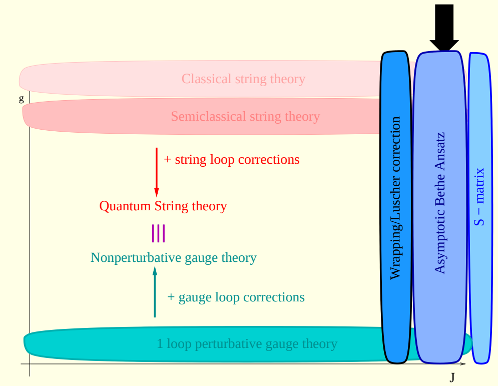

Infinite volume spectrum:

$E(p_1,\ldots,p_n)=\sum_i E(p_i)$ $\qquad E(p)=\sqrt{1+(4g\sin\frac{p}{2})^2}$ $\qquad p_i\in[-\pi,\pi]$

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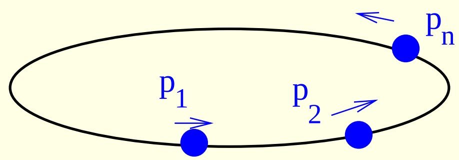

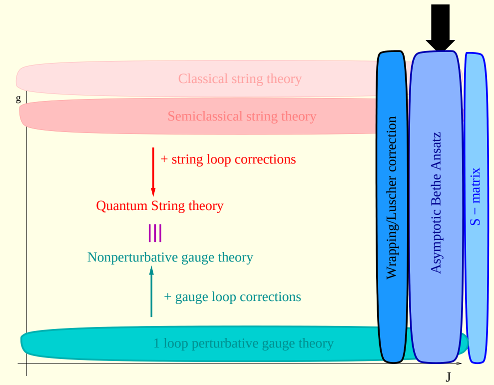

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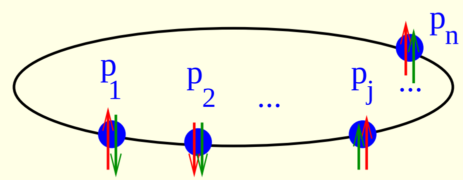
Polynomial volume corrections:

Asymptotic Bethe Ansatz; p_i quantized, .

$e^{ip_jL}\mathcal{S}(p_j,p_1)\dots\mathcal{S}(p_j,p_n)\Psi=-\Psi \quad S(0)=-\hat{P}$



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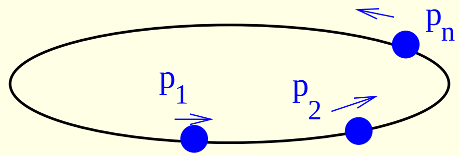

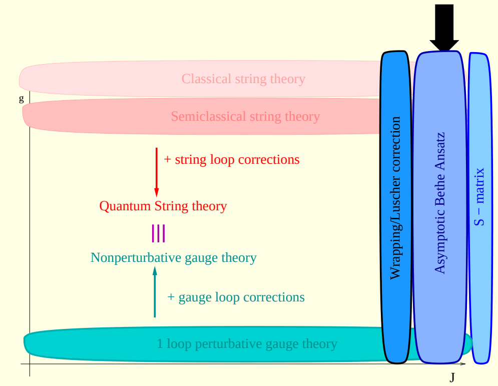

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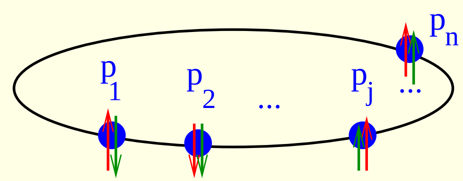
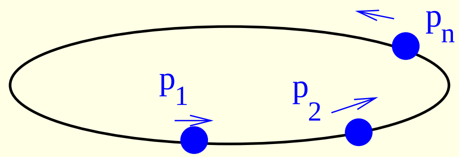
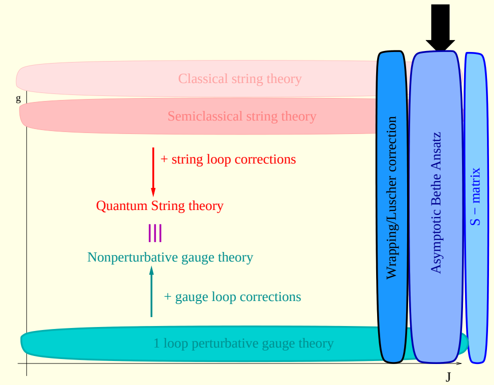


Inhomogenous Hubbard^2 spin-chain: $e^{iLp_j}S_0^2(u_j)\dfrac{Q_4^{++}(u_j)}{Q_4^{--}(u_j)}T(u_j)\dot{T}(u_j)=-1$

Bethe-Yang=Asymptotic Bethe Ansatz

Finite volume spectrum





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Asymptotic Bethe Ansatz; p_i quantized, .

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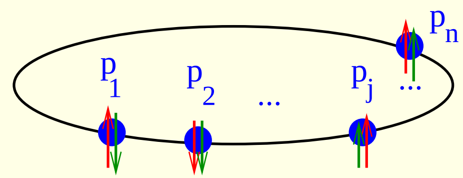


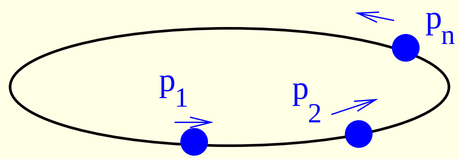
Inhomogenous Hubbard[2] spin-chain: $e^{iLp_j}S_0^2(u_j)\dfrac{Q_4^{++}(u_j)}{Q_4^{--}(u_j)}T(u_j)\dot{T}(u_j)=-1$

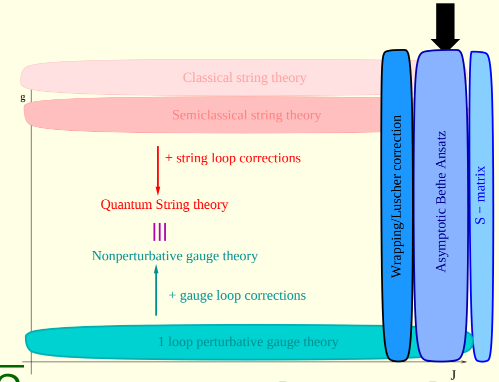
$$T(u)=\frac{B_1^-B_3^+R_4^{-(+)}}{B_1^+B_3^-R_4^{-(-)}}\left[\frac{Q_2^{--}Q_3^+}{Q_2Q_3^-}-\frac{R_4^{-(-)}Q_3^+}{R_4^{-(+)}Q_3^-}+\frac{Q_2^{++}Q_1^-}{Q_2Q_1^+}-\frac{B_4^{+(+)}Q_1^-}{B_4^{+(-)}Q_1^+}\right]$$ [Beisert,Staudacher]

$Q_j(u)=-R_j(u)B_j(u)$ and $R_j^{(\pm)}=\prod_k\frac{x(u)-x^{\mp}_{j,k}}{\sqrt{x^{\mp}_{j,k}}}$ $\qquad B_j^{(\pm)}=\prod_k\frac{\frac{1}{x(u)}-x^{\mp}_{j,k}}{\sqrt{x^{\mp}_{j,k}}}$

Bethe-Yang=Asymptotic Bethe Ansatz

Finite volume spectrum





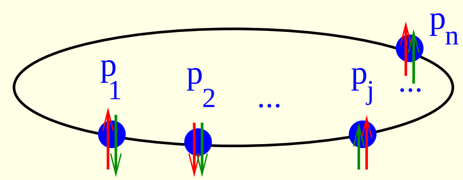
Infinite volume spectrum:

$E(p_1,\dots,p_n)=\sum_i E(p_i)$ $\qquad E(p)=\sqrt{1+(4g\sin\frac{p}{2})^2}$ $\qquad p_i\in[-\pi,\pi]$

Polynomial volume corrections:

Asymptotic Bethe Ansatz; p_i quantized, .

$e^{ip_jL}\mathcal{S}(p_j,p_1)\dots\mathcal{S}(p_j,p_n)\Psi=-\Psi \qquad S(0)=-\hat{P}$



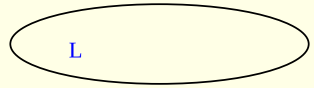
Inhomogenous Hubbard^2 spin-chain: $e^{iLp_j}S_0^2(u_j)\frac{Q_4^{++}(u_j)}{Q_4^{--}(u_j)}T(u_j)\dot{T}(u_j)=-1$

$$T(u)=\frac{B_1^-B_3^+R_4^{-(+)}}{B_1^+B_3^-R_4^{-(-)}}\left[\frac{Q_2^{--}Q_3^+}{Q_2Q_3^-}-\frac{R_4^{-(-)}Q_3^+}{R_4^{-(+)}Q_3^-}+\frac{Q_2^{++}Q_1^-}{Q_2Q_1^+}-\frac{B_4^{+(+)}Q_1^-}{B_4^{+(-)}Q_1^+}\right]$$ [Beisert,Staudacher]

$Q_j(u)=-R_j(u)B_j(u)$ and $R_j^{(\pm)}=\prod_k\frac{x(u)-x_{j,k}^{\mp}}{\sqrt{x_{j,k}^{\mp}}}$ $\qquad B_j^{(\pm)}=\prod_k\frac{\frac{1}{x(u)}-x_{j,k}^{\mp}}{\sqrt{x_{j,k}^{\mp}}}$

Bethe Ansatz: $\frac{Q_2^+B_4^{(-)}}{Q_2^-B_4^{(+)}}|_1=1 \quad \frac{Q_2^{--}Q_1^+Q_3^+}{Q_2^{++}Q_1^-Q_3^-}|_2=-1 \quad \frac{Q_2^+R_4^{(-)}}{Q_2^-R_4^{(+)}}|_3=1$

Thermodynamic Bethe Ansatz: AdS



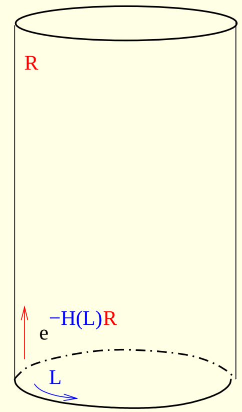
Ground-state energy exactly

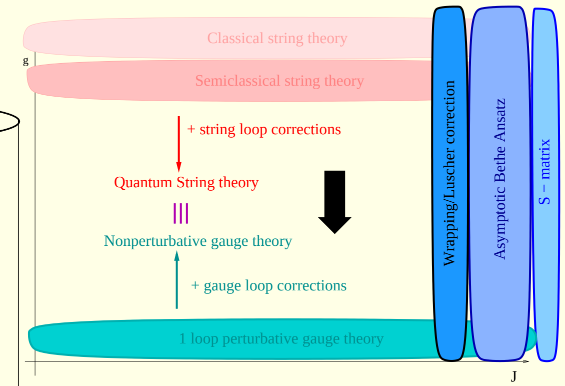
[Bombardelli,Tateo,Fioravanti,Frolov,Arutyunov,
Gromov,Kazakov,Viera,Kozak]

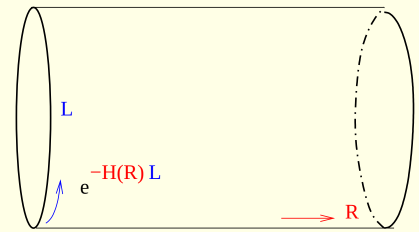
Euclidien $E^2 + (4g\sin\frac{p}{2})^2 = 1$ partition function:

$$Z(L,R) =_{R\to\infty} e^{-E_0(L)R}(1 + e^{-\Delta E\, R})$$

$$Z(L,R) =_{R\to\infty} \mathrm{Tr}(e^{-\tilde{H}(R)L}) =_{R\to\infty} \sum_n e^{-\tilde{E}_n(L)R}$$







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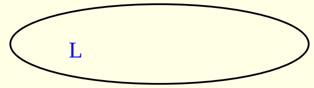

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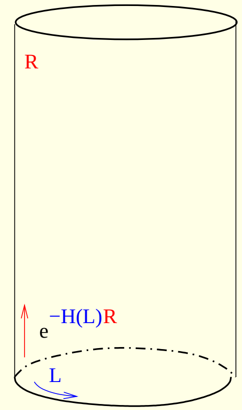

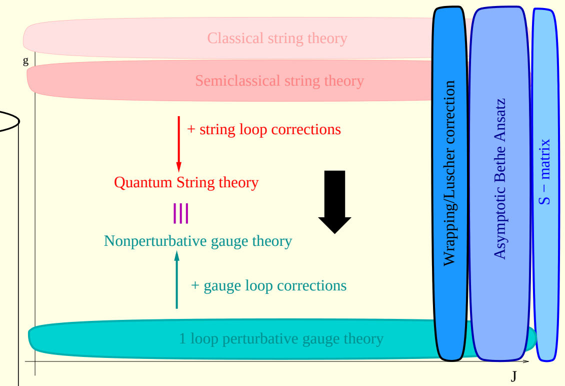

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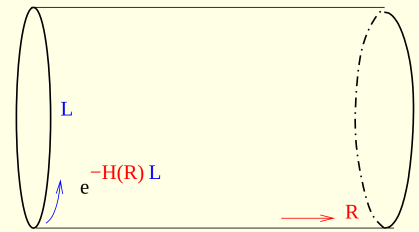
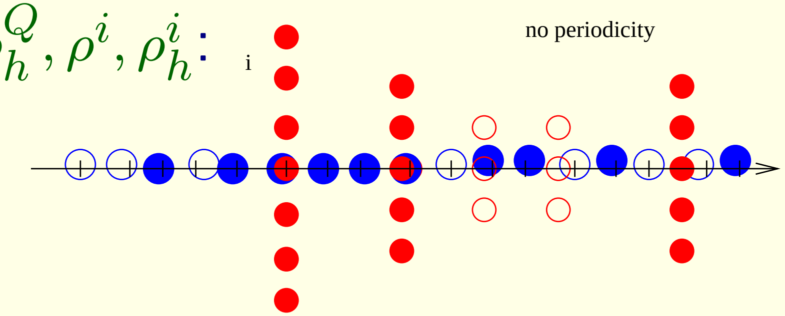


Finite particle/hole + Bethe root density $\rho^Q, \rho^Q_h, \rho^i, \rho^i_h$:



Thermodynamic Bethe Ansatz: AdS

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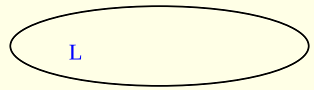

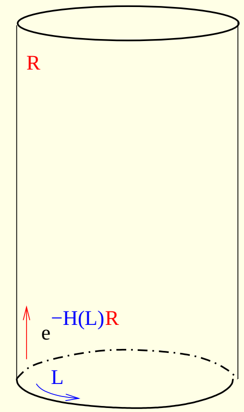

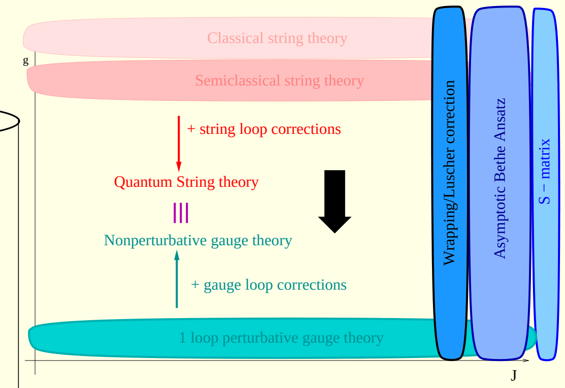

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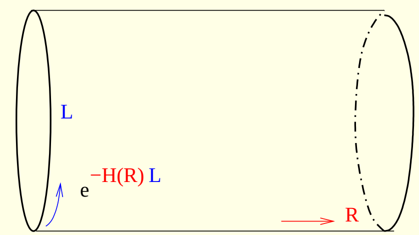

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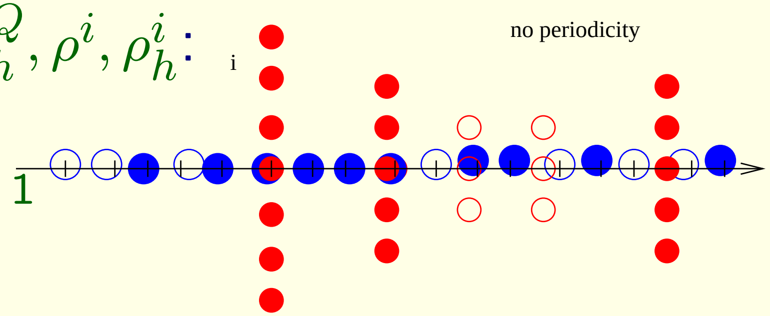

$$\tilde{E}_n(R)=\sum_{i,Q}\tilde{E}_Q(\tilde{p}_i)\to\sum_Q\int\tilde{E}_Q(\tilde{p})\rho^Q(\tilde{p})d\tilde{p}$$

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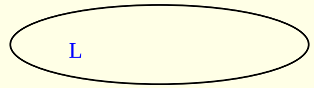
Thermodynamic Bethe Ansatz: AdS

Ground-state energy exactly



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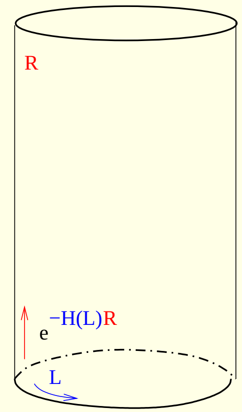

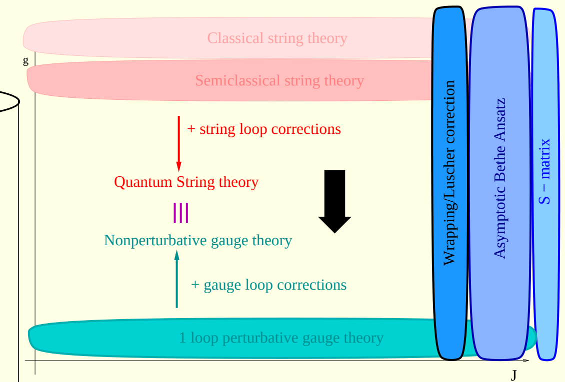

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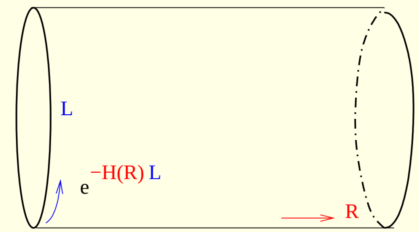

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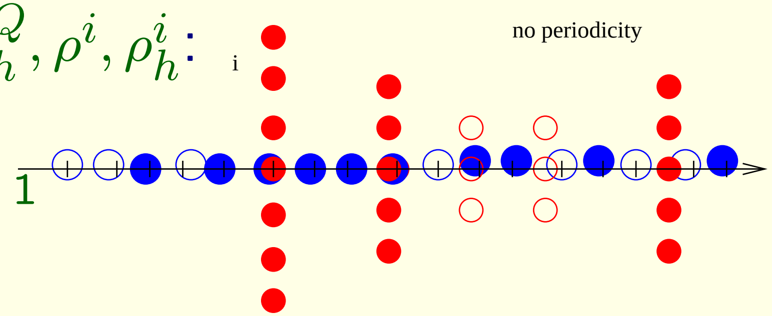

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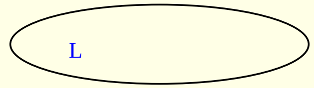
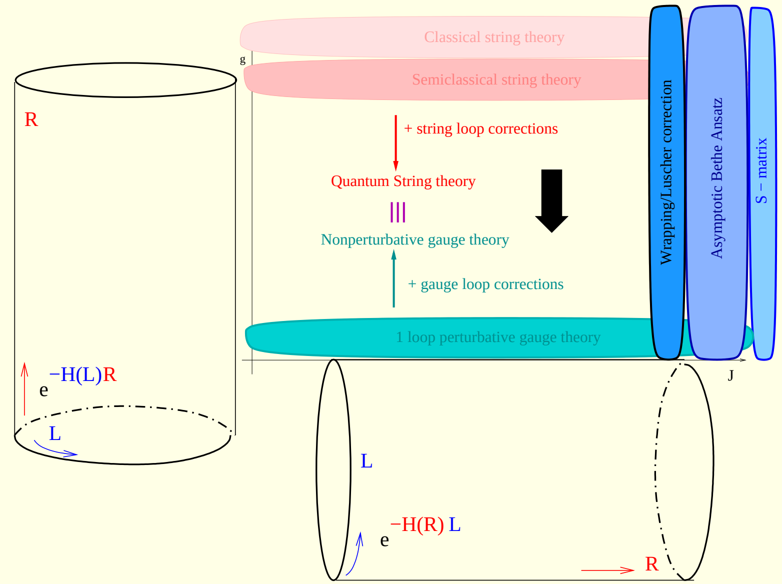
Thermodynamic Bethe Ansatz: AdS





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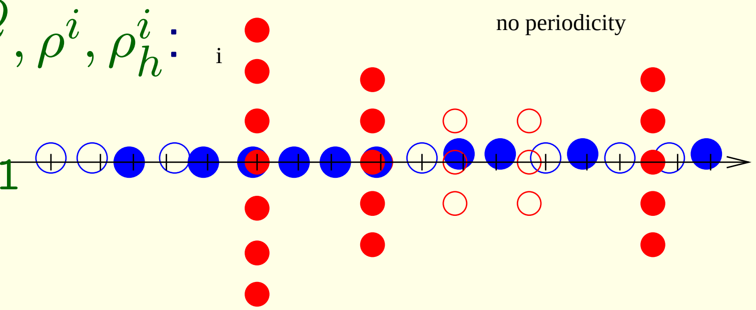

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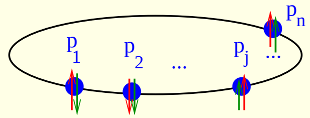
Saddle point : $\epsilon^i(\tilde p)=-\ln\frac{\rho^i(\tilde p)}{\rho^i_h(\tilde p)}$

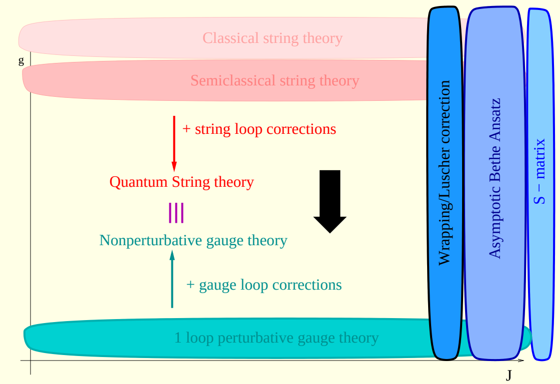
$$\boxed{\epsilon^j(\theta)=\delta^j_Q\tilde E_Q(\tilde p)L-\int K^j_i(\tilde p,\tilde p')\log(1+e^{-\epsilon^i(\tilde p')})d\tilde p'}$$

Ground state energy exactly:

$$\boxed{E_0(L)=-\sum_Q\int\frac{d\tilde p}{2\pi}\log(1+e^{-\epsilon_Q(\tilde p)})d\tilde p}$$

Excited states TBA, Y-system: AdS

Excited states exactly 



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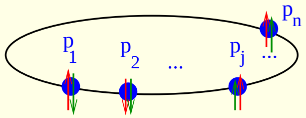
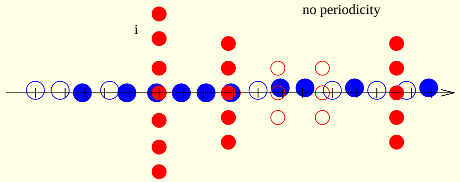
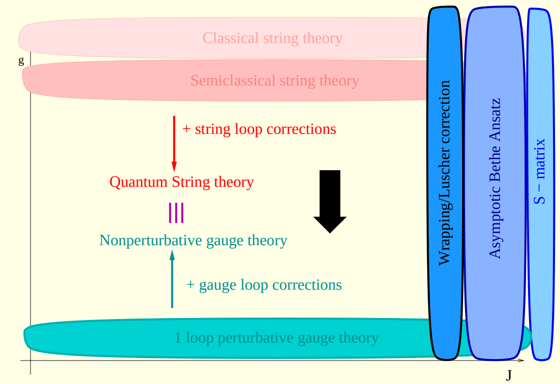
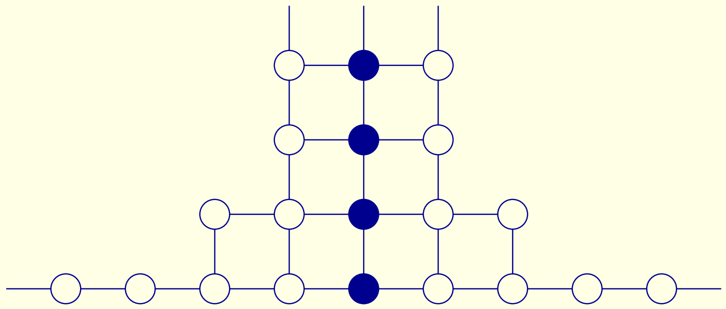






Y-system: AdS [Gromov,Kazakov,Viera]

$$\frac{Y_{a,s}(\theta+\frac{i}{2})Y_{a,s}(\theta-\frac{i}{2})}{Y_{a+1,s}Y_{a-1,s}}=\frac{(1+Y_{a,s-1})(1+Y_{a,s+1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$



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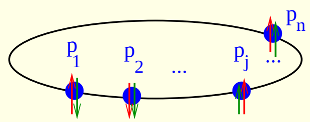

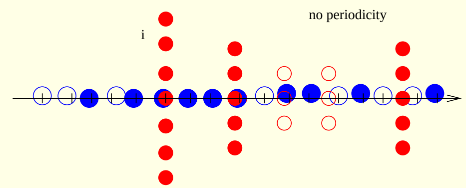

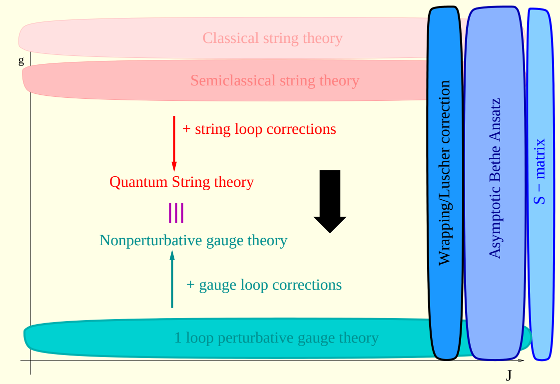

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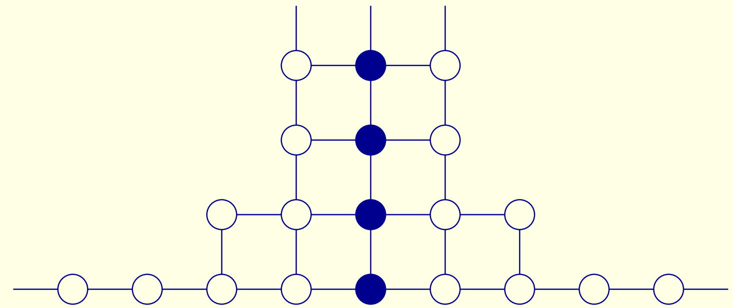

Excited states TBA: analyticity from Lüscher [Gromov,Kazakov,Kozak,Viera,Arutyunov, Frolov,Suzuki]

Quantum spectral curve formulations:

find P_a μ_{ab} $a,b=1..4$ which satisfy:

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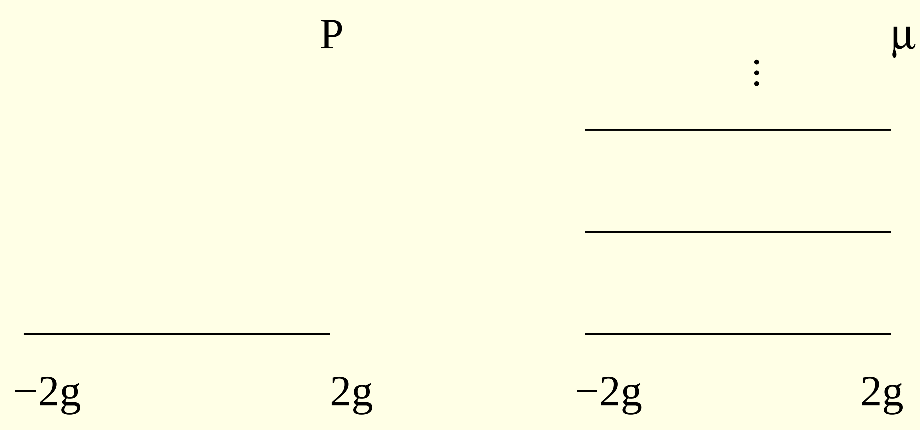

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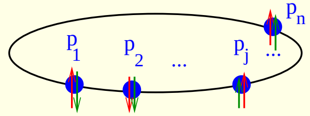

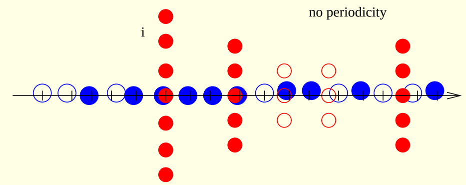

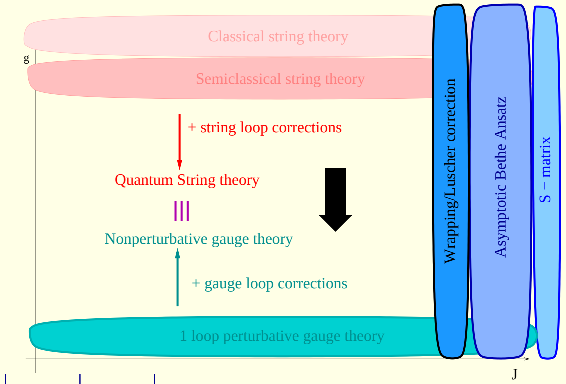

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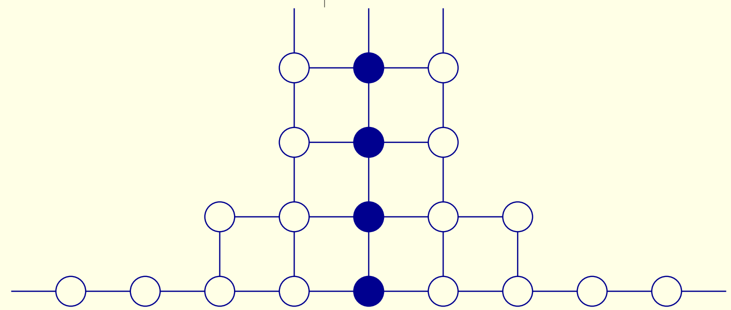

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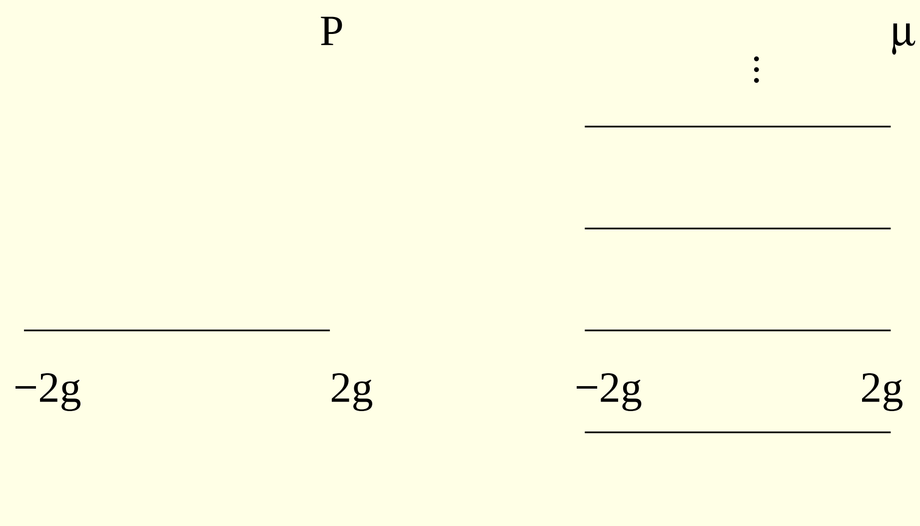
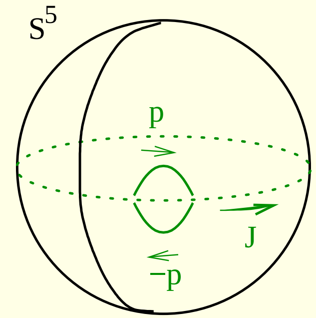
Lattice regularization: ?



Konishi scaling dimension



CFT 2pt function: $\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{|x|^{2\Delta_i}}$ scaling dimension: Δ_i

Konishi op. $\mathcal{O}_K = \mathrm{Tr}(\Phi_i^2)$

$$\Delta = 2 + 12g^2 - 48g^4 + 336g^6 +$$

loop	4	5	6	7	8	9
Δ	$96(-26+6\zeta_3 -15\zeta_5)$	$-96(-158-73\zeta_3+ 54\zeta_3^2+90\zeta_5-315\zeta_7)$				
gauge	[Fiamberti,Sieg, A. Santambrogio, Zanon 08] [Velizhanin09]	[Eden, Heslop, Korchemsky, Smirnov, Sokatchev 12]	[Smirnov ?]			
Lüscher	[Bajnok, Janik 08]	[Bajnok,Hegedus, Janik,Lukowski 09]	[Bajnok, Janik 12]			
TBA	[Kazakov, Gromov, Vieira 09]	[Balog, Hegedűs 10]				
FiNLIE	[Leurent, Serban, Volin 12]			[Leurent, Volin 13]		[Volin 1