Matrix Workshop: Integrability in Low-Dimensional Quantum systems

Creswick 2017 June-July

Finite volume diagonal form factors and AdS/CFT

Z. Bajnok

MTA Wigner Research Center for Physics,

Holographic QFT Group, Budapest

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Saleur-Pozsgay-Takacs conjecture:

$$\langle\theta_1,\dots,\theta_n|\mathcal{O}|\theta_n,\dots,\theta_1\rangle_L = \frac{\sum_{\alpha\cup\bar{\alpha}} F^c_{\alpha}\rho_{\bar{\alpha}}}{\rho_n} = \frac{\sum_{\alpha\cup\bar{\alpha}} F^s_{\alpha}\rho^s_{\bar{\alpha}}}{\rho_n}$$

LeClair-Mussardo conjecture:

$$\langle 0|\mathcal{O}|0\rangle_L = \sum_n \frac{1}{n!}\prod_{j=1}^{n}\int\frac{d\theta_i}{2\pi}\frac{e^{-\epsilon(\theta_i)}}{1+e^{-\epsilon(\theta_i)}}F^c_n(\theta_1,\dots,\theta_n)$$

Prologue: QFT as the continuum limit of XXZ, Form factors

S-matrix bootstrap, finite volume energy spectrum

Form factor bootstrap, finite volume form factors

Form factors in AdS/CFT

Prologue: inhomogenous XXZ

Consider the inhomogenous XXZ spin chain

$$R(\lambda) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sinh(\lambda)}{\sinh(\lambda - i\gamma)} & \frac{\sinh(-i\gamma)}{\sinh(\lambda - i\gamma)} & 0 \\ 0 & \frac{\sinh(-i\gamma)}{\sinh(\lambda - i\gamma)} & \frac{\sinh(\lambda)}{\sinh(\lambda - i\gamma)} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\vec{\xi} = \{ \quad \xi_-, \qquad \xi_+, \quad ..., \qquad \xi_+ \quad \},$$

$$T(\lambda|\vec{\xi}) = \; R_{01}(\lambda - \xi_1)\, R_{02}(\lambda - \xi_2) ... \qquad ... R_{0N}(\lambda - \xi_N) = \begin{pmatrix} A(\lambda) & B(\lambda) \\ C(\lambda) & D(\lambda) \end{pmatrix}$$

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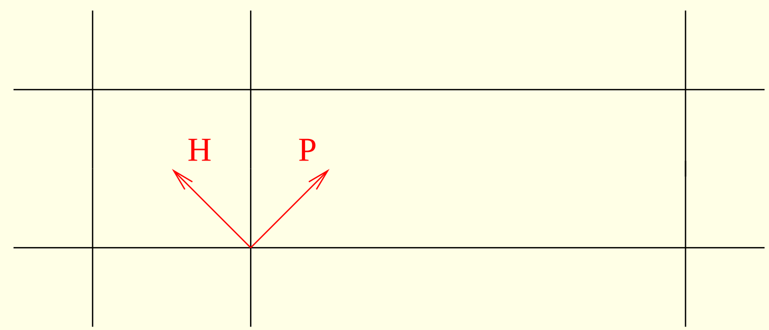
Integrability: $\mathcal{T}(\lambda|\vec{\xi}) = \mathrm{Tr}_0 \, T(\lambda|\vec{\xi})$ commute $\left[\mathcal{T}(\lambda|\vec{\xi}), \mathcal{T}(\lambda'|\vec{\xi})\right] = 0$

conserved charges $U_\pm = \mathcal{T}(\xi_\pm|\vec{\xi}) = e^{i\frac{2}{a}(H \pm P)}$



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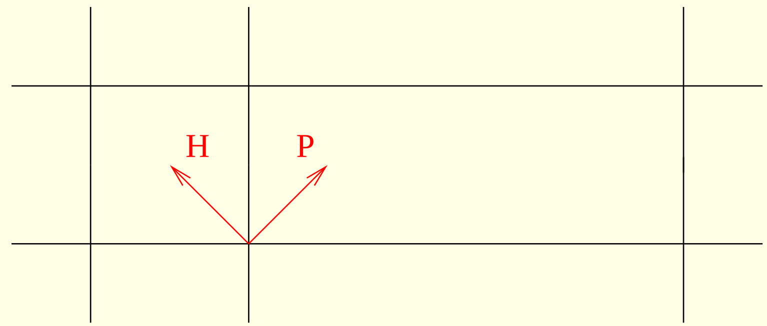


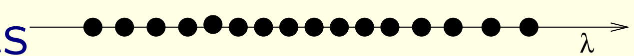
Eigenvectors: $|\vec{\lambda}\rangle = B(\lambda_1)\, B(\lambda_2) ... B(\lambda_m)\, |0\rangle,$

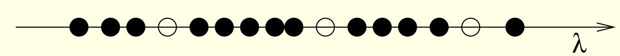
Bethe Ansatz:

$$\prod_{i=1}^{N} \frac{\sinh(\lambda_a - \xi_i - i\gamma)}{\sinh(\lambda_a - \xi_i)} \prod_{b=1}^{m} \frac{\sinh(\lambda_a - \lambda_b + i\gamma)}{\sinh(\lambda_a - \lambda_b - i\gamma)} = -1$$

Alternating inhomogeneities

$$\xi_\pm = \pm \frac{\gamma}{\pi}\Lambda - i\frac{\gamma}{2}$$

Grd st. $\frac{N}{2}$ roots 

Exc. st. +holes: 

Prologue: QFT as a continuum limit

Counting function $(-1)^{\delta}\, e^{i\, Z_\lambda(\lambda)} = \prod_{i=1}^{N} \frac{\sinh(\lambda-\xi_i-i\gamma)}{\sinh(\lambda-\xi_i)} \prod_{b=1}^{m} \frac{\sinh(\lambda-\lambda_b+i\gamma)}{\sinh(\lambda-\lambda_b-i\gamma)}$

Bethe Ansatz: $e^{i\, Z_\lambda(\lambda_a)} = -1$ take $\delta = 0$ and redefine $Z_N(\theta) = Z_\lambda(\frac{\gamma}{\pi}\theta)$, which satisfies

$$Z_N(\theta) = \frac{N}{2}\left\{ \begin{array}{c} \arctan\left[\sinh(\theta-\Lambda)\right] + \\ \arctan\left[\sinh(\theta+\Lambda)\right] \end{array} \right\} + \sum_{k=1}^{m_H} \chi(\theta-\theta_k) + 2\Im m \int_{-\infty}^{\infty} \frac{d\theta'}{2\pi i}\, G(\theta-\theta'-i\eta) \ln\left(1 + e^{i\, Z_N(\theta'+i\eta)}\right)$$

[Klümper, Pearce, Destri, de Vega,...]

$$G(\theta) = -i\partial_\theta \log S(\theta) = \int_{-\infty}^{\infty} d\omega\, e^{-i\,\omega\theta}\, \frac{\sinh(\frac{(p-1)\,\pi\omega}{2})}{2\cosh(\frac{\pi\omega}{2})\,\sinh(\frac{p\,\pi\,\omega}{2})}$$

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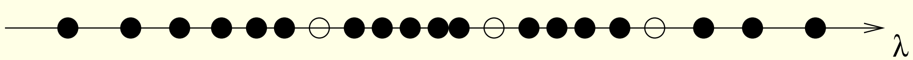
QFT = Scaled continuum limit $N \to \infty$: $\Lambda = \ln\frac{4}{\mathcal{M}\, a} = \ln\frac{2\,N}{\mathcal{M}\, L} \to \infty$

$$Z(\theta) = \mathcal{M}L \sinh\theta - i \sum_{k=1}^{m_H} \log S(\theta-\theta_k) + \int_{-\infty}^{\infty} \frac{d\theta'}{2\pi i}\, G(\theta-\theta'-i\eta) \ln\left(1+e^{i\, Z(\theta'+i\eta)}\right)$$

$$E \pm P = \mathcal{M} \sum_{k=1}^{m_H} e^{\pm\theta_k} \mp 2\mathcal{M}\, \Re e \int_{-\infty}^{\infty} \frac{d\theta}{2\pi i}\, e^{\pm\theta+i\eta} \ln\left(1+e^{i\, Z(\theta+i\eta)}\right)$$



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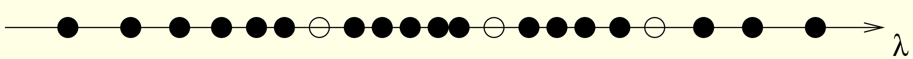

large volume equation: $e^{iZ(\theta_k)} = e^{i\mathcal{M}L\sinh\theta_k} \prod_{j=1}^{m_H} S(\theta_k-\theta_j) = -1$

relativistic energy spectrum: $E = \sum_{j=1}^{m_H} \mathcal{M}\cosh\theta_k$

Prologue: QFT finite volume form factors

topological charge $Q = \sum_n (\sigma^z_{2n} + \sigma^z_{2n-1}) \longleftrightarrow \int_0^L J_0(x,t)dx = \int_0^L \partial_x \Phi(x,t)dx$

we need the continuum limit of $\langle \sigma^z_n \rangle_\lambda = \frac{\langle \vec{\lambda} | \sigma^z_n | \vec{\lambda} \rangle}{\langle \vec{\lambda} | \vec{\lambda} \rangle}$ emptiness formation prob.

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QIS: $$e_n = \frac{1}{2}(1_n - \sigma^z_n) = \prod_{i=1}^{n-1} (A+D)(\xi_i)\; D(\xi_n) \prod_{i=n+1}^{N} (A+D)(\xi_i)$$

Slavnov: $\langle \vec{\mu} | \vec{\lambda} \rangle = \frac{\det H(\vec{\mu}|\vec{\lambda})}{\prod_{j>k} \sinh(\mu_k - \mu_j) \sinh(\lambda_j - \lambda_k)}$

Gaudin: $\langle \vec{\lambda} | \vec{\lambda} \rangle = \frac{\prod_{j,k} \sinh(\lambda_j - \lambda_k - i\gamma)}{\prod_{j>k} \sinh(\lambda_k - \lambda_j) \sinh(\lambda_j - \lambda_k)} \cdot \det \Phi(\vec{\lambda}) \qquad \Phi_{ab}(\vec{\lambda}) = -i \frac{\partial}{\partial \lambda_b} Z_\lambda(\lambda_a | \vec{\lambda})$

leads to $\langle e_n \rangle_\lambda = \sum_{A=1}^{m} \left(\Phi^{-1}(\vec{\lambda}) \cdot \hat{\mathcal{H}}(\vec{\mu}^{(A)}|\vec{\lambda}) \right)_{AA} = \sum_{a=1}^{m} \sum_{b=1}^{m} \Phi^{-1}_{ab}(\vec{\lambda})\, \mathcal{V}_b = \sum_{a=1}^{m} \mathcal{S}_a$

solve: $\sum_{b=1}^{m} \Phi_{ab}(\vec{\lambda})\, S_b = \mathcal{V}_a \rightarrow$ integral equation

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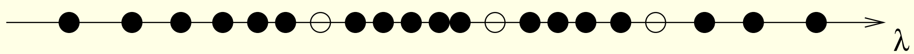
[Hegedűs 17]



$$\mathcal{S}(\lambda)=-\frac{\pi}{\gamma}\frac{1}{\cosh\left(\frac{\pi}{\gamma}(\lambda-\rho_n)\right)}+2\Im m\int_{-\infty}^{\infty}d\lambda'\,G_\lambda(\lambda-\lambda'+i\,\eta)\,\mathcal{S}(\lambda'+i\,\alpha\,\eta)\,\frac{e^{i\,Z_\lambda(\lambda'+i\eta)}}{1+e^{i\,Z_\lambda(\lambda'+i\eta)}}$$

$$\frac{1}{2}\langle\sigma^z_n\rangle_\lambda=-\frac{1}{2(1-\frac{\gamma}{\pi})}\left\{2\Im m\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}\,\mathcal{S}(\lambda+i\eta)\,\frac{e^{i\,Z_\lambda(\lambda+i\eta)}}{1+e^{i\,Z_\lambda(\lambda+i\eta)}}\right\}$$

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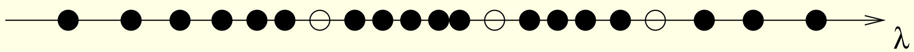
[Hegedűs 17] 

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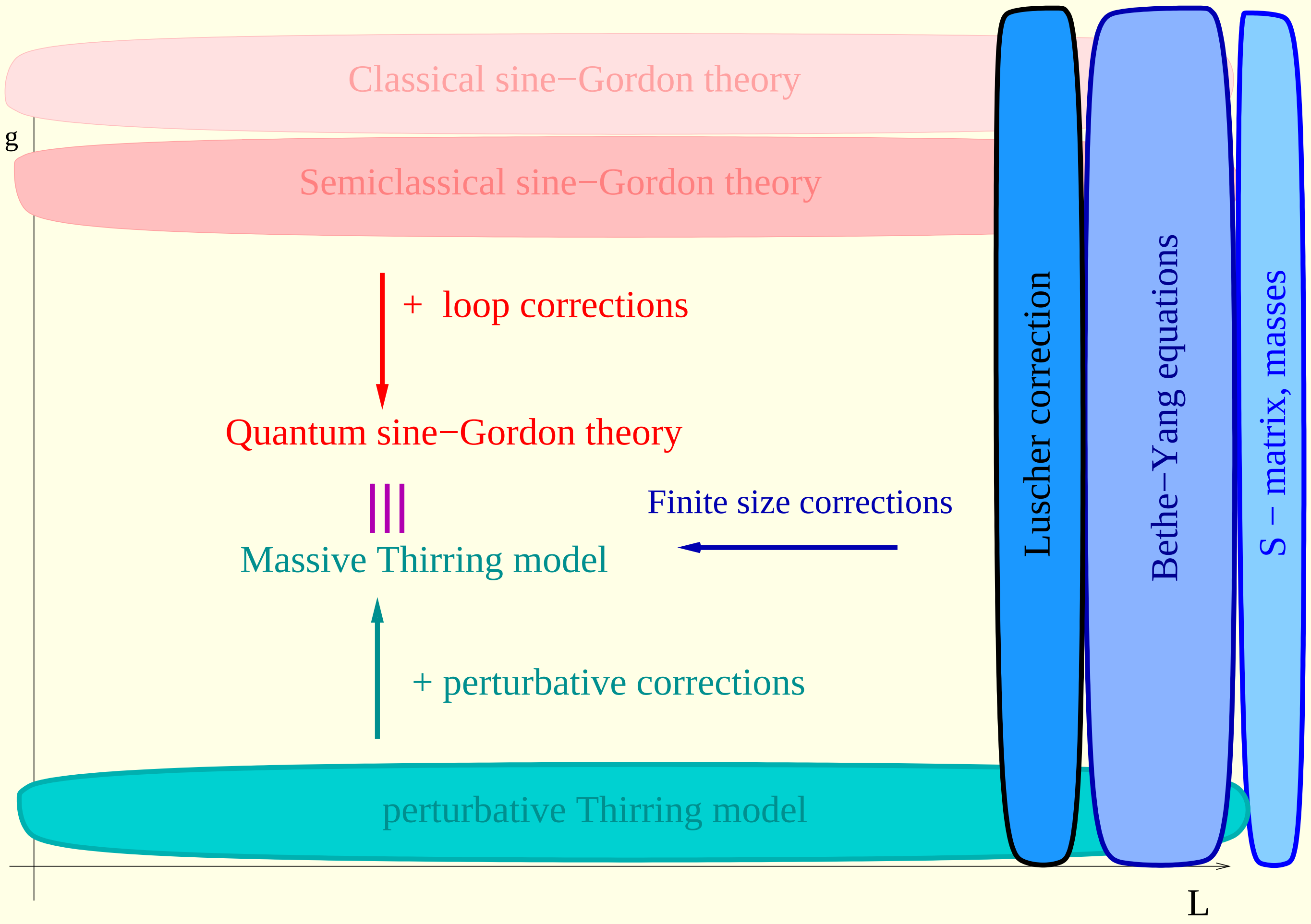
In the continuum limit the volume expansion is like the LeClair-Mussardo formula

$$\langle J_\mu(x)\rangle_0 \;=\; \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{1}{n!\,m!}\int\prod_{i=1}^{N}\frac{d\theta_i}{2\pi}\prod_{i=1}^{n}\frac{e^{i\,Z_\lambda(\lambda+i\eta)}}{1+\;e^{i\,Z_\lambda(\lambda+i\eta)}}\prod_{i=n+1}^{N}\frac{e^{-i\,Z_\lambda(\lambda-i\eta)}}{1+\;e^{-i\,Z_\lambda(\lambda-i\eta)}}\times$$
$$F_c^{J_\mu}(\theta_1+i\,\eta,...,\theta_n+i\,\eta,\theta_{n+1}-i\,\eta,...,\theta_N-i\,\eta)$$

Sine-Gordon/massive Thirring duality

$$\mathcal{L}_{SG} = \frac{1}{2}\partial_\nu \Phi \partial^\nu \Phi + \frac{m^2}{\beta^2} : \cos(\beta\Phi) : \qquad 0 < \beta^2 < 8\pi,$$



strong-weak duality:

$$1 + \frac{g}{4\pi} = \frac{4\pi}{\beta^2} = \frac{p+1}{2p}$$

$$\mathcal{L}_{MT} = \bar\Psi(i\gamma_\nu\partial^\nu + m_0)\Psi - \frac{g}{2}\bar\Psi\gamma^\nu\Psi\bar\Psi\gamma_\nu\Psi$$

Perturbative QFT: sinh-Gordon

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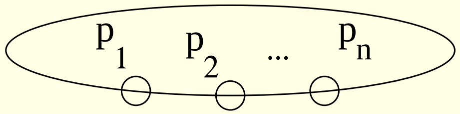
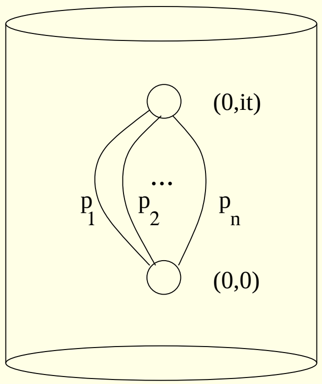
The simplest interacting QFT in 1+1 D: $\mathcal{L} = \frac{1}{2}(\partial_t\varphi)^2 - \frac{1}{2}(\partial_x\varphi)^2 - \frac{m^2}{b^2}(\cosh b\varphi - 1)$

interesting observables: finite size spectrum,



finite size correlators ${}_L\langle 0|\mathcal{O}(it)\mathcal{O}(0)|0\rangle_L = \sum_n |{}_L\langle 0|\mathcal{O}(0)|n\rangle_L|^2 e^{-E_n t}$



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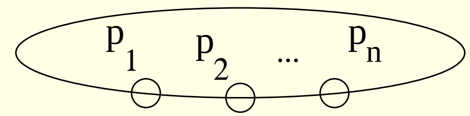

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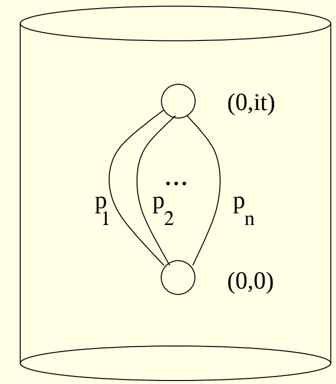
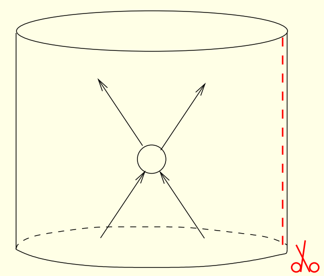
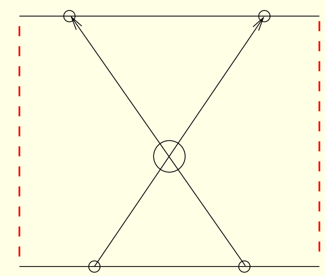


Too difficult, instead

Infinite volume $\to$ LSZ reduction formula





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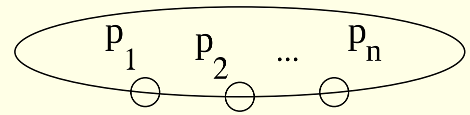

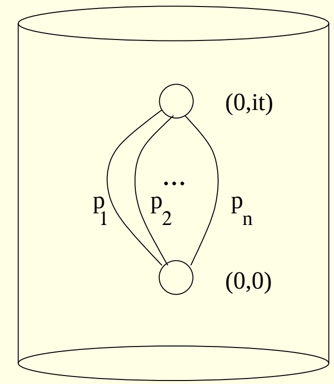

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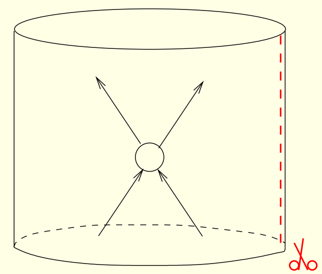

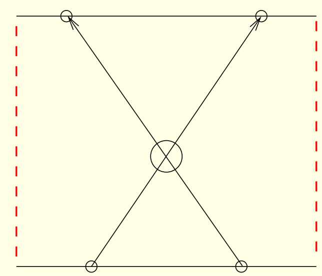

$\langle p_1', p_2'|\mathcal{O}|p_1, p_2\rangle = \bar{\mathcal{D}}_1' \bar{\mathcal{D}}_2' \mathcal{D}_1 \mathcal{D}_2 \langle 0|T(\mathcal{O}\varphi(1)\varphi(2)\varphi(3)\varphi(4))|0\rangle$

where $\mathcal{D}_j = i\int d^2x_j e^{ip_j x - i\omega_j t}\left\{\partial_t^2 - \partial_x^2 + m^2\right\}$ amputates a leg + puts it onshell

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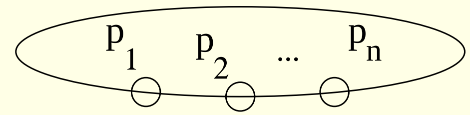

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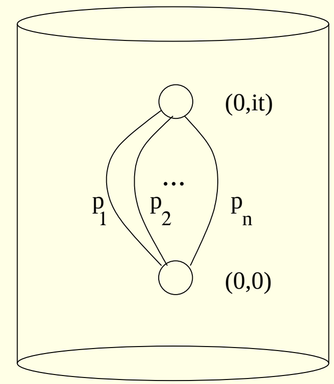

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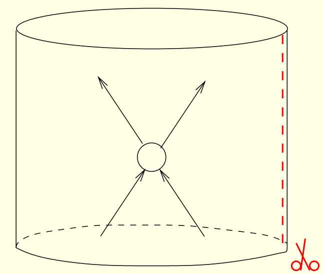
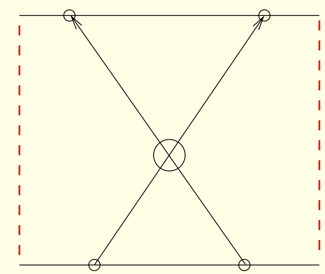




$\langle p_1', p_2'|\mathcal{O}|p_1, p_2\rangle = \bar{\mathcal{D}}_1'\bar{\mathcal{D}}_2'\mathcal{D}_1\mathcal{D}_2\langle 0|T(\mathcal{O}\varphi(1)\varphi(2)\varphi(3)\varphi(4))|0\rangle$

where $\mathcal{D}_j = i\int d^2x_j e^{ip_jx - i\omega_j t}\left\{\partial_t^2 - \partial_x^2 + m^2\right\}$ amputates a leg + puts it onshell

Consequence: perturbative definition, calculational tool: [Arefyeva et al]

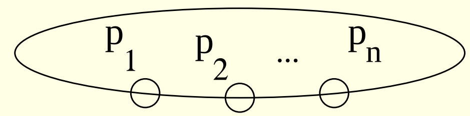
$S(\theta) = 1 - \frac{1}{4}ib^2\text{csch}\theta - \frac{b^4(\text{csch}\theta(\pi\text{csch}\theta - i))}{32\pi} + \frac{ib^6\text{csch}\theta(\pi\text{csch}\theta - i)^2}{256\pi^2} + O\left(b^8\right)$

Mandelstam variable $s = 4m^2\cosh^2\frac{\theta}{2}$ where $\theta = \theta_1 - \theta_2$ rapidity: $p_i = m\sinh\theta_i$

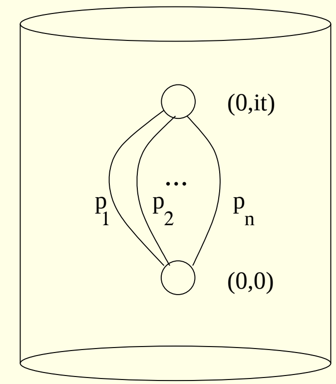
Perturbative QFT: sinh-Gordon

The simplest interacting QFT in 1+1 D: $\mathcal{L} = \frac{1}{2}(\partial_t\varphi)^2 - \frac{1}{2}(\partial_x\varphi)^2 - \frac{m^2}{b^2}(\cosh b\varphi - 1)$

interesting observables: finite size spectrum,



finite size correlators ${}_L\langle 0|\mathcal{O}(it)\mathcal{O}(0)|0\rangle_L = \sum_n |{}_L\langle 0|\mathcal{O}(0)|n\rangle_L|^2 e^{-E_n t}$



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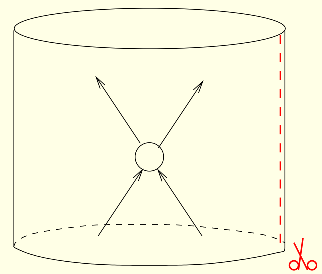

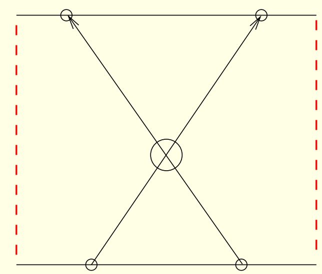

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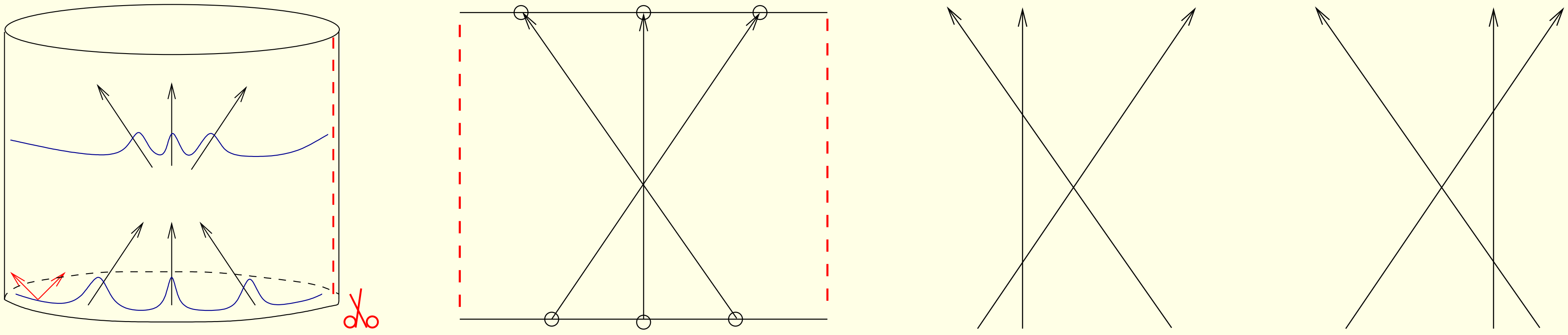
known analytical properties: unitarity, crossing $S(\theta) = S(-\theta)^{-1} = S(i\pi - \theta)$

S-matrix bootstrap

S-matrix bootstrap

S-matrix bootstrap: Calculate the two particle S-matrix [Zamolodchikov^2 '79,Dorey]



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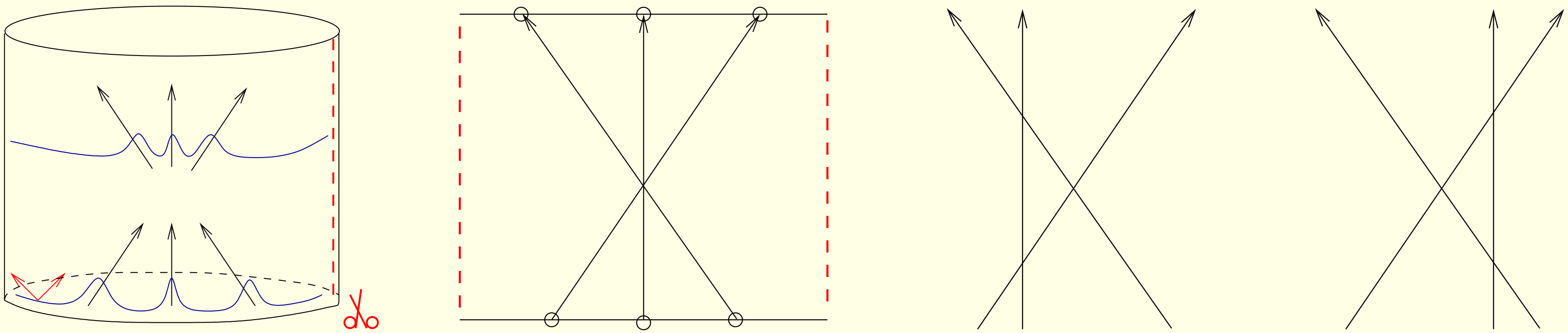
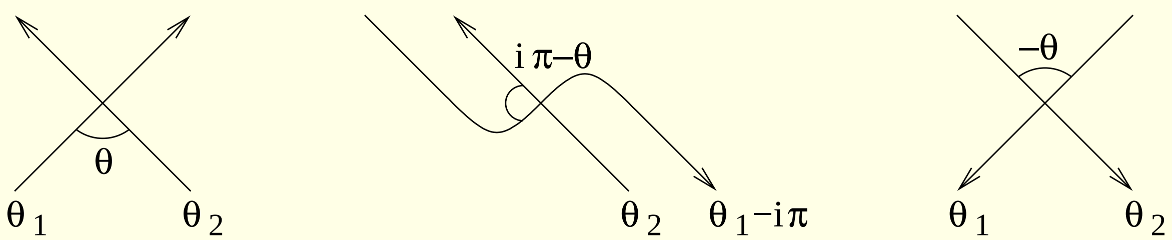


Infinite volume → crossing symmetry, $\theta \to i\pi - \theta$ in rapidity

$(E(\theta), p(\theta)) = m(\cosh\theta, \sinh\theta)$



$S(\theta_1 - \theta_2) = S(\theta) = S(i\pi - \theta) = S(-\theta)^{-1}$:

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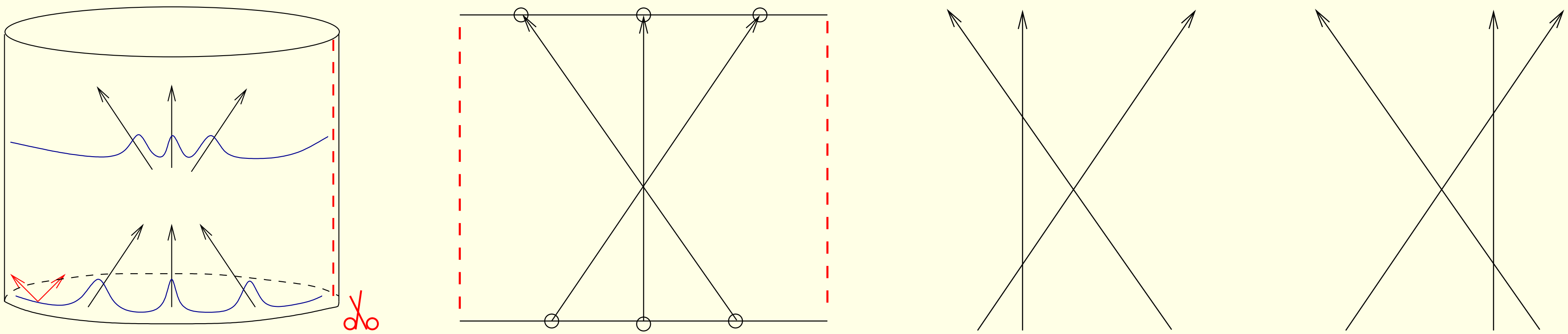

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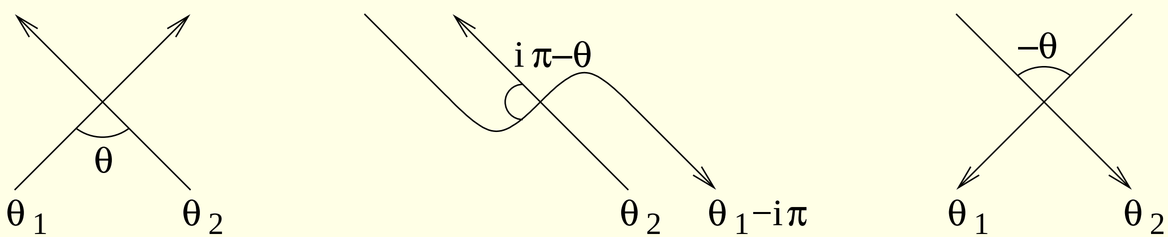


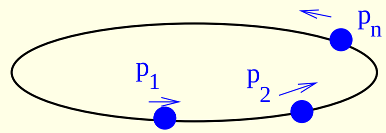
Simplest solution:
sinh-Gordon

$$S(\theta) = \frac{\sinh\theta - i\sin a}{\sinh\theta + i\sin a} \qquad a = \frac{\pi b^2}{8\pi + b^2}$$

$S(\theta_1 - \theta_2) = S(\theta) = S(i\pi - \theta) = S(-\theta)^{-1}$:

Spectral problem

Finite volume spectrum



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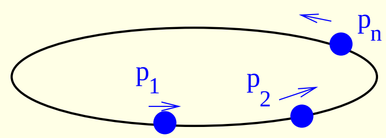

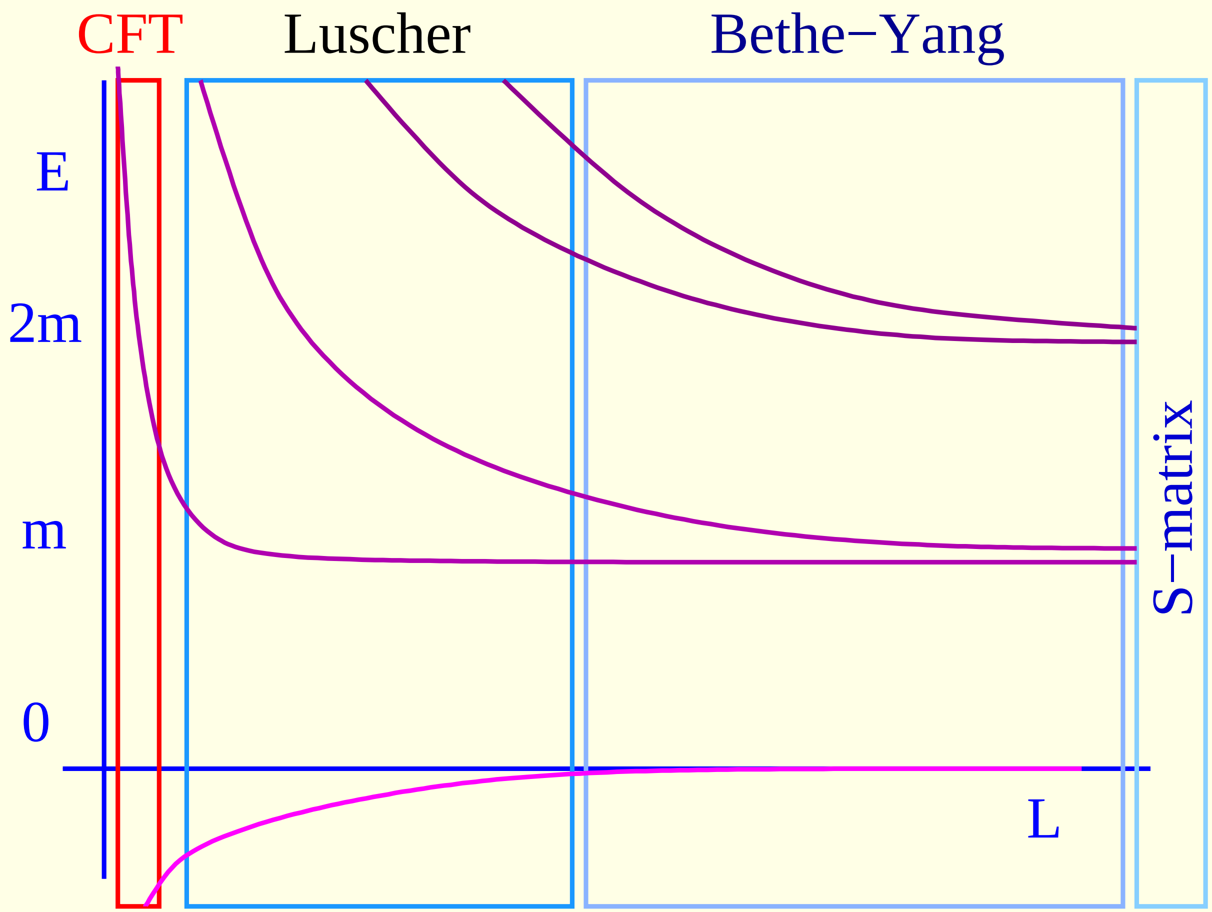

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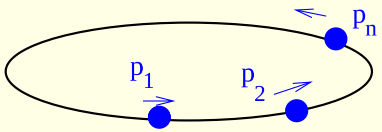
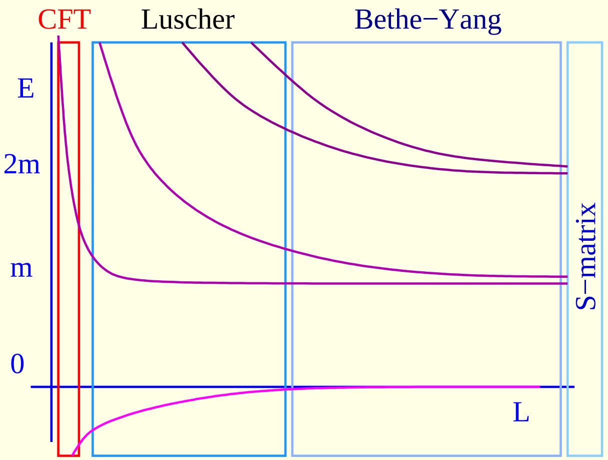
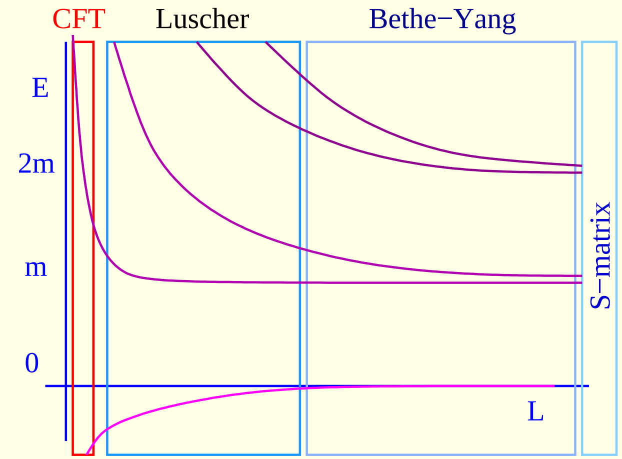




Polynomial volume corrections:

$E(p_1,\dots,p_n)=\sum_i E(p_i)$

Spectral problem



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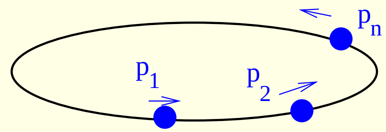

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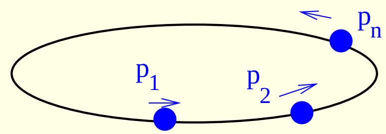
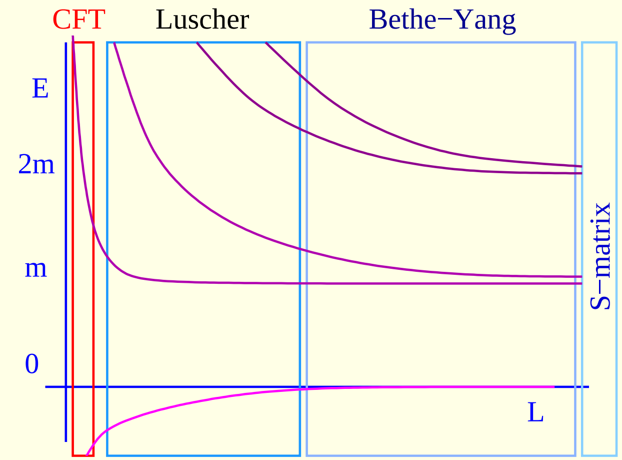
Spectral problem

Finite volume spectrum





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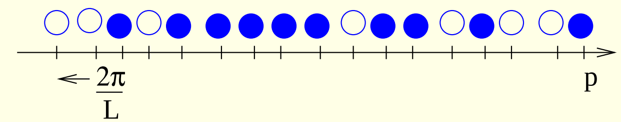

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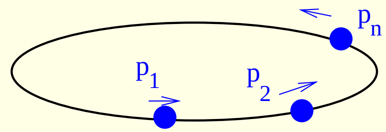
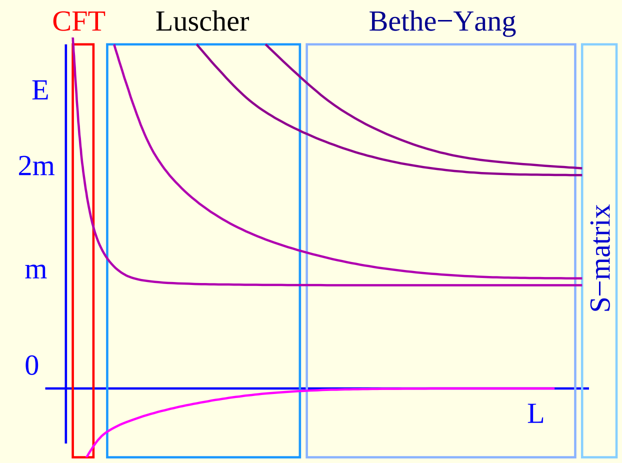
Spectral problem

Finite volume spectrum





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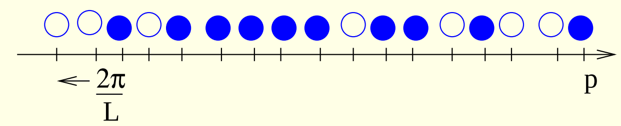

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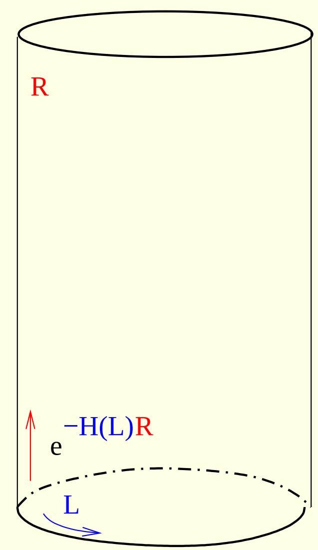


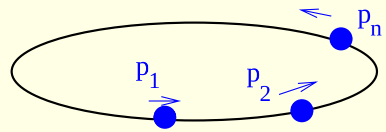
Ground-state energy from

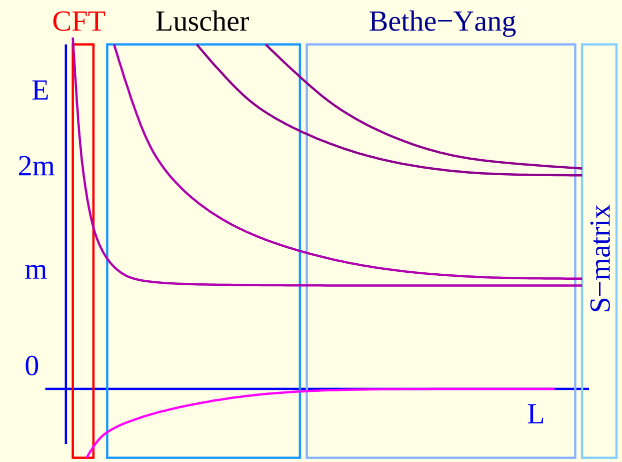
Euclidian partition function:

$Z(L,R)=_{R\to\infty}\mathrm{Tr}(e^{-H(L)R})=e^{-E_0(L)R}+\dots$

Spectral problem

Finite volume spectrum





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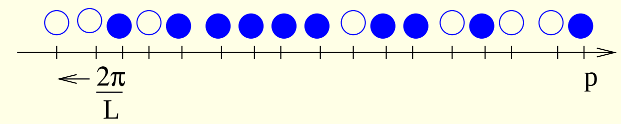

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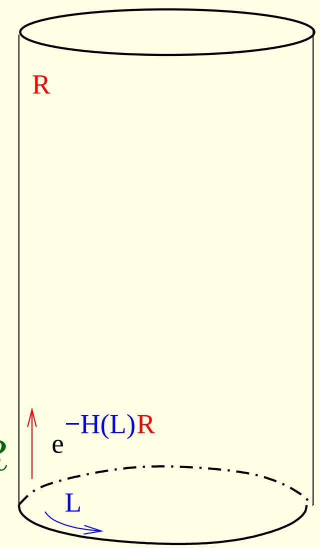
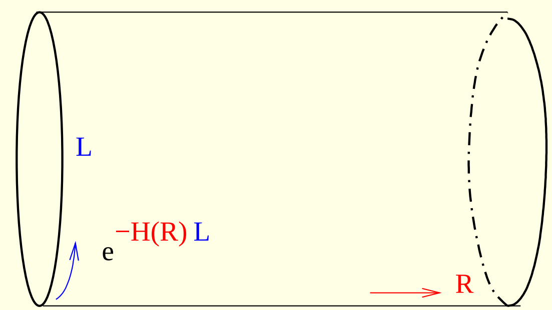
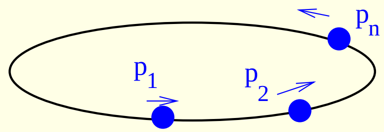
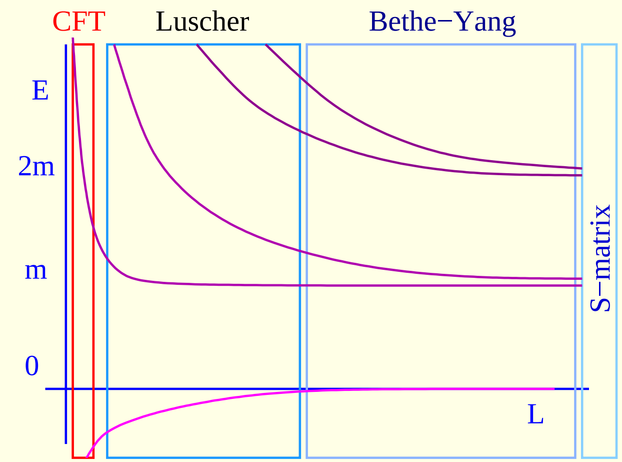




Spectral problem

Finite volume spectrum





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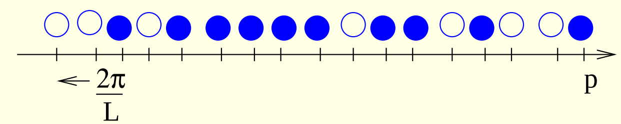

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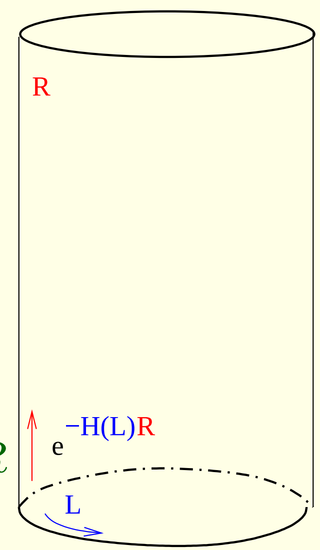
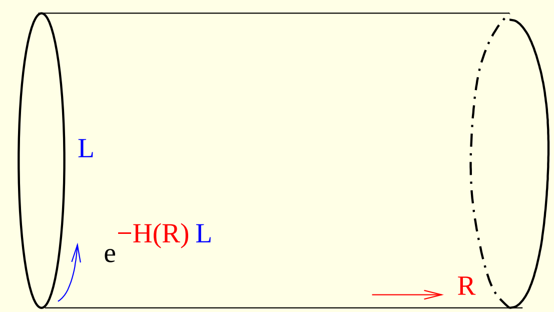
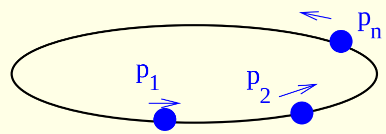


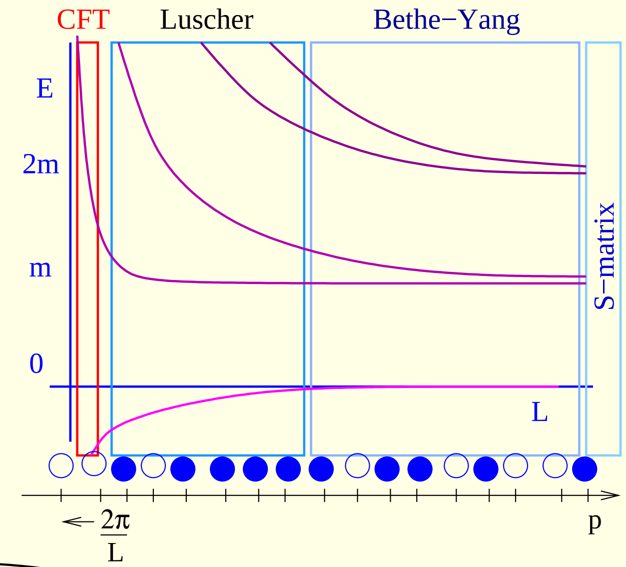


Main contribution: finite density ρ, ρ_h

Spectral problem

Finite volume spectrum





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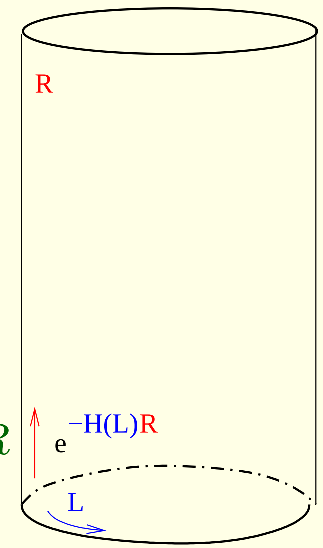
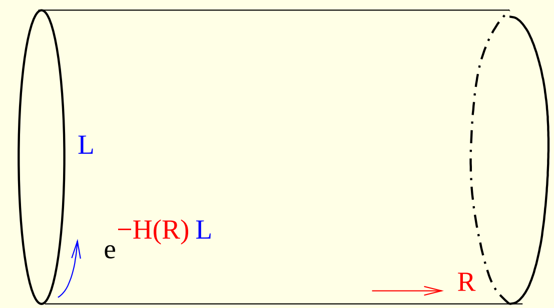




Main contribution: finite density ρ,ρ_h

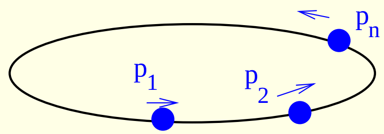
Large volume: Bethe-Yang can be used

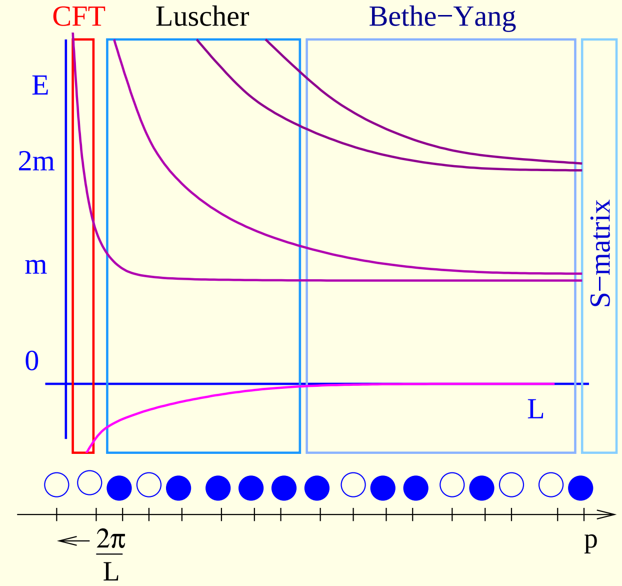
$p_jR+\sum_k \frac{1}{i}\log S(p_j,p_k)=(2n+1)\pi \quad\longrightarrow\quad R+\int(-id_p\log S(p,p'))\rho(p')dp'=2\pi(\rho+\rho_h)$

$Z(L,R)=\int d[\rho,\rho_h]e^{-LE(R)-\int((\rho+\rho_h)\ln(\rho+\rho_h)-\rho\ln\rho-\rho_h\ln\rho_h)dp}$

Spectral problem

Finite volume spectrum





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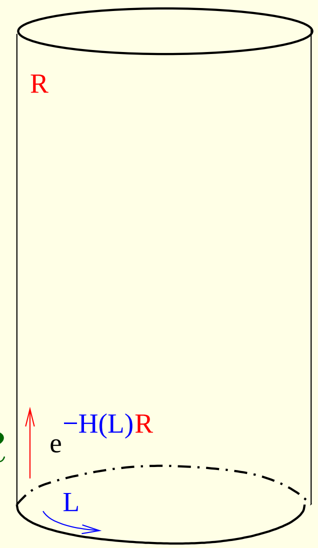
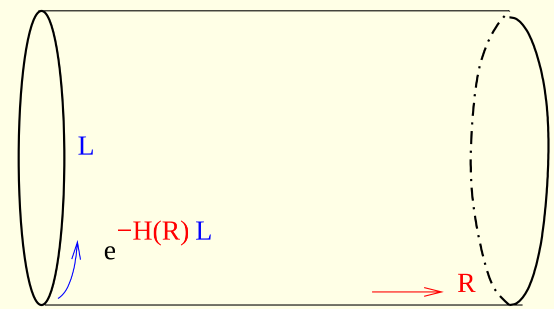




Main contribution: finite density ρ, ρ_h

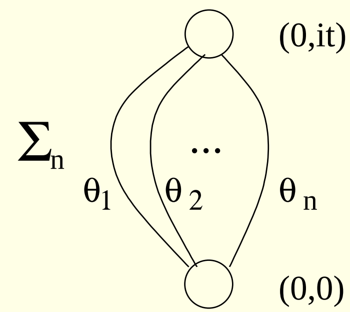
Saddle point : $\epsilon(p) = \ln \frac{\rho_h(p)}{\rho(p)}$ $\epsilon(p) = E(p)L + \int \frac{dp}{2\pi} i d_p \log S(p',p) \log(1+e^{-\epsilon(p')})$

Ground state energy exactly: $E_0(L) = -\int \frac{dp}{2\pi} \log(1+e^{-\epsilon(p)})$ [Zamolodchikov]

Form factor bootstrap

Form factor bootstrap

Correlation functions: [Smirnov, Karowszki] $\langle 0|\mathcal{O}(it)\mathcal{O}(0)|0\rangle =$



$\sum_n \frac{1}{n!}\int\frac{d\theta_1}{2\pi}\cdots\int\frac{d\theta_n}{2\pi}|\langle 0|\mathcal{O}(0)|\theta_1,\dots,\theta_n\rangle|^2 e^{-m(\sum_i \cosh\theta_i)t}$

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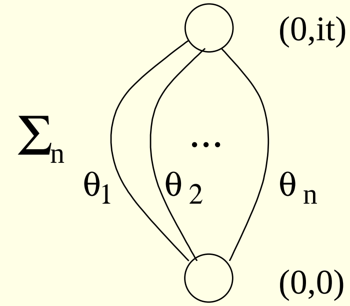

$$\sum_n \frac{1}{n!}\int \frac{d\theta_1}{2\pi}\cdots\int\frac{d\theta_n}{2\pi}|\langle 0|\mathcal{O}(0)|\theta_1,\dots,\theta_n\rangle|^2 e^{-m(\sum_i \cosh\theta_i)t}$$

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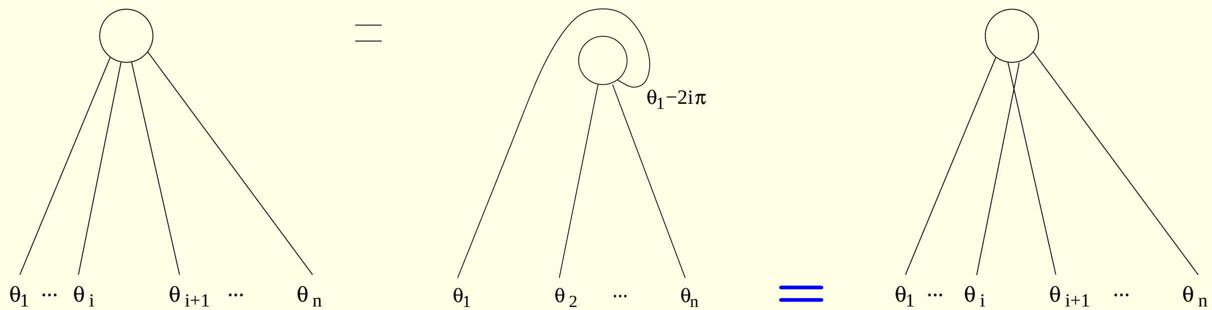
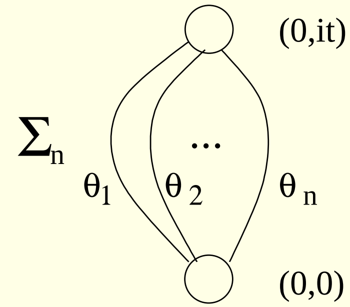


$\langle 0|\mathcal{O}|\theta_1,\dots,\theta_n\rangle =$

$F(\theta_1,\dots,\theta_n) = \quad F(\theta_2,\dots,\theta_n,\theta_1-2i\pi\rangle \quad = S(\theta_i-\theta_{i+1})F(\theta_1,\dots,\theta_{i+1},\theta_i,\dots,\theta_n)$

Form factor bootstrap

Correlation functions: [Smirnov, Karowszki] $\langle 0|\mathcal{O}(it)\mathcal{O}(0)|0\rangle =$



$\sum_n \frac{1}{n!}\int \frac{d\theta_1}{2\pi}\cdots\int \frac{d\theta_n}{2\pi}|\langle 0|\mathcal{O}(0)|\theta_1,\dots,\theta_n\rangle|^2 e^{-m(\sum_i \cosh\theta_i)t}$

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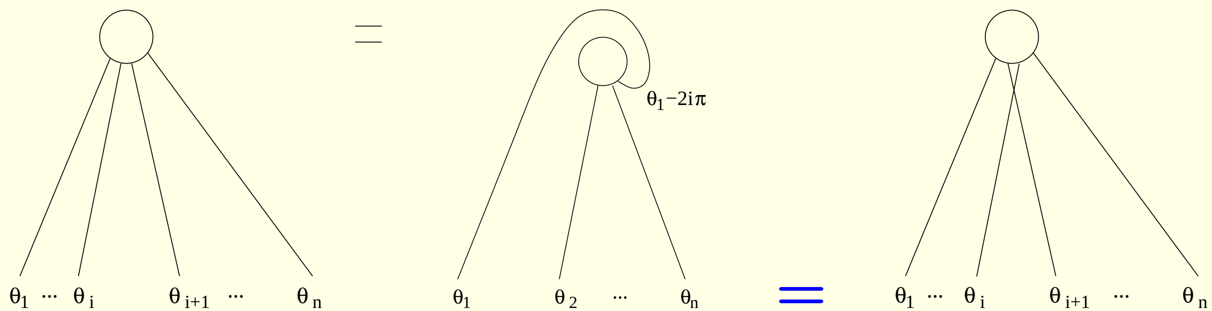
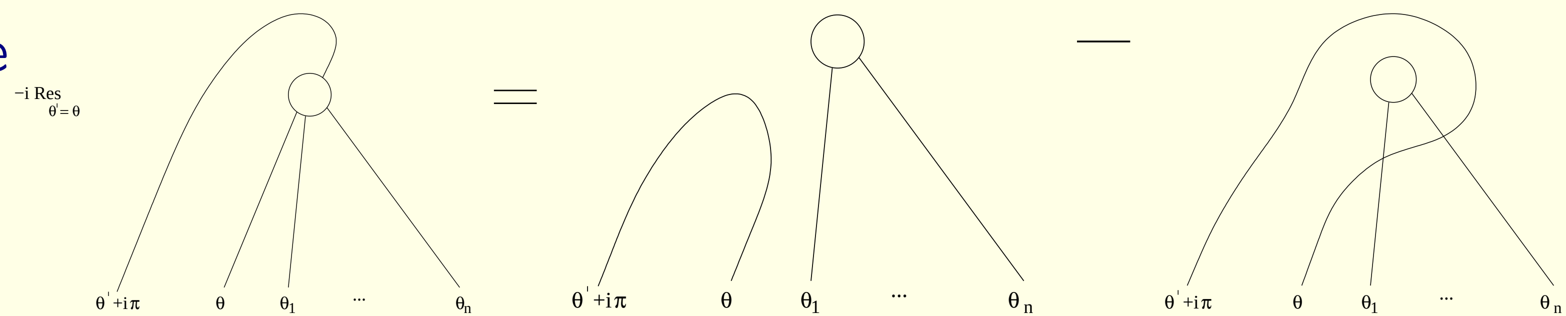


$\langle 0|\mathcal{O}|\theta_1,\dots,\theta_n\rangle =$

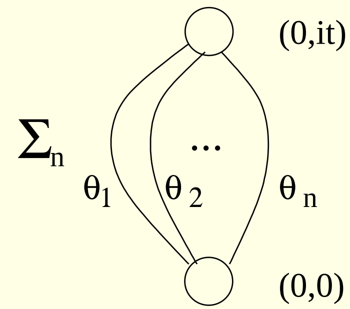
$F(\theta_1,\dots,\theta_n) = \quad F(\theta_2,\dots,\theta_n,\theta_1-2i\pi\rangle \quad = S(\theta_i-\theta_{i+1})F(\theta_1,\dots,\theta_{i+1},\theta_i,\dots,\theta_n)$

Singularity stucture



$$-i\text{Res}_{\theta'=\theta}F(\theta'+i\pi,\theta,\theta_1\dots,\theta_n) = (1-\prod_i S(\theta-\theta_i))F(\theta_1,\dots,\theta_n)$$

Form factor bootstrap



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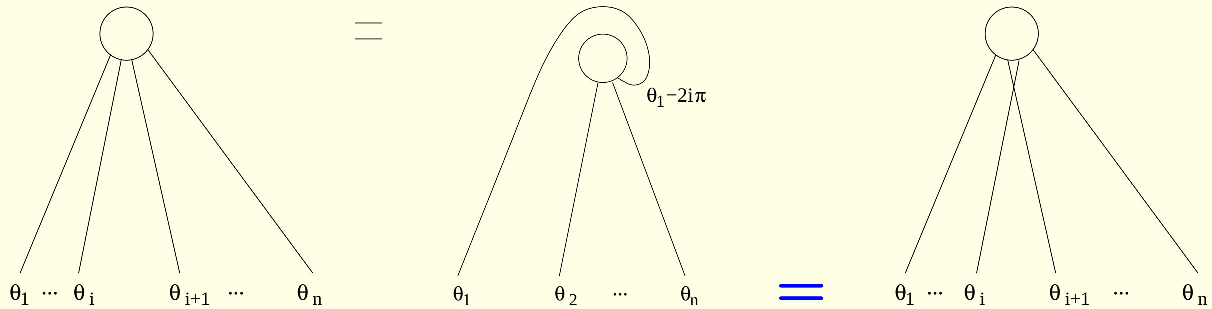

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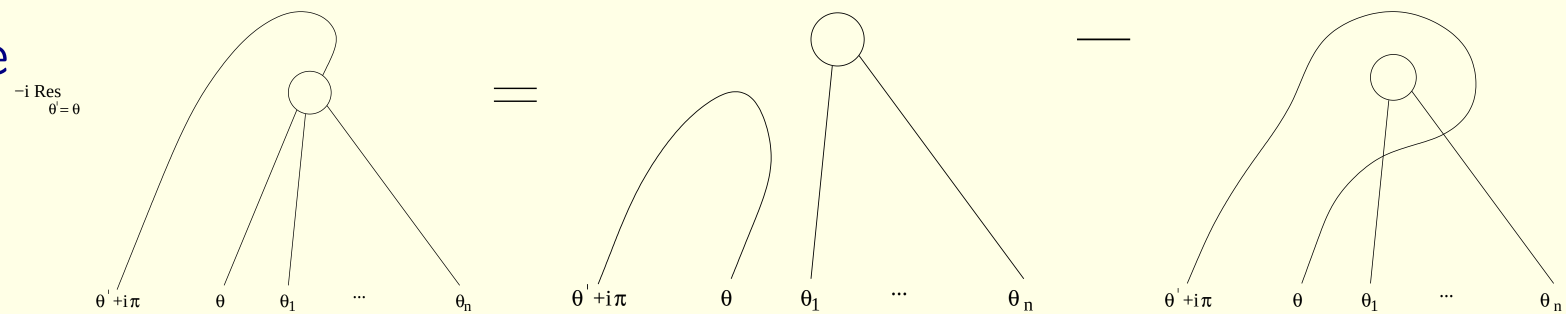

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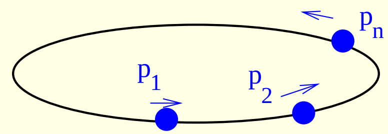
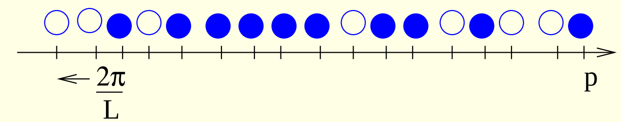
Solution for sinh-Gordon: $f(\theta_1-\theta_2) = e^{(D+D^{-1})^{-1}\log S}$; $Df(\theta) = f(\theta+i\pi)$
[Fring, Mussardo, Simonetti]

Finite volume form factors

Finite volume state $|\{\theta_i\}\rangle_L \equiv |\{n_i\}\rangle$





Polynomial volume corrections: $Q_j = p(\theta_j)L + \sum_k \frac{1}{i}\log S(\theta_j - \theta_k) = (2n_j + 1)\pi$

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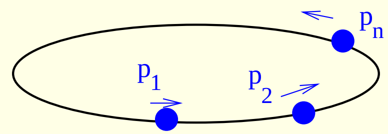

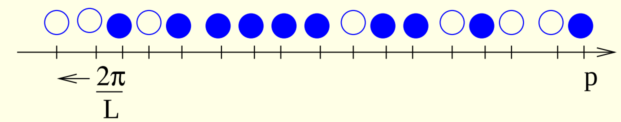

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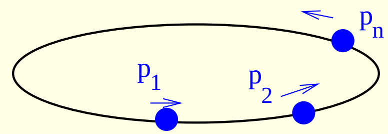

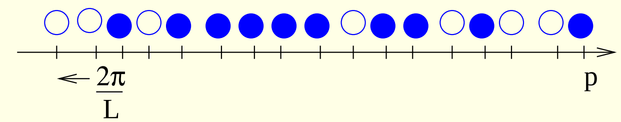

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[proved Pozsgay, Takacs]

crossing $\bar\theta = \theta + i\pi$

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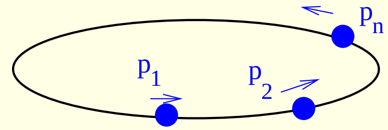

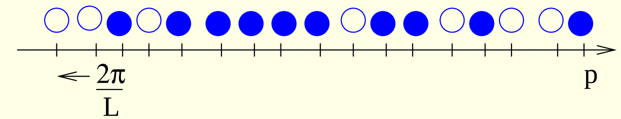

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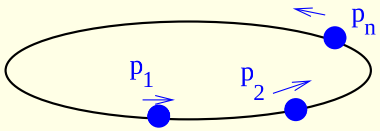
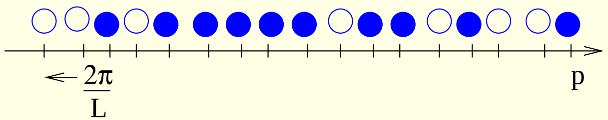
Finite volume state $|\{\theta_i\}\rangle_L \equiv |\{n_i\}\rangle$ 



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Finite volume form factors

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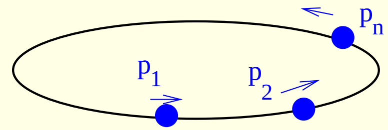

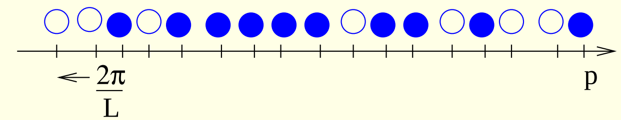

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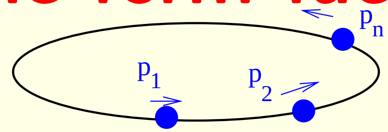
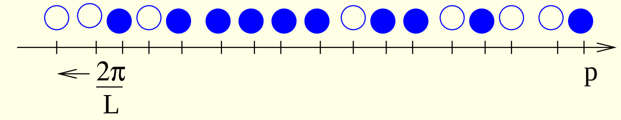
BY: $Q_1 = p_1 L - i\log S(\theta_1 - \theta_2) = 2\pi n_1$ and $Q_1 = p_2 L - i\log S(\theta_2 - \theta_1) = 2\pi n_2$

Finite volume form factors





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$$\rho_2(\theta_1,\theta_2) = \begin{vmatrix} E_1L+\phi & -\phi \\ -\phi & E_2L+\phi \end{vmatrix} = E_1E_2L^2 + \phi(E_1+E_2)L \qquad \phi(\theta) = -i\partial_\theta \log S(\theta)$$

and $\rho_1(\theta_1) = E_1L + \phi \quad ; \qquad \rho_1(\theta_2) = E_2L + \phi$

Connected form factors

We need $F(\bar{\theta}_1 + \epsilon_1, \ldots, \bar{\theta}_n + \epsilon_n, \theta_n, \ldots, \theta_1)$

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BUT: kinematical singularity: $F(\bar{\theta}+\epsilon,\theta,\theta_1\ldots,\theta_n)=\frac{i}{\epsilon}(1-\prod_i S(\theta-\theta_i))F(\theta_1,\ldots,\theta_n)$

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Graphical representation: $F(\bar{\theta}_1+\epsilon_1,\dots,\bar{\theta}_n+\epsilon_n,\theta_n,\dots,\theta_1)=\sum_{\text{graphs}}F_{\text{graphs}}$
[Pozsgay, Takacs]

graphs: oriented, tree-like, at each vertex only at most one outgoing edge

contributions: $(i_1,\dots,i_k)$ with no outgoing edges $F^c(\theta_{i_1},\dots,\theta_{i_k})$,
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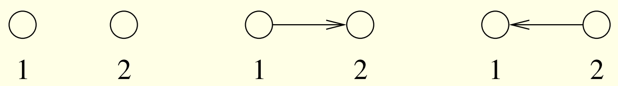

which gives $F_4(\bar{\theta}_1+\epsilon_1,\bar{\theta}_2+\epsilon_2,\theta_2,\theta_1)=F_4^c(\theta_1,\theta_2)+\frac{\epsilon_1}{\epsilon_2}\phi_{12}F_2^c(\theta_1)+\frac{\epsilon_2}{\epsilon_1}\phi_{21}F_2^c(\theta_2)$

Proof of $\frac{\sum_{\alpha\cup\bar{\alpha}} F^c_\alpha \rho_{\bar{\alpha}}}{\rho_n}$

Idea: take $\langle\theta,\theta'_1,\ldots,\theta'_n|\mathcal{O}|\theta_n,\ldots,\theta_1\rangle_L = \frac{F_{2n+1}(\bar{\theta},\bar{\theta}'_1,\ldots,\bar{\theta}'_n,\theta_n,\ldots,\theta_1)}{\sqrt{\rho_{n+1}\rho_n}}$ and $\theta\to\infty$

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graphs: oriented, tree-like, at each vertex only at most one outgoing edge,
vertex with no outgoing edge can be black or white

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Proof of $\frac{\sum_{\alpha\cup\bar{\alpha}} F^c_\alpha \rho_{\bar{\alpha}}}{\rho_n}$

Idea: take $\langle\theta,\theta'_1,\ldots,\theta'_n|\mathcal{O}|\theta_n,\ldots,\theta_1\rangle_L = \frac{F_{2n+1}(\bar{\theta},\bar{\theta}'_1,\ldots,\bar{\theta}'_n,\theta_n,\ldots,\theta_1)}{\sqrt{\rho_{n+1}\rho_n}}$ and $\theta\to\infty$

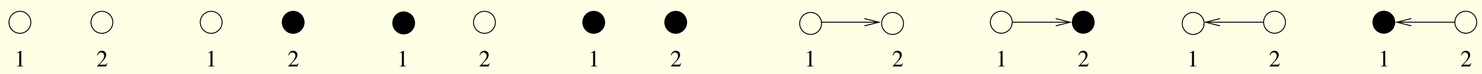
with $Q'_j \equiv p(\theta'_j)L - i\sum_{k:k\neq j}\log S(\theta'_j-\theta'_k) - i\log S(\theta'_j-\theta) = 2\pi n_j$

$Q'_j - Q_j = E_j L\epsilon_j + \sum_{k:k\neq j}\phi_{jk}(\epsilon_j-\epsilon_k) - \delta_j = \partial_j Q_k\epsilon_k - \delta_j = 0$

Graphical rep.: $F(\bar{\theta},\bar{\theta}_1+\epsilon_1,\ldots,\bar{\theta}_n+\epsilon_n,\theta_n,\ldots,\theta_1) = \sum_{\text{colored graphs}} F_{\text{graphs}}$

graphs: oriented, tree-like, at each vertex only at most one outgoing edge,
vertex with no outgoing edge can be black or white

contributions: $(i_1,\ldots,i_k)$ with no outgoing white edges $F^c(\theta_{i_1},\ldots,\theta_{i_k})$,
for each edge from i to j : $\frac{\epsilon_j}{\epsilon_i}\phi(\theta_i-\theta_j)$, i^{th} black edge $\frac{\delta_i}{\epsilon_i}$:



$F^c_4(\theta_1,\theta_2) + \frac{\epsilon_2}{\epsilon_1}\phi_{21}\frac{\delta_2}{\epsilon_2} + \frac{\epsilon_1}{\epsilon_2}\phi_{12}\frac{\delta_1}{\epsilon_1} + \frac{\delta_1}{\epsilon_1}\frac{\delta_2}{\epsilon_2} + \frac{\epsilon_1}{\epsilon_2}\phi_{12}F^c_2(\theta_1) + \frac{\delta_2}{\epsilon_2}F^c_2(\theta_1) + \frac{\epsilon_2}{\epsilon_1}\phi_{21}F^c_2(\theta_2) + \frac{\delta_1}{\epsilon_1}F^c_2(\theta_2)$

Proof of $\frac{\sum_{\alpha\cup\bar{\alpha}} F^c_\alpha \rho_{\bar{\alpha}}}{\rho_n}$

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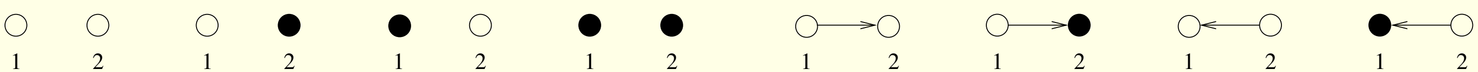


$F^c_4(\theta_1, \theta_2) + \frac{\epsilon_2}{\epsilon_1}\phi_{21}\frac{\delta_2}{\epsilon_2} + \frac{\epsilon_1}{\epsilon_2}\phi_{12}\frac{\delta_1}{\epsilon_1} + \frac{\delta_1}{\epsilon_1}\frac{\delta_2}{\epsilon_2} + \frac{\epsilon_1}{\epsilon_2}\phi_{12}F^c_2(\theta_1) + \frac{\delta_2}{\epsilon_2}F^c_2(\theta_1) + \frac{\epsilon_2}{\epsilon_1}\phi_{21}F^c_2(\theta_2) + \frac{\delta_1}{\epsilon_1}F^c_2(\theta_2)$

use BY to eliminate δ: $\frac{\epsilon_1\phi_{12} + \delta_1}{\epsilon_2} = E_2 L + \phi_{21} = \rho_1(\theta_2)$ and $\frac{\delta_1}{\epsilon_1}\frac{\delta_2}{\epsilon_2} + \frac{\epsilon_1}{\epsilon_2}\phi_{12}\frac{\delta_1}{\epsilon_1} + \frac{\epsilon_2}{\epsilon_1}\phi_{21}\frac{\delta_2}{\epsilon_2} = \rho_2(\theta_1, \theta_2)$

giving: $F^c_4(\theta_1, \theta_2) + \rho_1(\theta_2)F^c_2(\theta_1) + \rho_1(\theta_1)F^c_2(\theta_2) + \rho_2(\theta_1, \theta_2)$

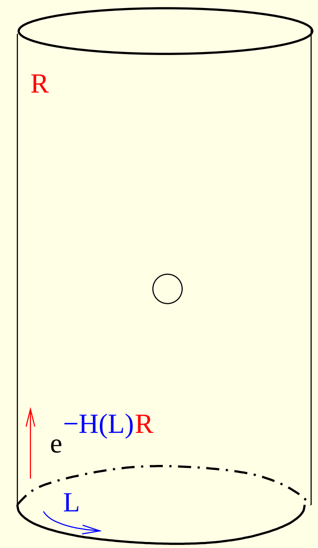
Proof of LM

Proof of LM

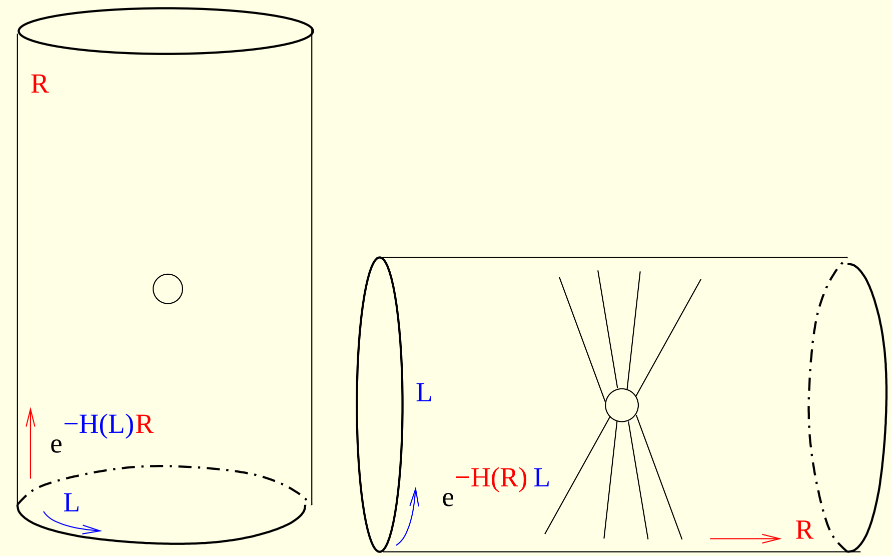
LeClair-Mussardo formula

from thermal evaluation:

$\langle 0|\mathcal{O}|0\rangle_L =_{R\to\infty} \mathrm{Tr}(\mathcal{O}e^{-H(L)R})/Z(L,R) + \ldots$



Proof of LM



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Exchange space and Euclidian time

$$_{R\to\infty}\mathrm{Tr}(\mathcal{O}e^{-H(L)R})/Z =_{R\to\infty} \mathrm{Tr}(e^{-H(R)L})/Z$$

$$=_{R\to\infty} \frac{\sum_n \langle n|\mathcal{O}|n\rangle e^{-E_n(L)R}}{\sum_n e^{-E_n(L)R}}$$

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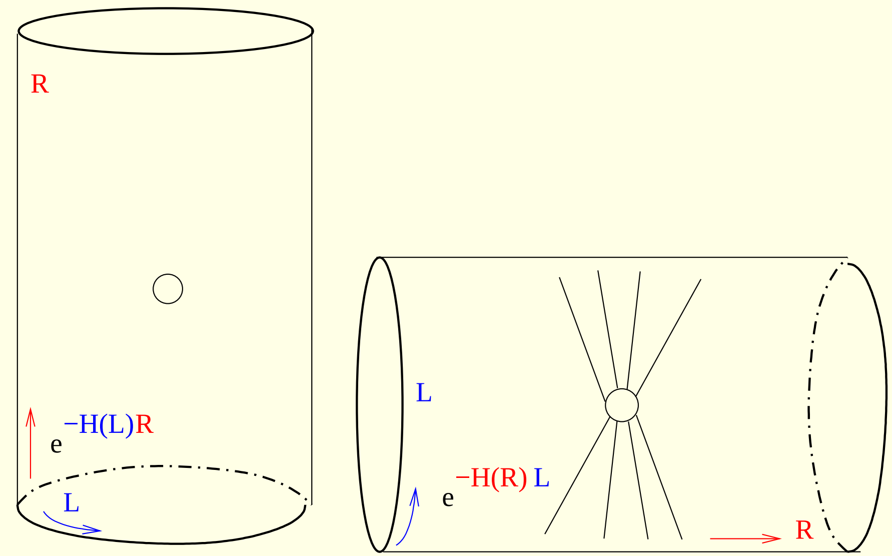

Main contribution: finite density ρ, ρ_h

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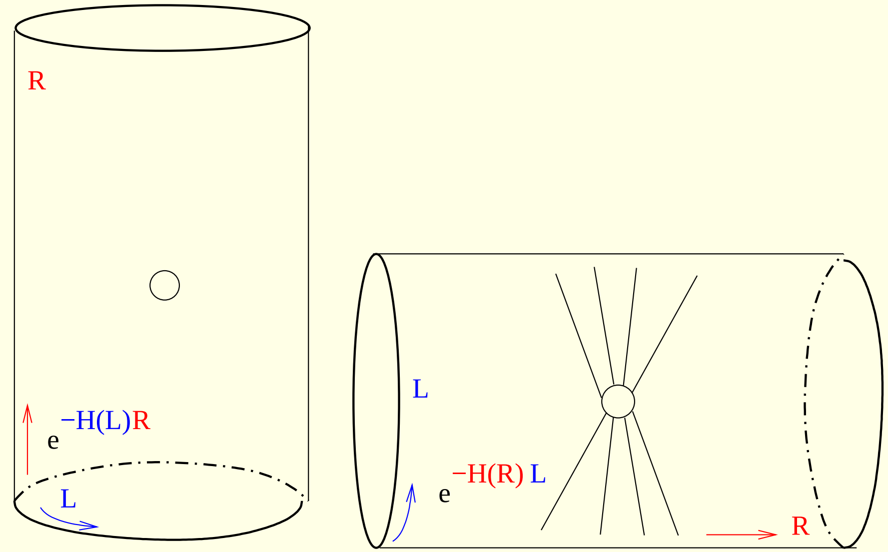

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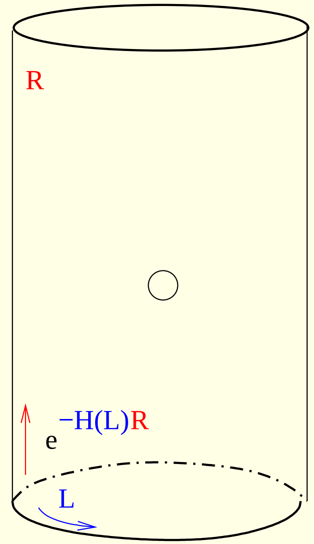
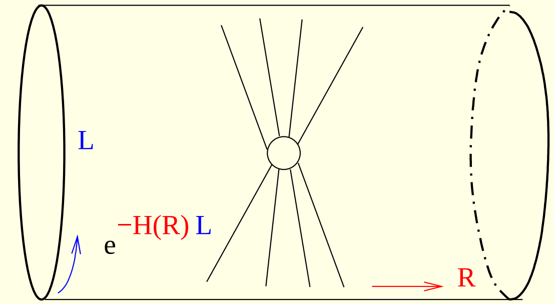
we need $\langle \rho, \rho_n|\mathcal{O}|\rho, \rho_n\rangle$ in a highly excited Bethe state [Pozsgay]

Large volume: asymptotic formula $\frac{\sum_{\alpha\cup\bar{\alpha}} F^c_\alpha \rho_{\bar{\alpha}}}{\rho_n}$ can be used as

$\frac{\sum_{\alpha\cup\bar{\alpha}} F^c_\alpha \rho_{\bar{\alpha}}}{\rho_n} = F + \lim_{n\to\infty} \int \frac{d\theta}{2\pi} F^c(\theta)\frac{\rho_{n-1}}{\rho_n} + \dots$ leading to $F_0 + \int \frac{d\theta}{2\pi} F^c(\theta)\frac{e^{-\epsilon(\theta)}}{1+e^{-\epsilon(\theta)}} + \dots$

Proof of LM





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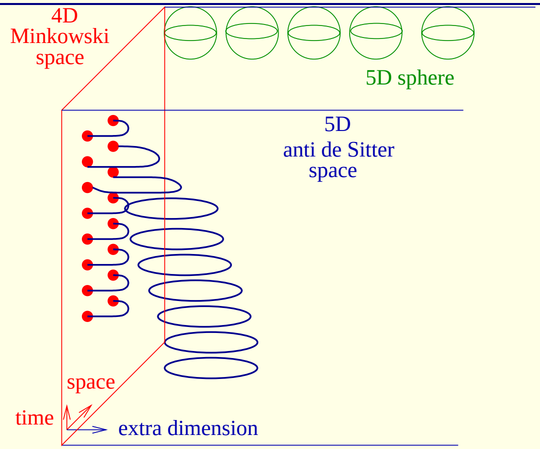
Saddle point : $\epsilon(p) = \ln\frac{\rho_h(p)}{\rho(p)}$ $\quad \epsilon(\theta) = E(\theta)L - \int \frac{d\theta}{2\pi}\phi(\theta-\theta')\log(1+e^{-\epsilon(\theta')})$

Finite volume expectation value: $\langle \mathcal{O}\rangle_L = \sum_n \frac{1}{n!}\prod_{j=1}^n \int \frac{d\theta_i}{2\pi}\frac{e^{-\epsilon(\theta)}}{1+e^{-\epsilon(\theta)}} F^c(\theta_1,\dots,\theta_n)$

[LeClair-Mussardo] excited states [Pozsgay]

AdS/CFT correspondence (Maldacena 1998)

II_B superstring on $AdS_5 \times S^5$



$\sum_1^6 Y_i^2 = R^2 \qquad - + + + + - = -R^2$

$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M \right) + \dots$

$\equiv$

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

$$\frac{2}{g_{YM}^2} \int d^4x \mathrm{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \overline{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \overline{\Psi} [\Phi, \Psi]$$

$\beta = 0$ superconformal $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

gaugeinvariants:$\mathcal{O} = \mathrm{Tr}(\Phi^2), \det(\ )$

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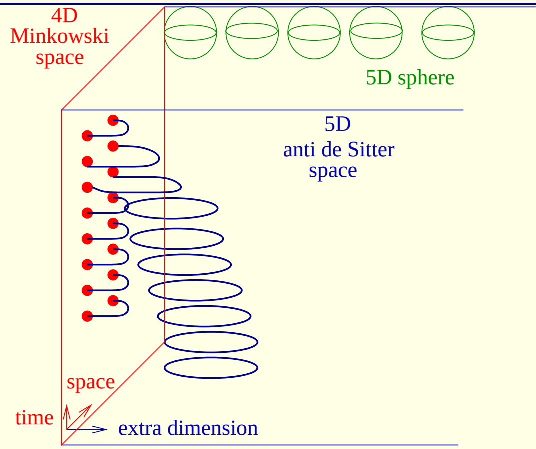

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Dictionary

Coupl.: $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, $g_s = \frac{\lambda}{N} \to 0$

2D QFT

String energy levels: $E(\lambda)$

$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$$

strong$\leftrightarrow$weak

$\Downarrow$

$\lambda = g_{YM}^2 N$, $N \to \infty$ planar

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Anomalous dim $\Delta(\lambda)$

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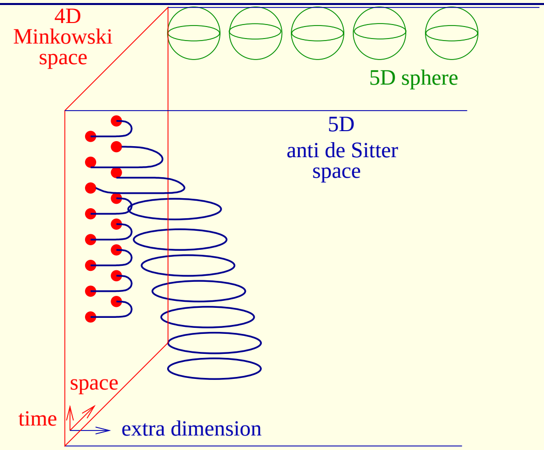

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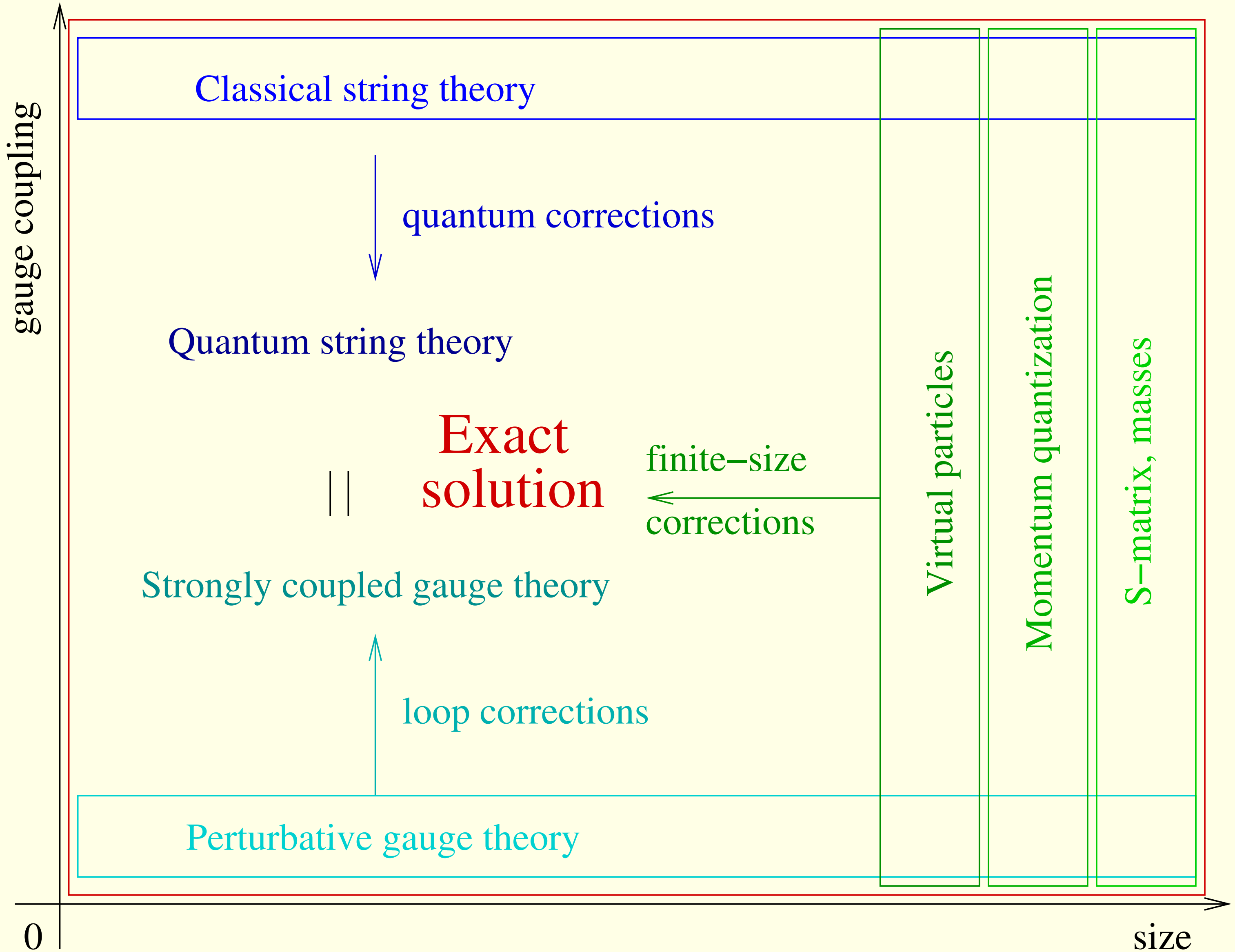
2D integrable QFT

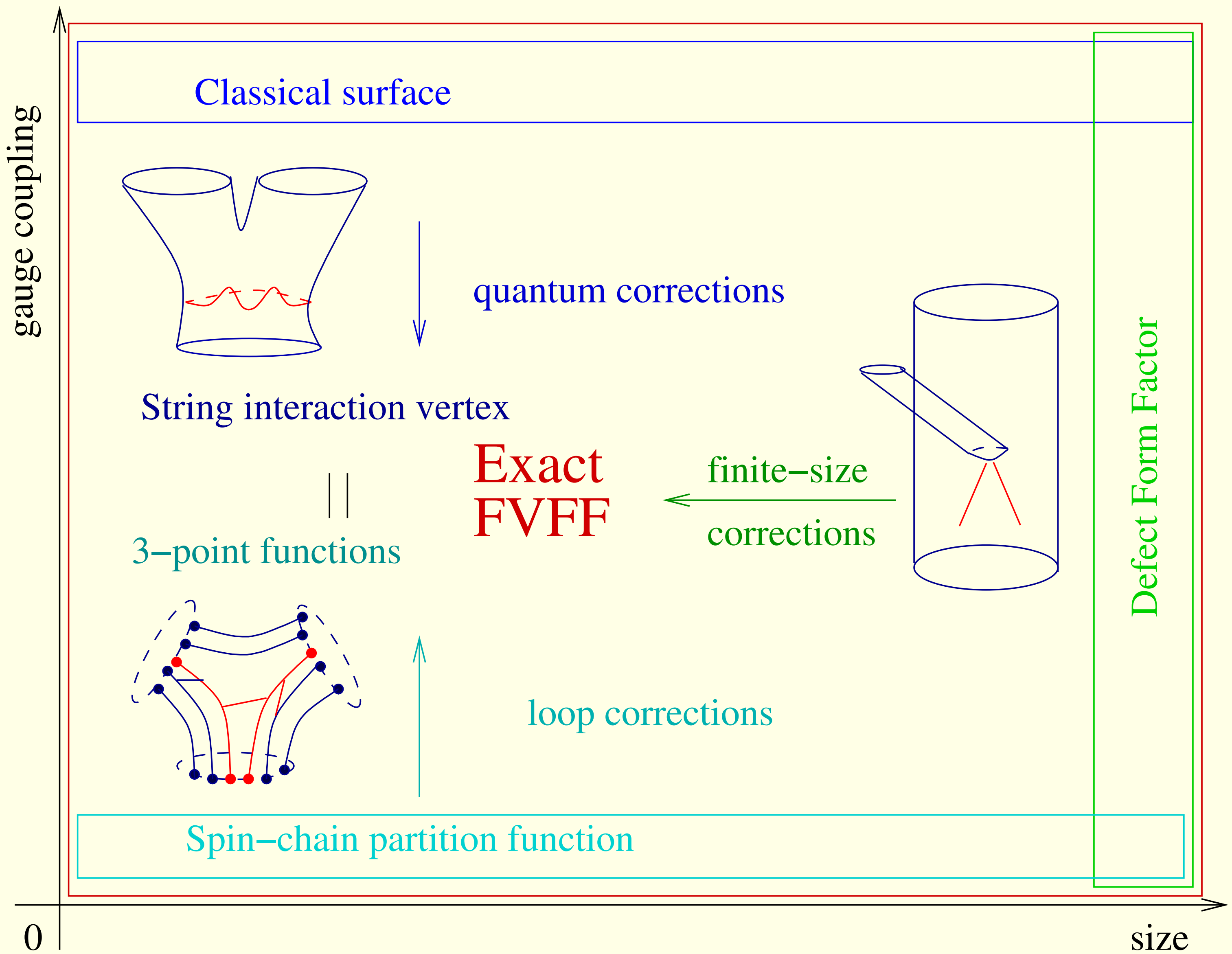
spectrum: $Q = 1, 2, \dots, \infty$ dispersion: $\epsilon_Q(p) = \sqrt{Q^2 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}}$

Exact scattering matrix: $S_{Q_1 Q_2}(p_1, p_2, \lambda)$

How integrability works:



String interaction, 3pt functions



Decompactification limit of the string vertex

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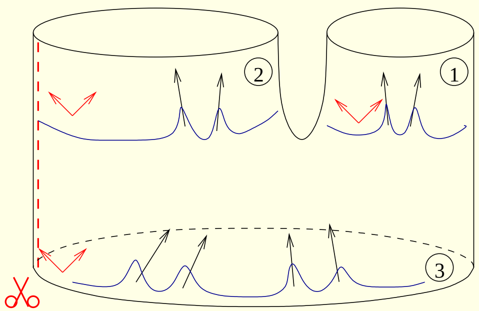
Decompactify string 2 & 3:



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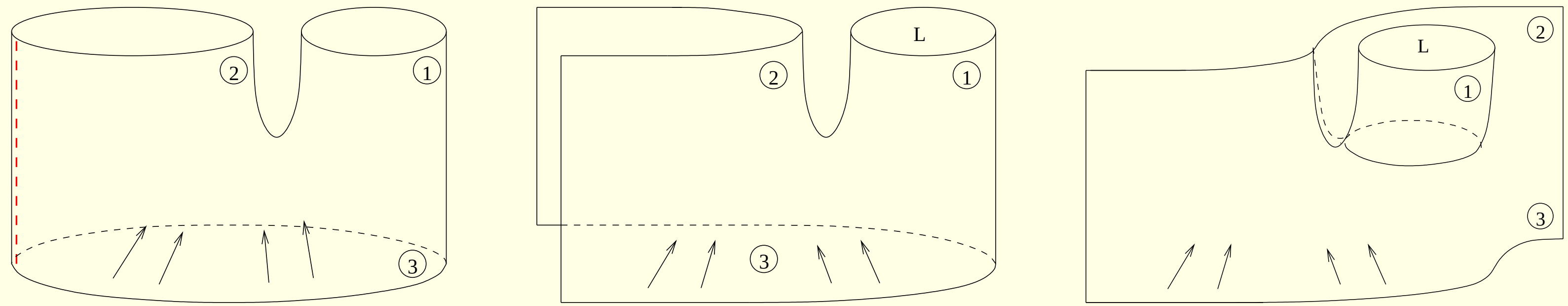

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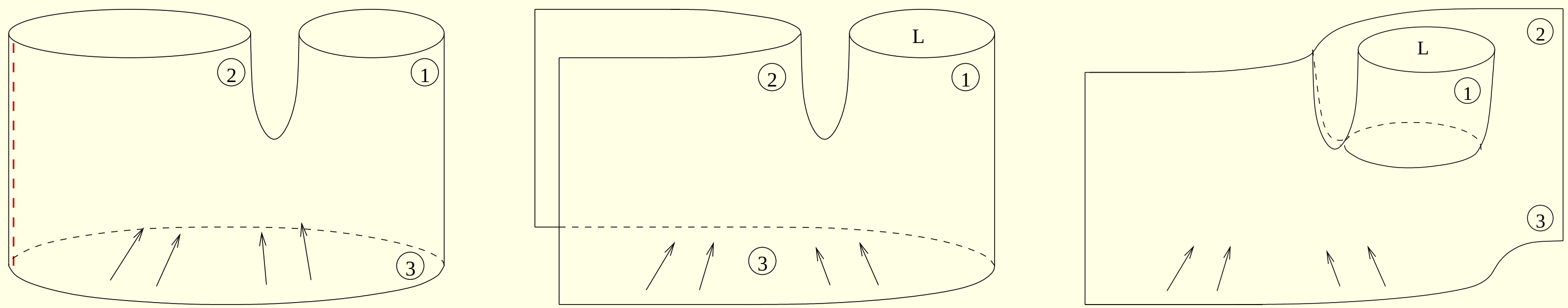
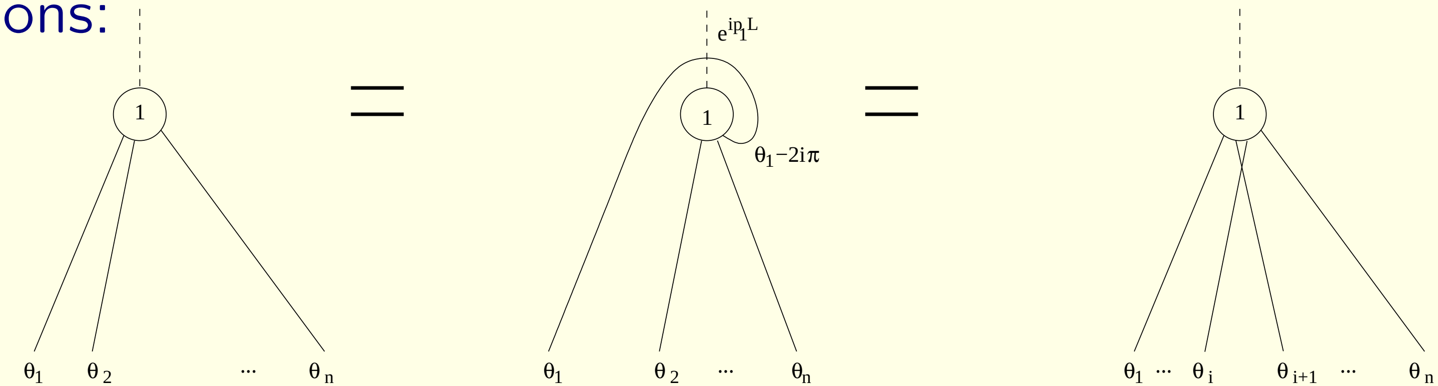


Form factor equations:



$$N_L(\theta_1,\dots,\theta_n\rangle = e^{-ip_1 L} N_L(\theta_2,\dots,\theta_n,\theta_1-2i\pi\rangle = S(\theta_i-\theta_{i+1}) N_L(\dots,\theta_{i+1},\theta_i,\dots)$$

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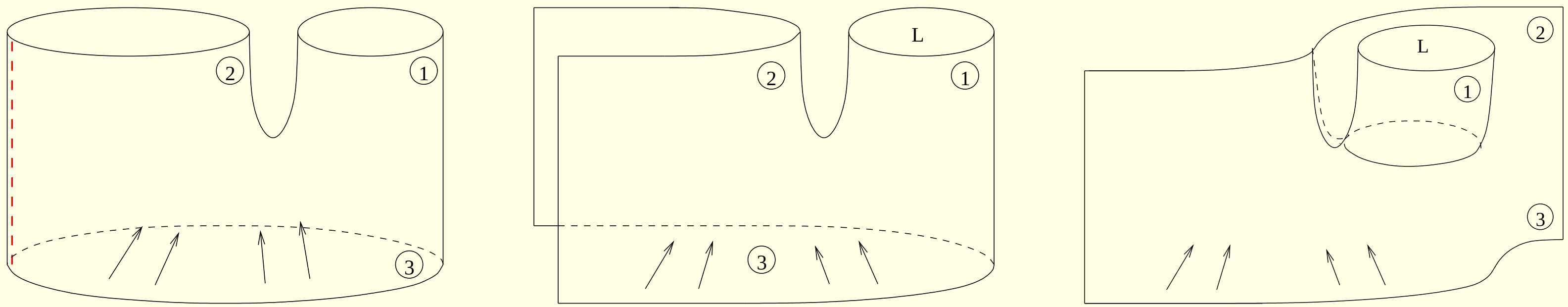

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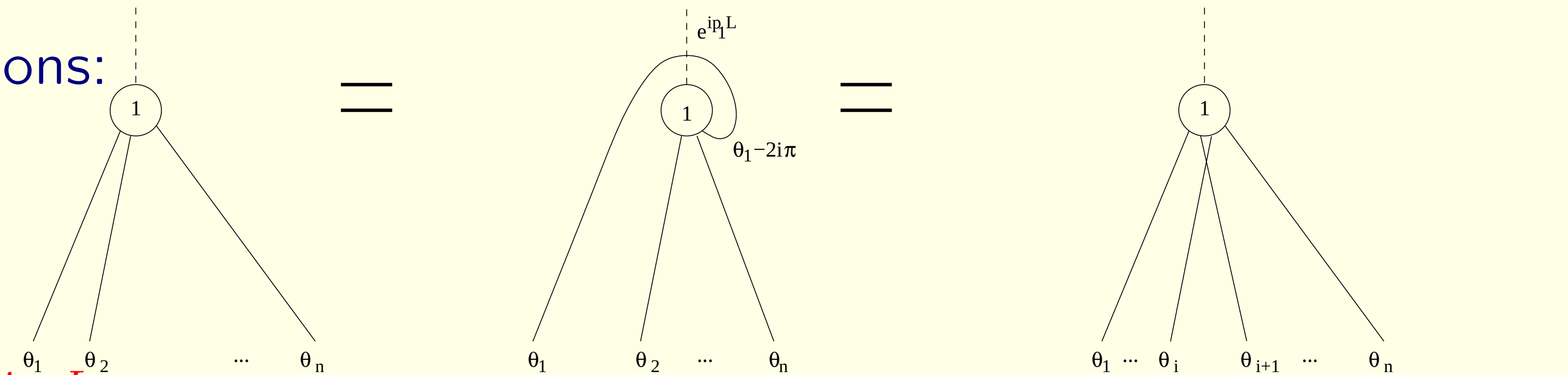

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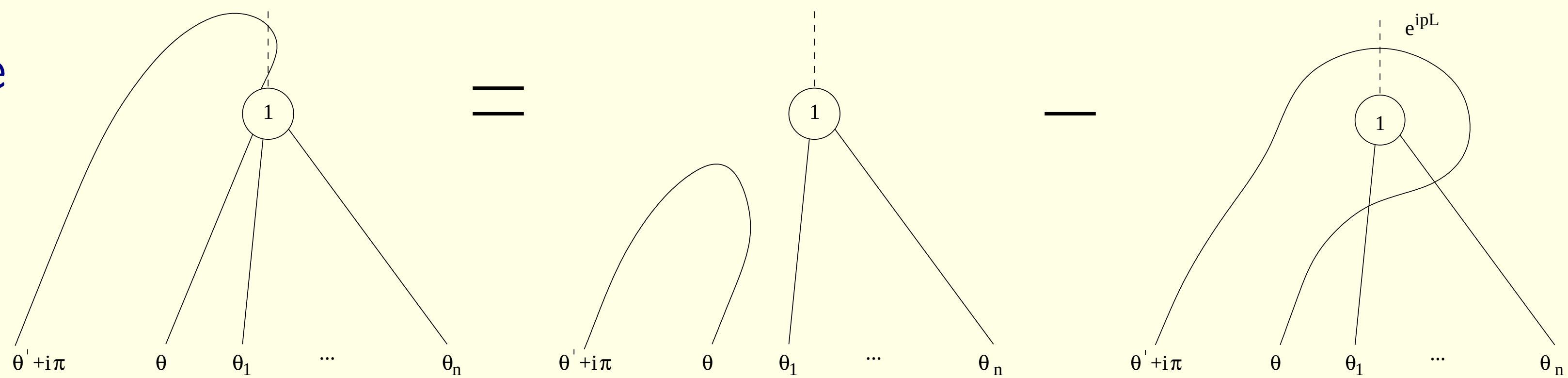
Singularity stucture

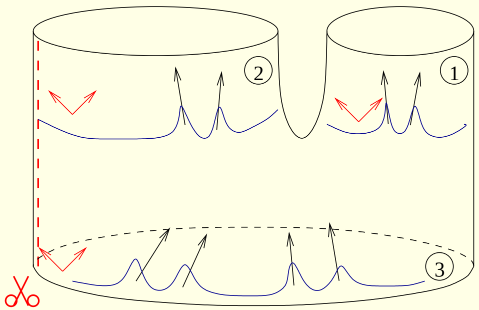


$$-i\text{Res}_{\theta'=\theta}N_L(\theta'+i\pi,\theta,\theta_1\dots,\theta_n) = (1-e^{ipL}\prod_i S(\theta-\theta_i))N_L(\theta_1,\dots,\theta_n)$$

The string vertex for $L_1 = 0$: diagonal form factor

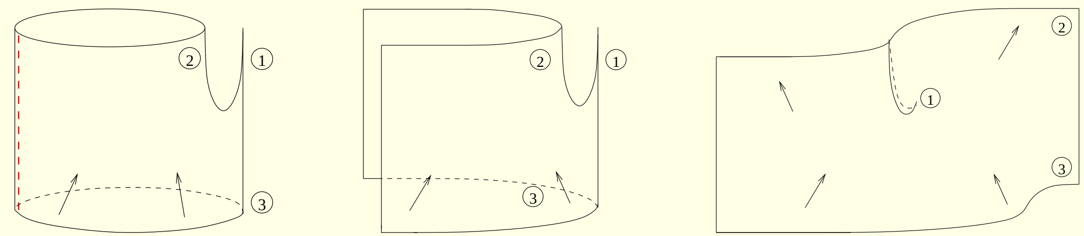
The string vertex for $L_1 = 0$: diagonal form factor

Decompactify string 2 & 3 but $L_1 = 0$:



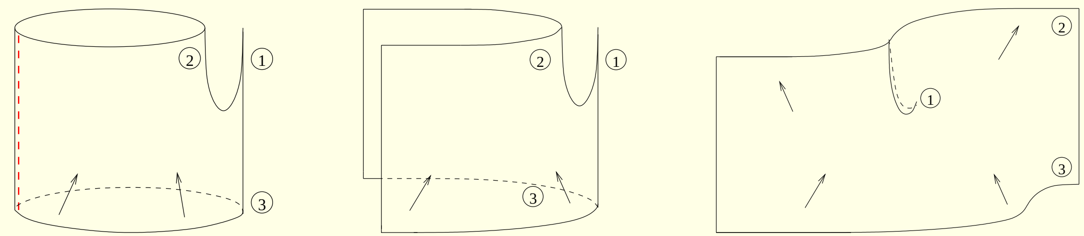
The string vertex for $L_1 = 0$: diagonal form factor

Decompactify string 2 & 3



The string vertex for $L_1 = 0$: diagonal form factor

Decompactify string 2 & 3



Local operator form factor equations:

$$N_0(\theta_1, \dots, \theta_n\rangle = N_0(\theta_2, \dots, \theta_n, \theta_1 - 2i\pi\rangle = S(\theta_i - \theta_{i+1}) N_0(\dots, \theta_{i+1}, \theta_i, \dots)$$

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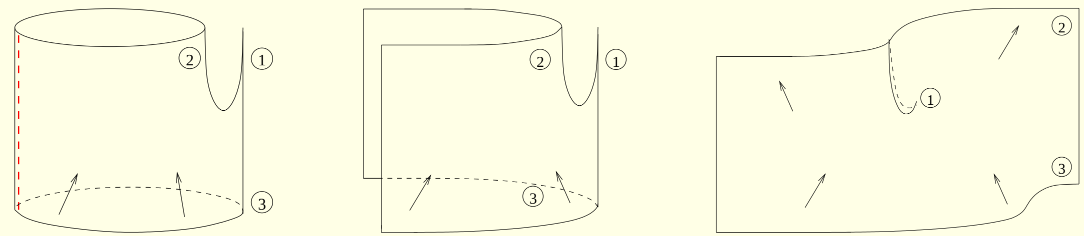

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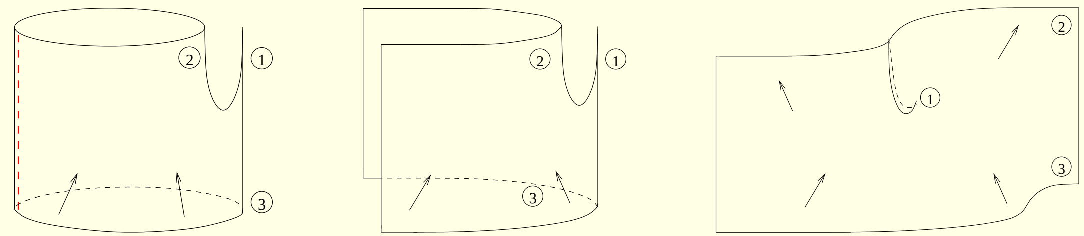
HeavyHeavyLight 3pt function strong coupling prescription

[Costa et al., Zarembo]: $C_{HHL} = \int_{\text{world sheet}} \mathcal{V}(X[\text{heavy solution}(\sigma, \tau)]) d^2\sigma$

The string vertex for $L_1 = 0$: diagonal form factor

Decompactify string 2 & 3



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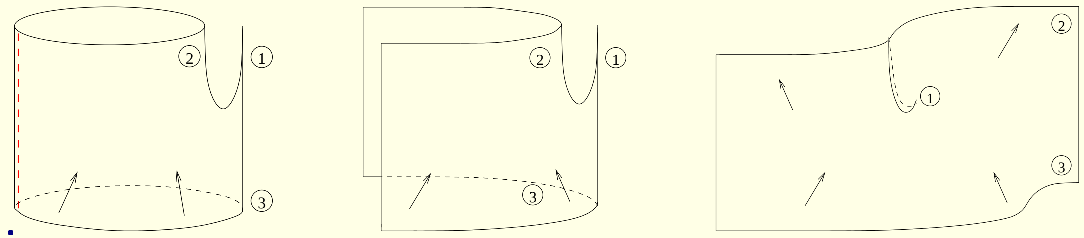
for multiparticle state: $C_{HHL} = \int_{\text{moduli space}\,\{y_i\}} \mathcal{V}(X[\text{heavy solution}(\mathrm{y_i})])d^n y$

The string vertex for $L_1 = 0$: diagonal form factor

Decompactify string 2 & 3



Local operator form factor equations:

$$N_0(\theta_1,\dots,\theta_n\rangle = N_0(\theta_2,\dots,\theta_n,\theta_1 - 2i\pi\rangle = S(\theta_i - \theta_{i+1})N_0(\dots,\theta_{i+1},\theta_i,\dots)$$

$$-i\text{Res}_{\theta'=\theta}N_0(\theta' + i\pi,\theta,\theta_1\dots,\theta_n) = (1 - \prod_i S(\theta - \theta_i))N_0(\theta_1,\dots,\theta_n)$$

HeavyHeavyLight 3pt function strong coupling prescription

[Costa et al., Zarembo]: $C_{HHL} = \int_{\text{world sheet}} \mathcal{V}(X[\text{heavy solution}(\sigma,\tau)])d^2\sigma$

for multiparticle state: $C_{HHL} = \int_{\text{moduli space}\,\{y_i\}} \mathcal{V}(X[\text{heavy solution}(\text{y}_\text{i})])d^n y$

classical diagonal form factors:

$$_L\langle\theta_2,\theta_1|\mathcal{V}|\theta_1,\theta_2\rangle_L = \frac{F_2^s(\theta_1,\theta_2)+\rho_1(\theta_1)F_1^s(\theta_2)+\rho_1(\theta_2)F_1^s(\theta_1)}{\rho_2(\theta_1,\theta_2)}$$

Explicitly checked at weak coupling [Hollo, Jiang, Petrovskii],
checked from hexagon [Basso, Komatsu, Vieira] by [Jiang]