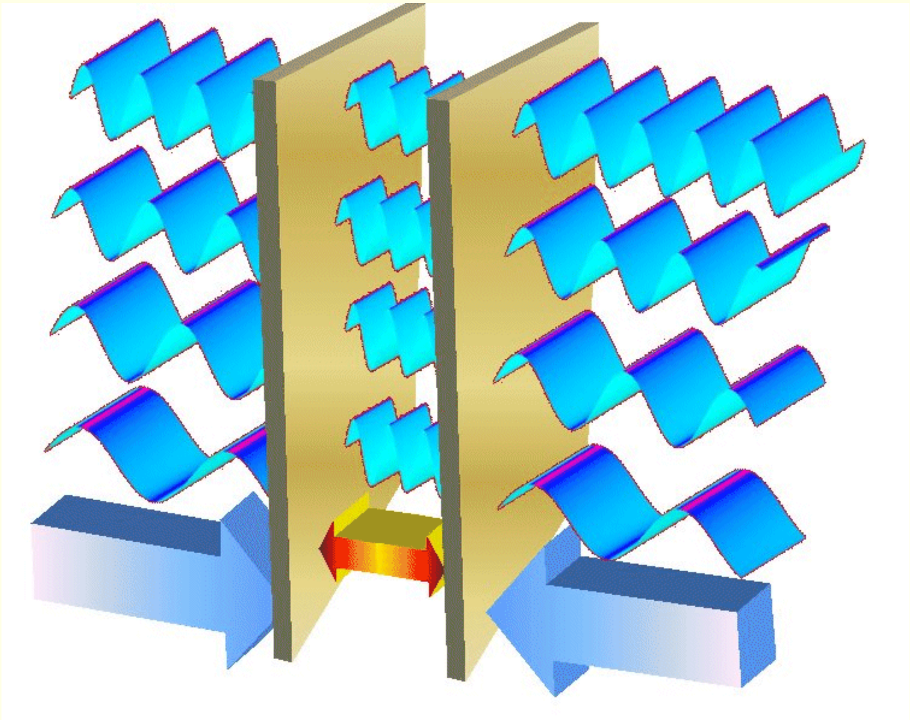
University of Miami, Miami, 19th of October, 2011

Casimir effect and boundary quantum field theories

Zoltán Bajnok,

Theoretical Physics Research Group of the Hungarian Academy of Sciences, Budapest

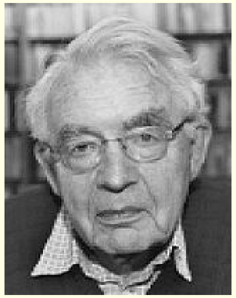
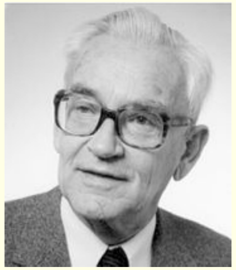
in collaboration with L. Palla and G. Takács



$$\frac{F(L)}{A} = -\frac{dE_0(L)}{AdL} = -\frac{\hbar c \pi^2}{240 L^4}$$

Planar Casimir energy $E_0(L)$ from finite size effects in *(integrable)* boundary QFT

Motivation: Casimir-Polder effect

Hendrik Casimir   Dirk Polder

colloidal solution: neutral atoms

force not like Van der Waals

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not a theoretical curiosity!

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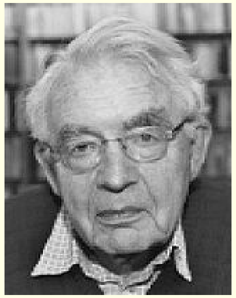
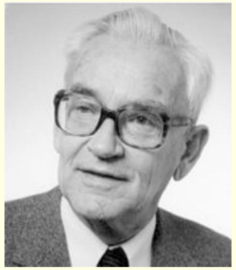

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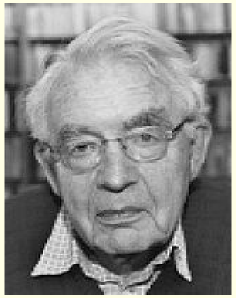

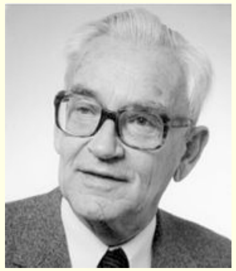

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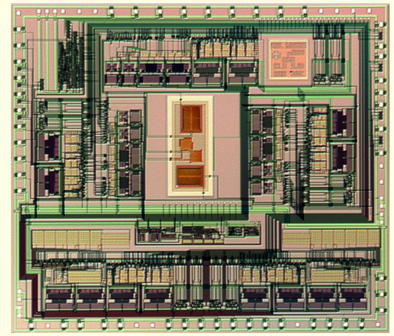
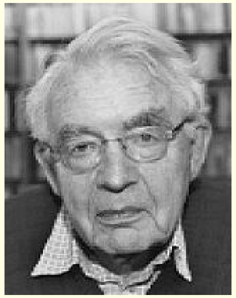
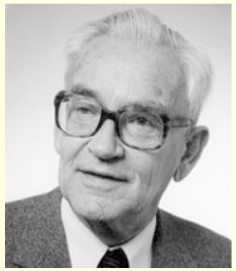


Airbag trigger chip

Motivation: Casimir-Polder effect



Hendrik Casimir



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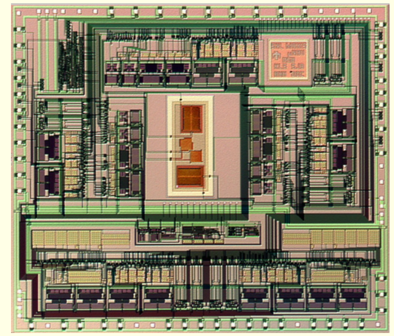

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micromechanical device: pieces stick friction, levitation

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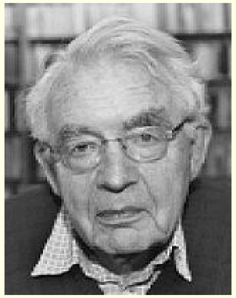

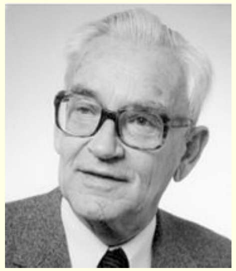

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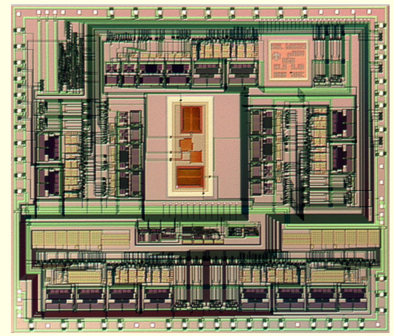

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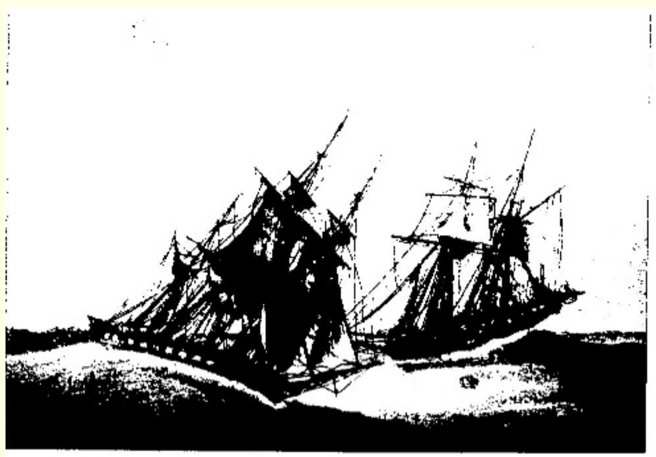
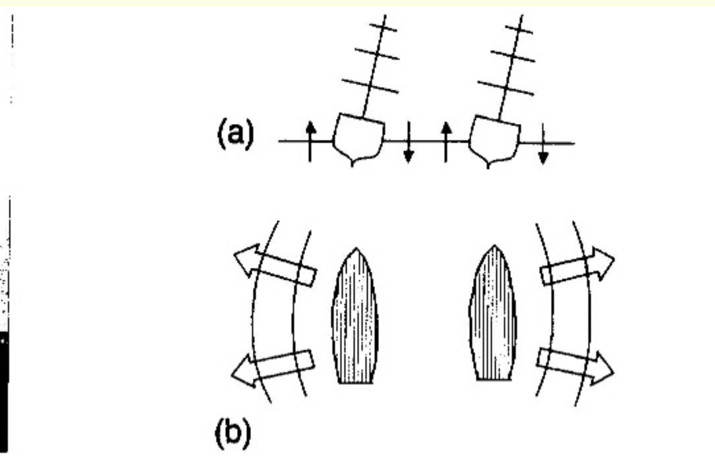
Maritime analogy:





Aim: understand/describe planar Casimir effect

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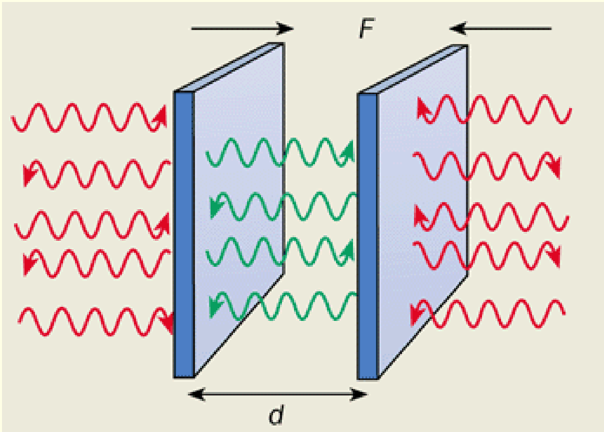

Usual explanation: energy of the vacuum: $E_0(L) = \frac{1}{2}\sum_{k(L,BC)} \omega(k) \propto \infty$

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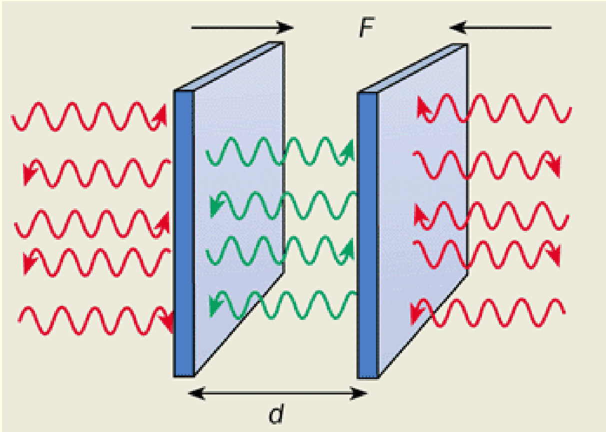

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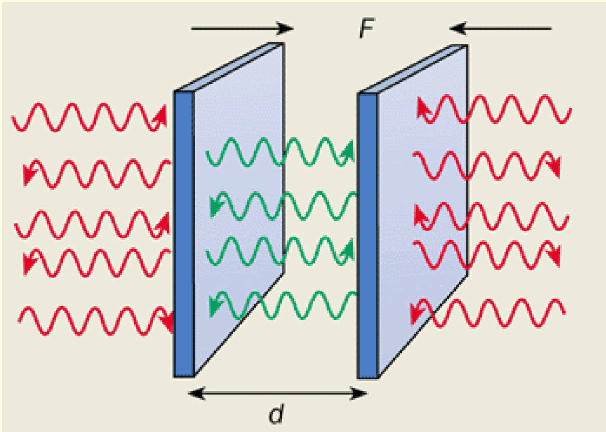

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Lifshitz formula: QED, Parallel dielectric slabs $(\epsilon_1, 1, \epsilon_2)$

$$\Delta E_0(L)/A = \sum_{i=\parallel,\perp} \int_0^\infty \frac{d^2q}{8\pi^2} d\zeta \log\left[1 - R_i^1(\zeta,q) R_i^2(\zeta,q) e^{-2L\sqrt{q^2+\zeta^2}}\right]$$

$$R_\perp(\omega = \sqrt{q^2+\zeta^2}, q) = \frac{\sqrt{\omega^2-q^2}-\sqrt{\epsilon\omega^2-q^2}}{\sqrt{\omega^2-q^2}+\sqrt{\epsilon\omega^2-q^2}} \quad R_\parallel(\omega,q) = \frac{\epsilon\sqrt{\omega^2-q^2}-\sqrt{\epsilon\omega^2-q^2}}{\epsilon\sqrt{\omega^2-q^2}+\sqrt{\epsilon\omega^2-q^2}}$$

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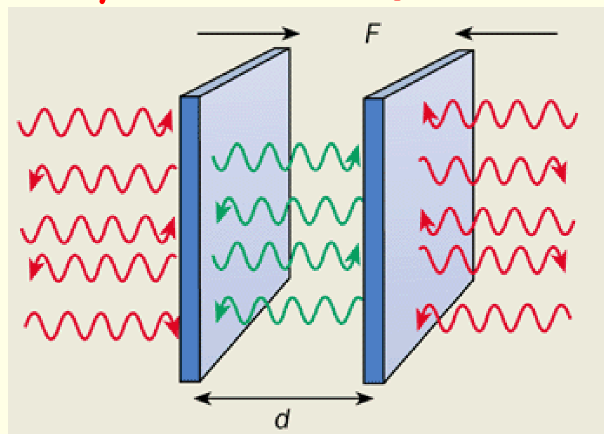

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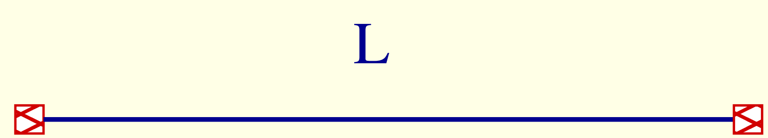
Physics can be understood in 1+1 D QFT



integrability helps to solve the problem even exactly → large volume expansion in any D

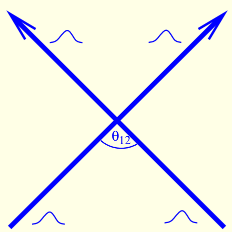
Plan of talk

Cylinder

Plan of talk

Cylinder

Infinite volume



S-matrix

Plan of talk

Cylinder

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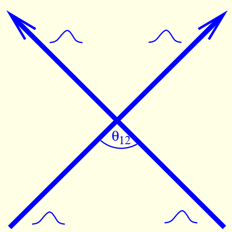
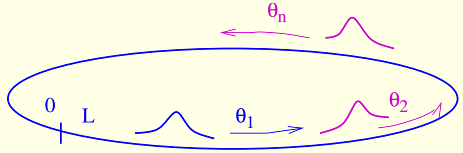


S-matrix

Large volumes



Bethe-Yang lines

Plan of talk

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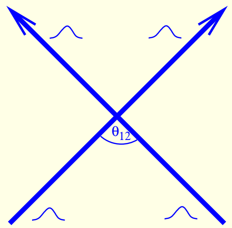

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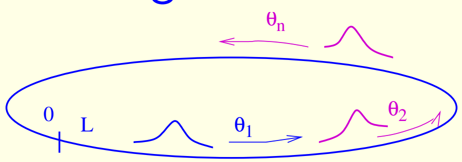

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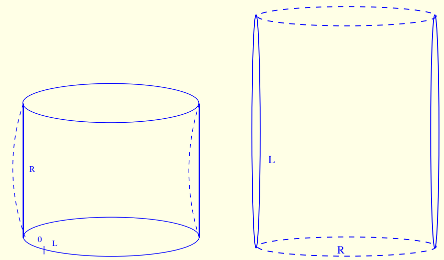
Lüscher correction



$$E_0(L) = -m\int \frac{d\theta}{2\pi}\cosh\theta\, e^{-m\cosh(\theta)L}$$

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Cylinder

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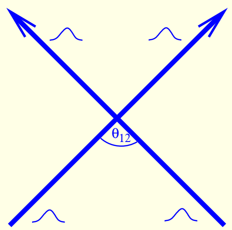

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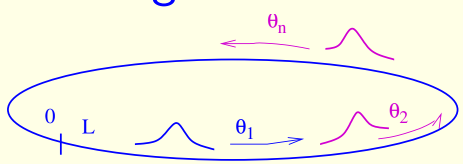

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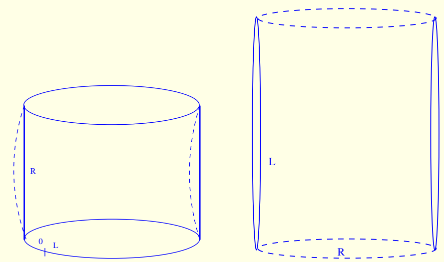

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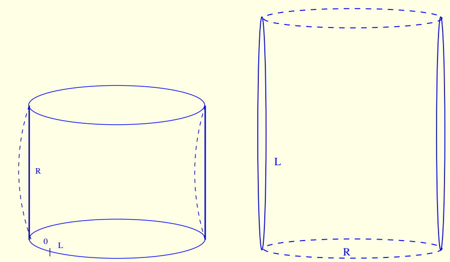
Exact groundstate



$$E_0(L) = -\frac{m}{2\pi} \int \cosh\theta \log(1 + e^{-\epsilon(\theta)}) d\theta$$

Plan of talk

Cylinder

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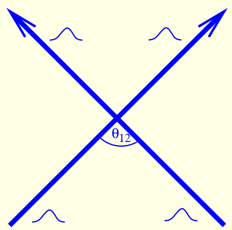

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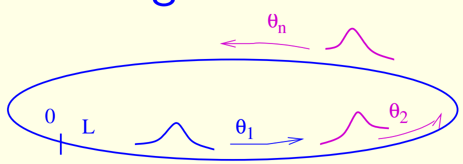

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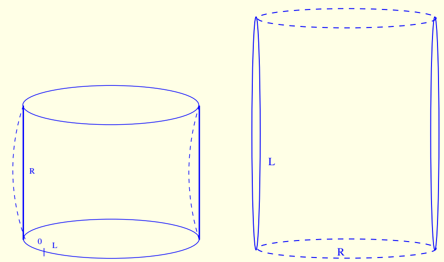

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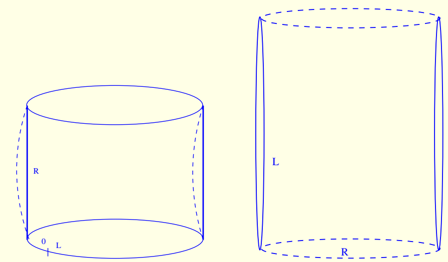

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Strip

Plan of talk

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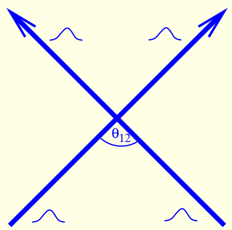

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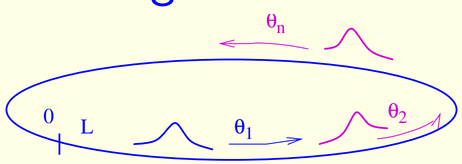

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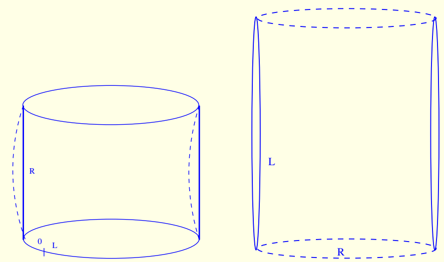

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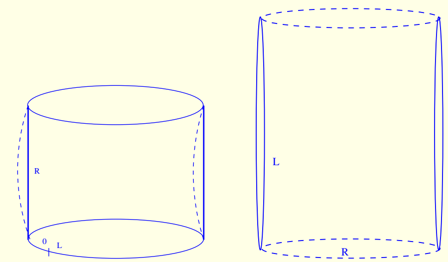

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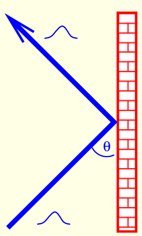
Strip

Semiinfinite volume

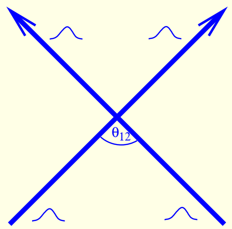


R-matrix

Plan of talk

Cylinder

Infinite volume



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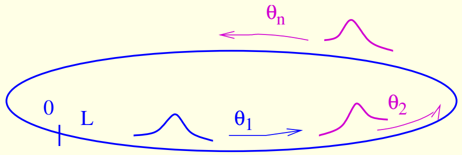

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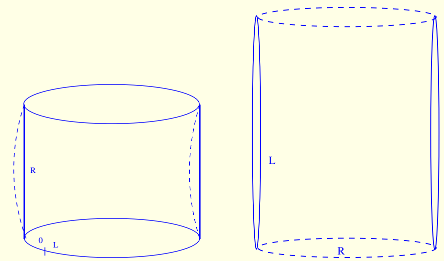

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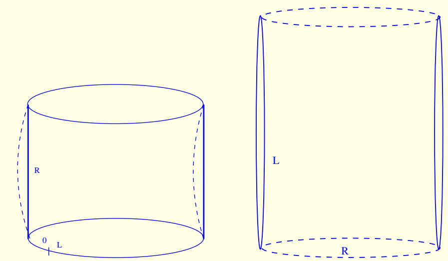

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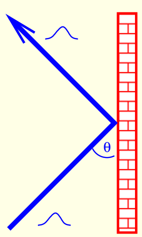
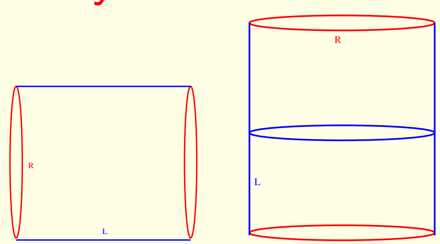


R-matrix

Boundary Lüscher correction

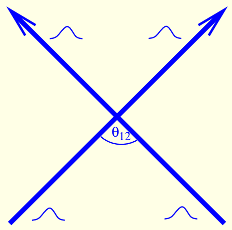


$$E_0(L) =$$
$$-\int \frac{md\theta}{4\pi} \cosh\theta K(\theta)\bar{K}(\theta) e^{-2mL\cosh(\theta)}$$
$$K(\theta) = R(\frac{i\pi}{2} - \theta)$$

Plan of talk

Cylinder

Infinite volume



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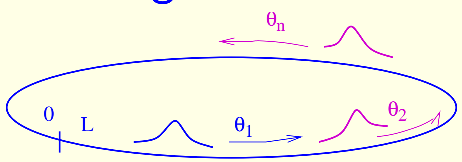

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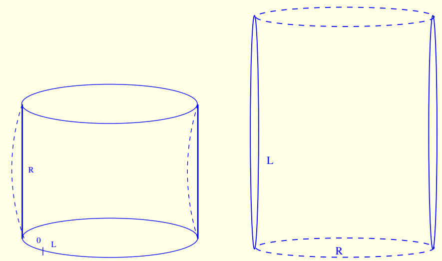

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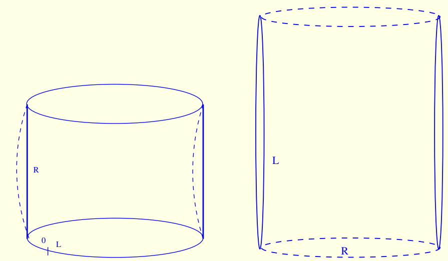

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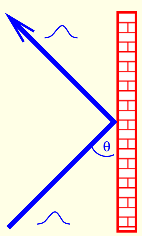

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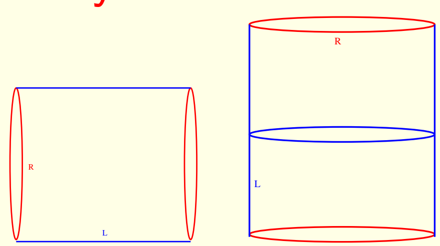

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$$K(\theta) = R(\frac{i\pi}{2}-\theta)$$

Boundary TBA

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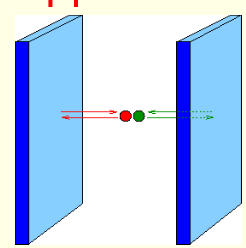
Application



Casimir effect

$$p_i = m \sinh \theta_i$$

$$E_i = m \cosh \theta_i$$

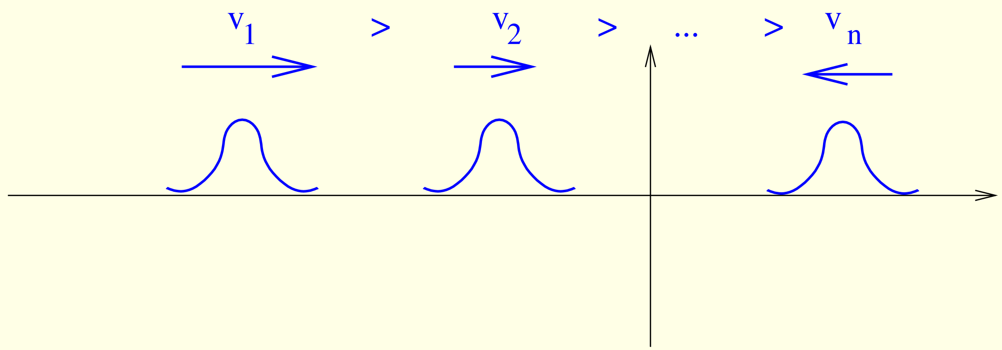
Integrable field theory: Bootstrap

Integrable field theory: Bootstrap

$$p_i = m \sinh \theta_i$$
$$E_i = m \cosh \theta_i$$

Bulk multiparticle state: with n particles

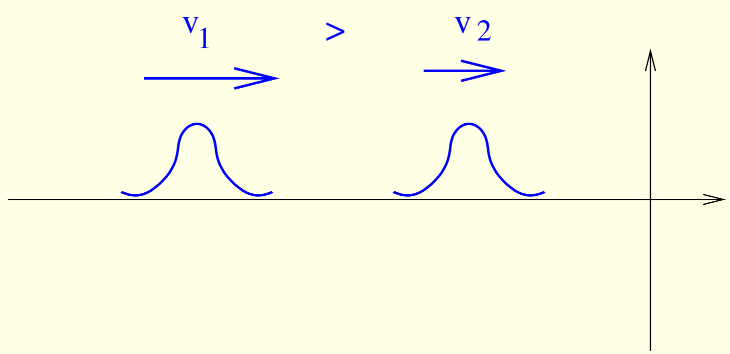
$$E(\theta_1, \theta_2, \ldots, \theta_n) = \sum_i m \cosh \theta_i$$



$$p_i = m \sinh \theta_i$$
$$E_i = m \cosh \theta_i$$

Integrable field theory: Bootstrap

Bulk twoparticle state:

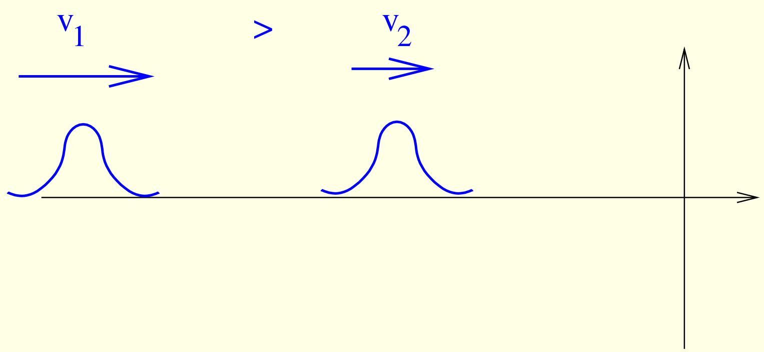


Integrable field theory: Bootstrap

$p_i = m \sinh \theta_i$

$E_i = m \cosh \theta_i$

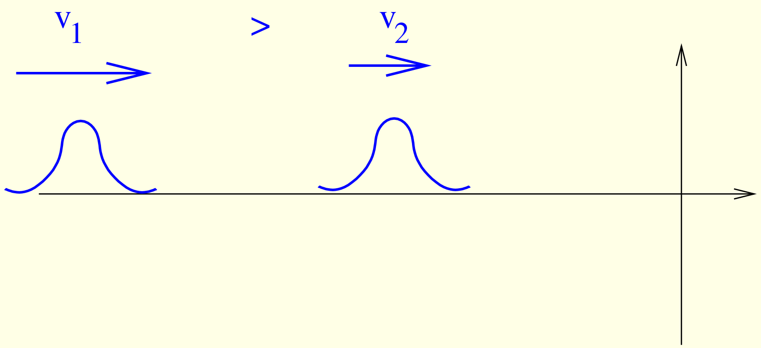
Bulk twoparticle in state: $t \to -\infty$



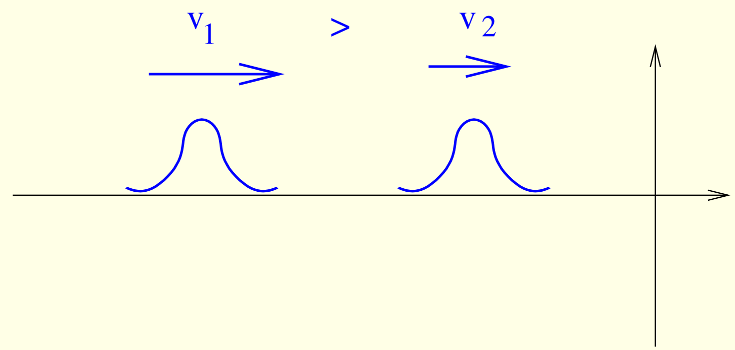
$$p_i = m \sinh \theta_i$$
$$E_i = m \cosh \theta_i$$

Integrable field theory: Bootstrap

Bulk twoparticle in state: $t \to -\infty$



times develop

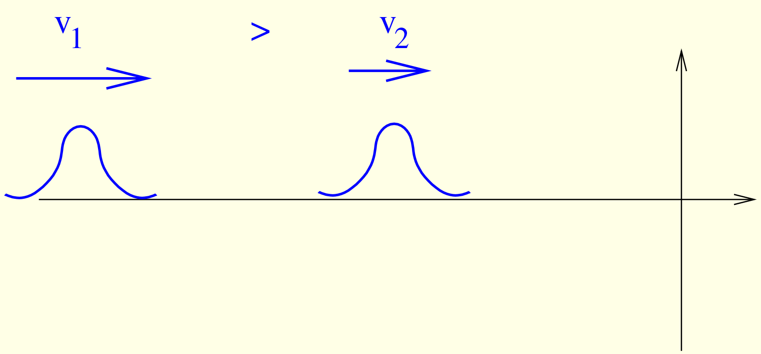


$p_i = m \sinh\theta_i$

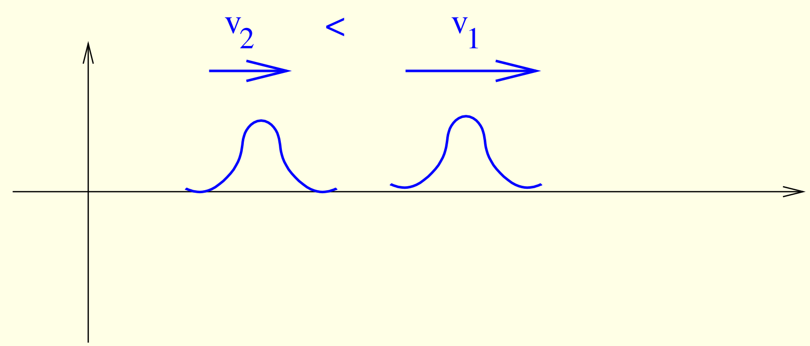
$E_i = m \cosh\theta_i$

Integrable field theory: Bootstrap

Bulk twoparticle in state: $t \to -\infty$



times develop further

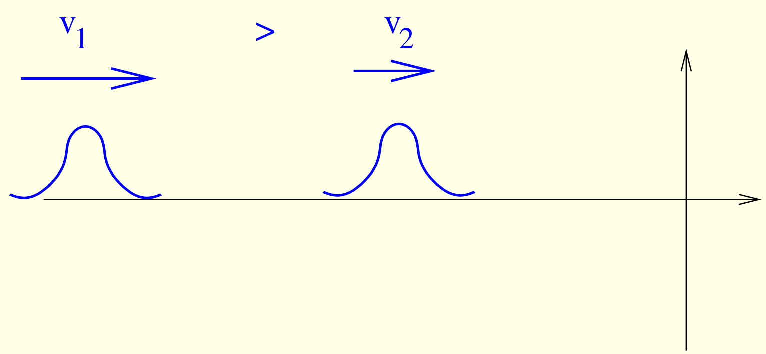


Integrable field theory: Bootstrap

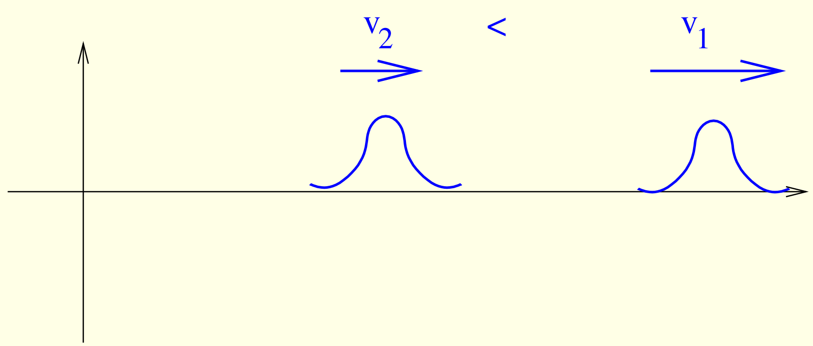
$p_i = m \sinh \theta_i$

$E_i = m \cosh \theta_i$

Bulk twoparticle in state: $t \to -\infty$



Bulk twoparticle out state: $t \to \infty$



Integrable field theory: Bootstrap

$p_i = m \sinh \theta_i$

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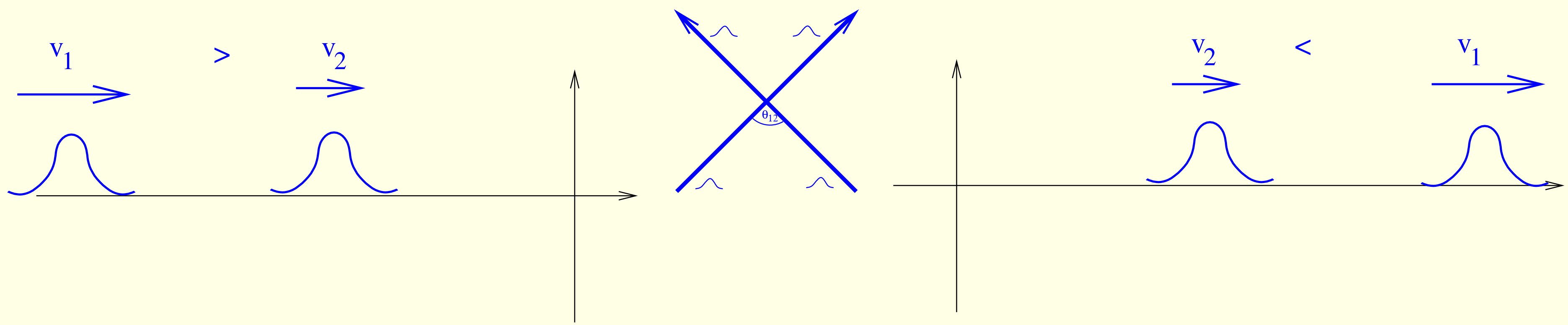
Bulk twoparticle in state: $t \to -\infty$

Bulk twoparticle out state: $t \to \infty$



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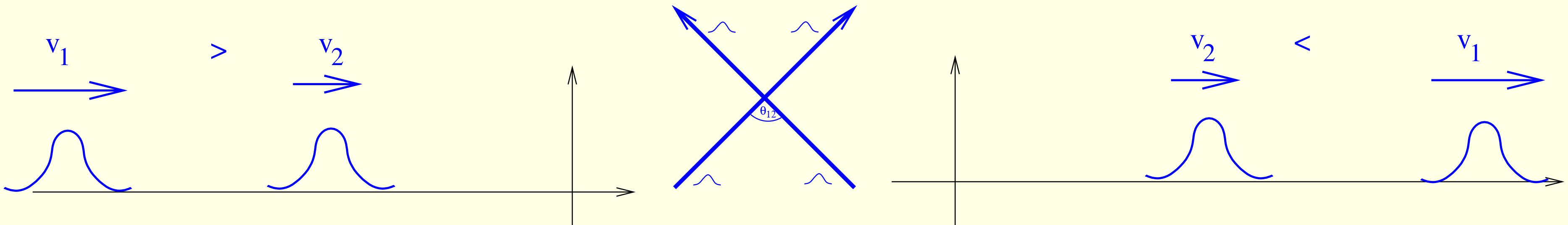
Integrable field theory: Bootstrap

Bulk twoparticle in state: $t \to -\infty$

Bulk twoparticle out state: $t \to \infty$



Free, noninteracting in particles

Free, noninteracting out particles

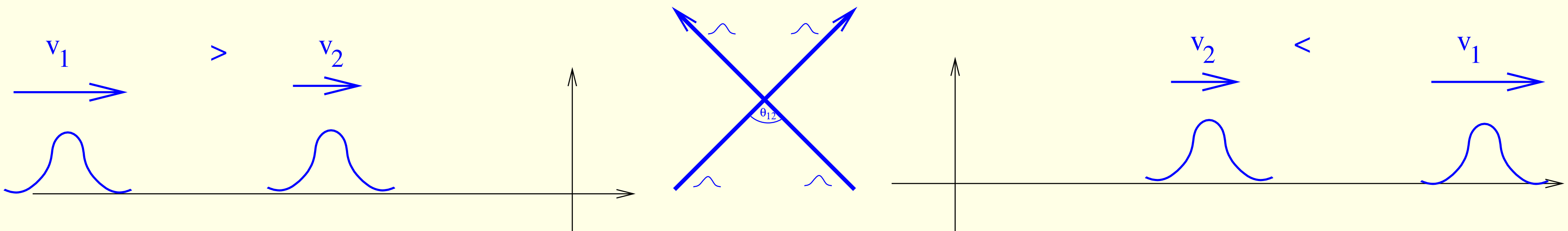
$p_i = m \sinh \theta_i$

$E_i = m \cosh \theta_i$

Integrable field theory: Bootstrap

Bulk twoparticle in state: $t \to -\infty$

Bulk twoparticle out state: $t \to \infty$



Free, noninteracting in particles $\longleftarrow$ **S-matrix** $\longrightarrow$ Free, noninteracting out particles

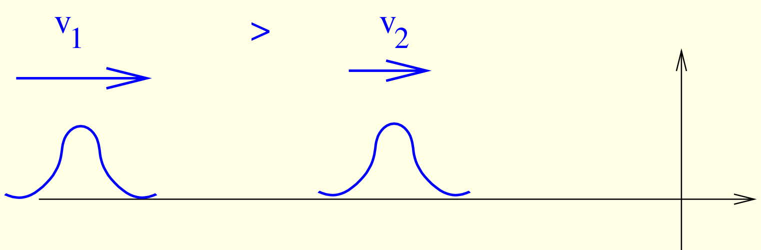
Integrable field theory: Bootstrap

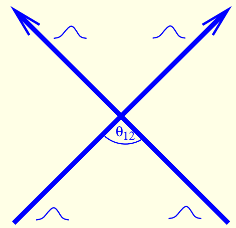
$p_i = m \sinh \theta_i$

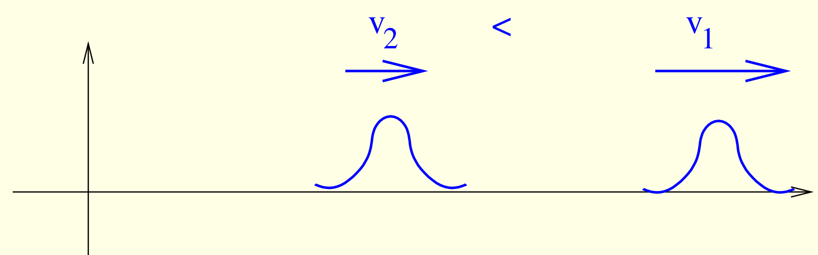
$E_i = m \cosh \theta_i$

Bulk twoparticle in state: $t \to -\infty$

Bulk twoparticle out state: $t \to \infty$







Free, noninteracting in particles $\leftarrow$ **S-matrix** $\rightarrow$ Free, noninteracting out particles

$$|\theta_1, \theta_2\rangle^{in} = S(\theta_1 - \theta_2)|\theta_1, \theta_2\rangle^{out}$$

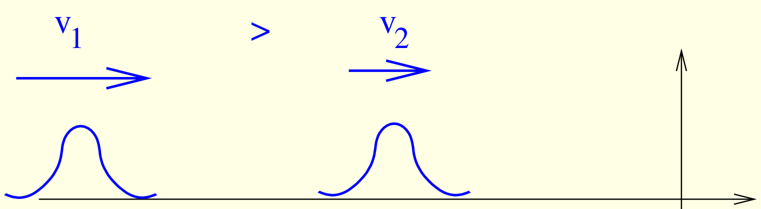
$p_i = m \sinh \theta_i$

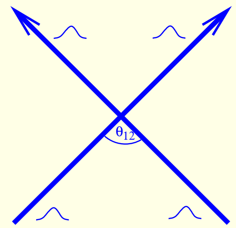
$E_i = m \cosh \theta_i$

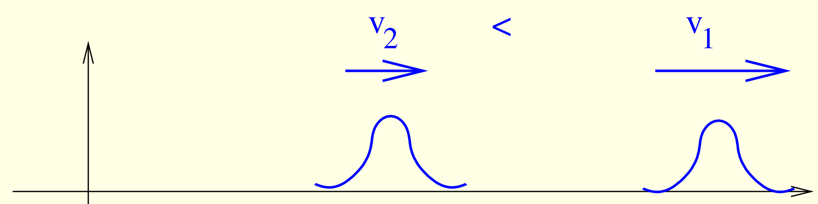
Integrable field theory: Bootstrap

Bulk twoparticle in state: $t \to -\infty$

Bulk twoparticle out state: $t \to \infty$

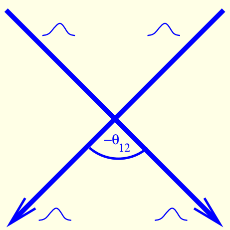




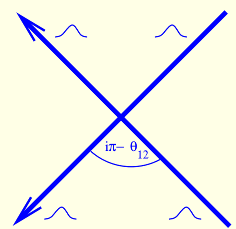


Free, noninteracting in particles $\leftarrow$ **S-matrix** $\rightarrow$ Free, noninteracting out particles

$$|\theta_1, \theta_2\rangle^{in} \quad = \quad S(\theta_1 - \theta_2)|\theta_1, \theta_2\rangle^{out}$$



Unitarity $S^{-1}(\theta_{12}) = S(\theta_{21})$



Crossing $S(\theta_{12}) = S(i\pi - \theta_{12})$

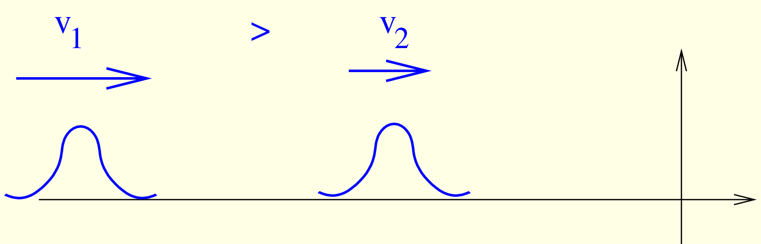
Integrable field theory: Bootstrap

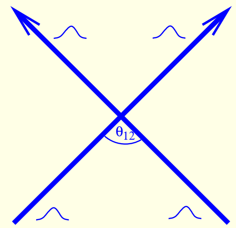
$p_i = m \sinh \theta_i$

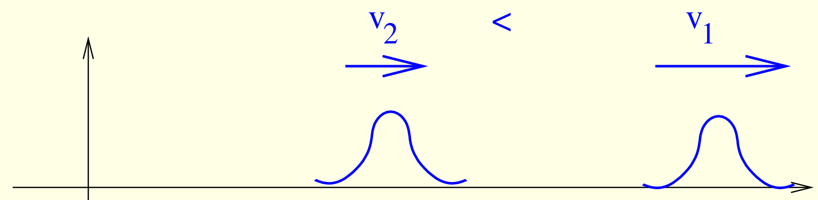
$E_i = m \cosh \theta_i$

Bulk twoparticle in state: $t \to -\infty$

Bulk twoparticle out state: $t \to \infty$

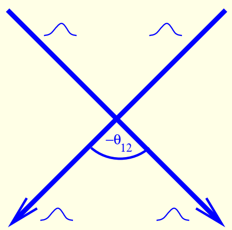




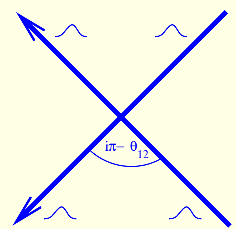


Free, noninteracting in particles $\leftarrow$ **S-matrix** $\rightarrow$ Free, noninteracting out particles

$$|\theta_1, \theta_2\rangle^{in} = S(\theta_1 - \theta_2)|\theta_1, \theta_2\rangle^{out}$$



Unitarity $S^{-1}(\theta_{12}) = S(\theta_{21})$



Crossing $S(\theta_{12}) = S(i\pi - \theta_{12})$

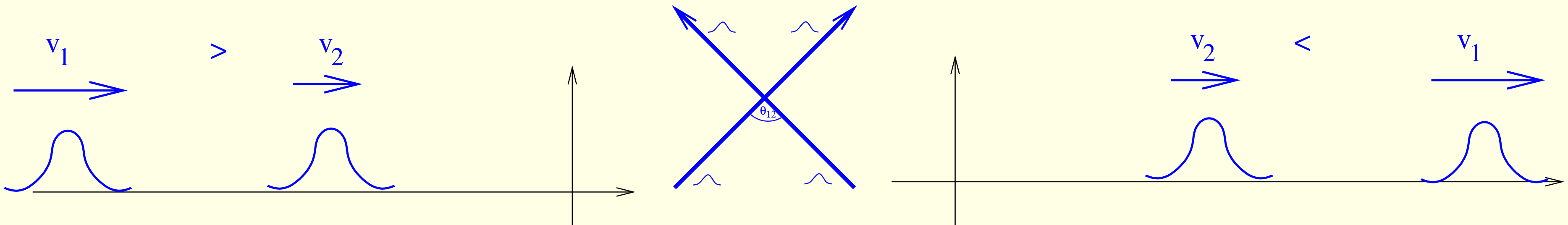
Integrability $\rightarrow$ factorizability: $|\theta_1, \theta_2, \ldots, \theta_n\rangle = \prod_{i<j} S(\theta_i - \theta_j)|\theta_n, \theta_{n-1}, \ldots, \theta_1\rangle$

$p_i = m \sinh \theta_i$

$E_i = m \cosh \theta_i$

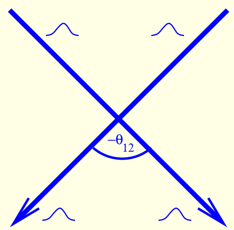
Integrable field theory: Bootstrap

Bulk twoparticle in state: $t \to -\infty$ Bulk twoparticle out state: $t \to \infty$



Free, noninteracting in particles $\leftarrow$ **S-matrix** $\rightarrow$ Free, noninteracting out particles

$$|\theta_1, \theta_2\rangle^{in} = S(\theta_1 - \theta_2)|\theta_1, \theta_2\rangle^{out}$$



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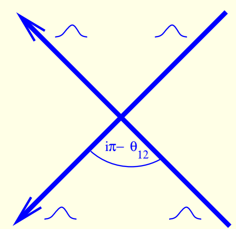


Crossing $S(\theta_{12}) = S(i\pi - \theta_{12})$

Integrability $\rightarrow$ factorizability: $|\theta_1, \theta_2, \ldots, \theta_n\rangle = \prod_{i<j} S(\theta_i - \theta_j)|\theta_n, \theta_{n-1}, \ldots, \theta_1\rangle$

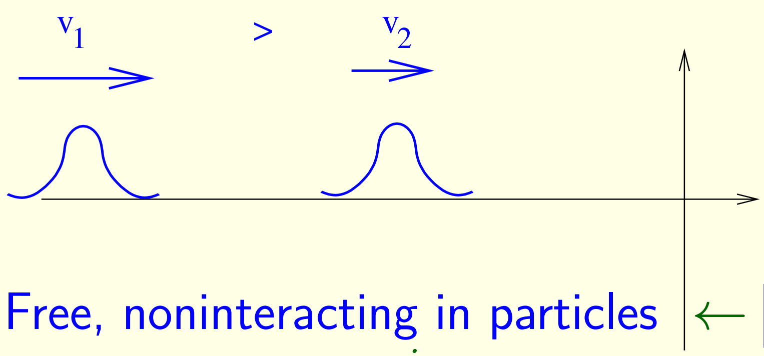
Minimal solutions: free boson $S = \pm 1$ sinh-Gordon $S(\theta) = \frac{\sinh \theta - i \sin \pi p}{\sinh \theta + i \sin \pi p}$, Lee-Yang $p = -\frac{1}{3}$

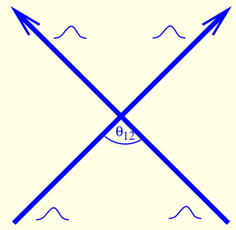
$p_i = m \sinh \theta_i$
$E_i = m \cosh \theta_i$

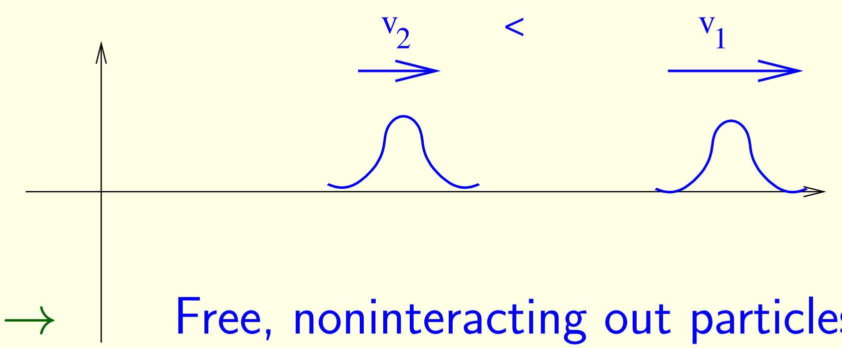
Integrable field theory: Bootstrap

Bulk twoparticle in state: $t \to -\infty$

Bulk twoparticle out state: $t \to \infty$

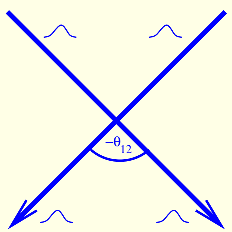




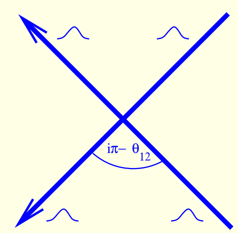


Free, noninteracting in particles $\leftarrow$ **S-matrix** $\rightarrow$ Free, noninteracting out particles

$$|\theta_1, \theta_2\rangle^{in} = S(\theta_1 - \theta_2)|\theta_1, \theta_2\rangle^{out}$$



Unitarity $S^{-1}(\theta_{12}) = S(\theta_{21})$



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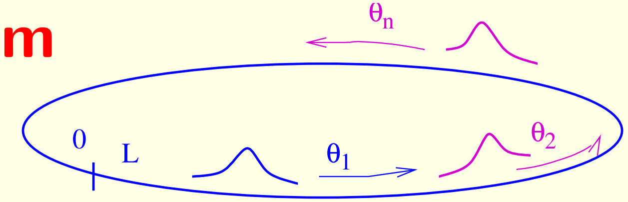
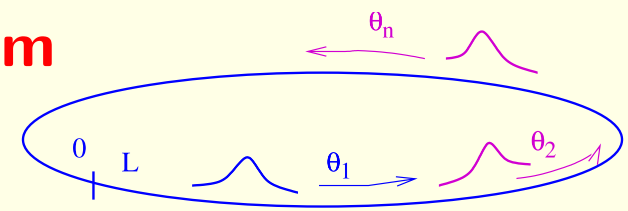
Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial \phi)^2 \quad - \mu(\cosh b\phi - 1) \qquad p = \frac{b^2}{8\pi + b^2}$

Very large volume spectrum



$$\frac{1}{2}(\partial\phi)^2 - \mu(\cosh b\phi - 1) \leftrightarrow S(\theta) = \frac{\sinh\theta - i\sin\pi p}{\sinh\theta + i\sin\pi p}$$

Very large volume spectrum



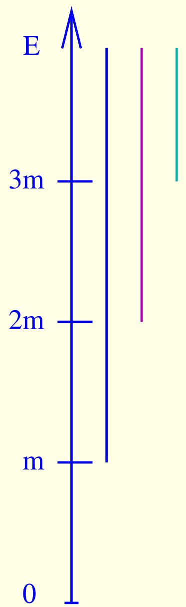
$$\frac{1}{2}(\partial\phi)^2 - \mu(\cosh b\phi - 1) \leftrightarrow S(\theta) = \frac{\sinh\theta - i\sin\pi p}{\sinh\theta + i\sin\pi p}$$

Infinite volume

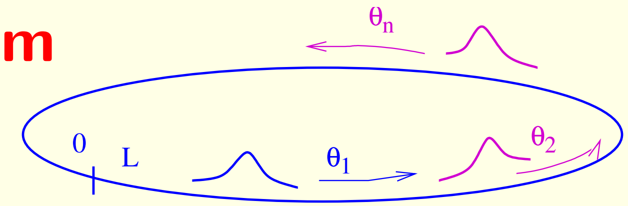
$$E(\theta) = m\cosh\theta$$

$$p(\theta) = m\sinh\theta$$

$$E(\theta_1, ..., \theta_n) = \sum_i E(\theta_i)$$



Very large volume spectrum



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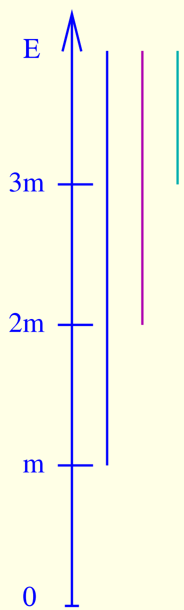
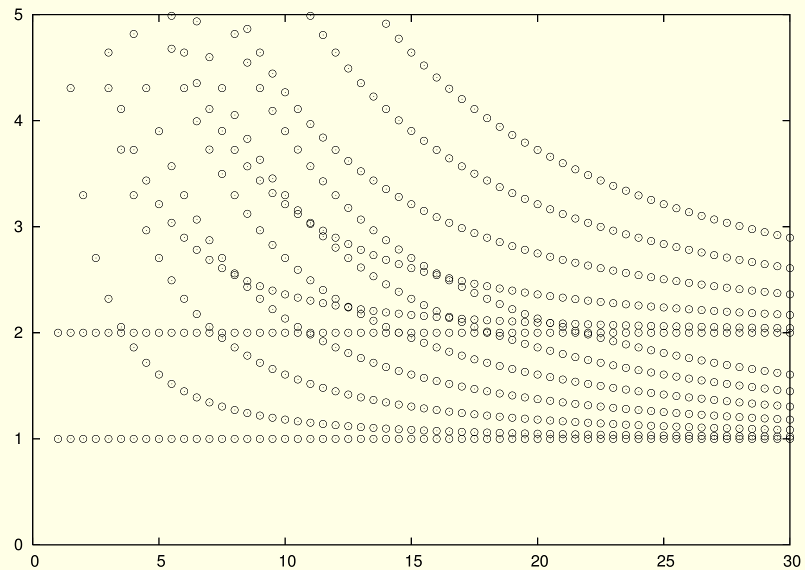


Finite volume: free particles

$e^{ip(\theta)L} = 1$ Quantization

$$p(\theta) \to p(k) = \frac{2\pi k}{L}$$

$$|\theta_1, \ldots \theta_n\rangle \to |k_1, \ldots, k_n\rangle$$



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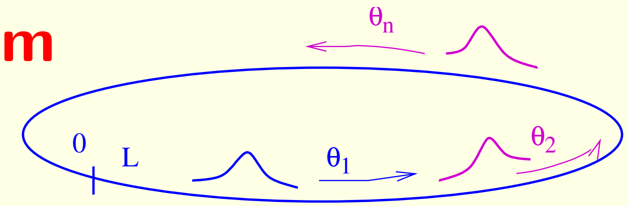

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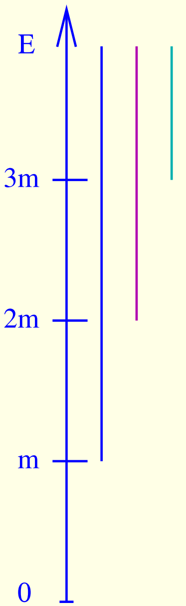

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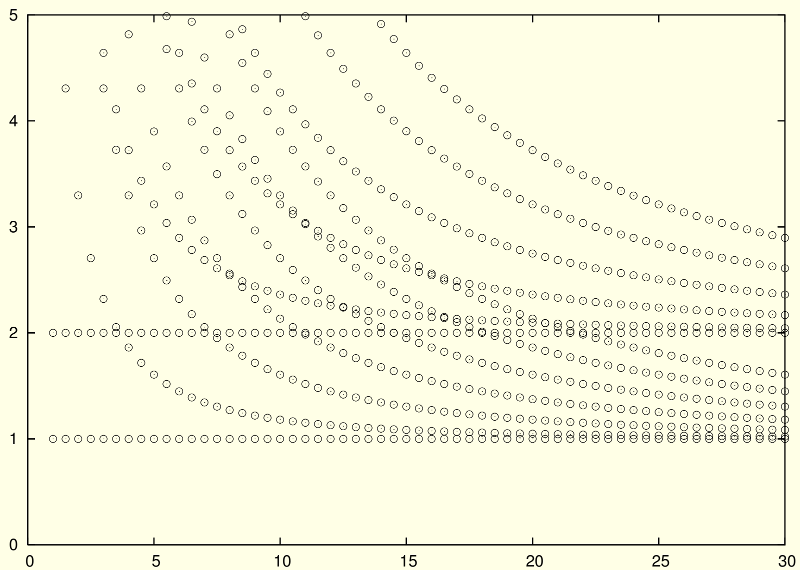


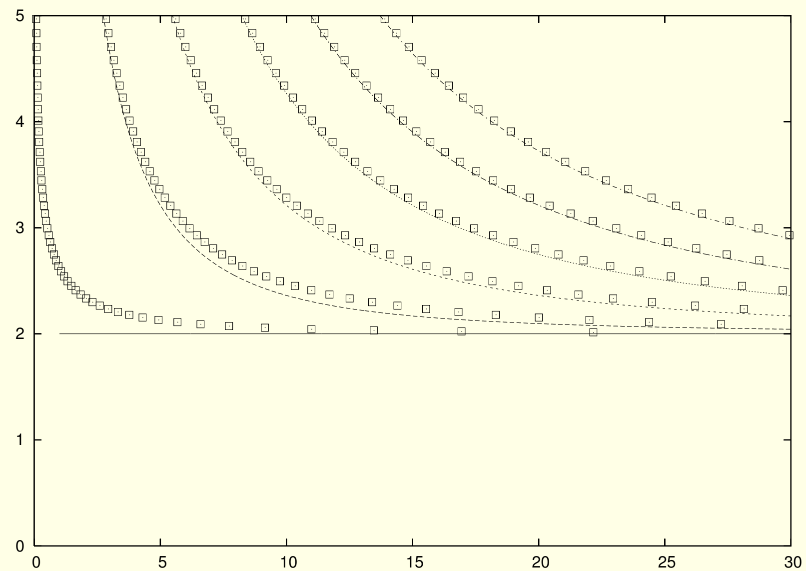
Very large volume, interacting particles

one particle $e^{ip(\theta)L} = 1; \theta \to p(k) = \frac{2\pi k}{L}$

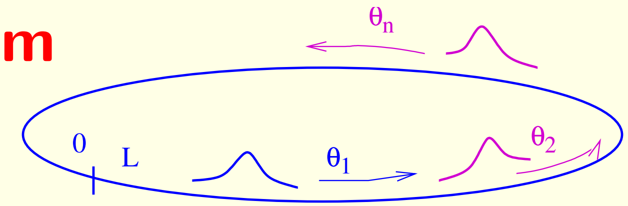
two particles $e^{ip(\theta_1)L} S(\theta_{12}) = 1$

$p(\theta_1)L + \varphi(\theta_{12}) = 2\pi n_1 \; ; \quad S = e^{i\varphi}$

n particles $p(\theta_i)L + \sum_j \varphi(\theta_{ij}) = 2\pi n_i$



Very large volume spectrum



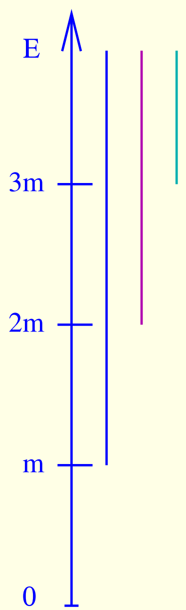
$$\frac{1}{2}(\partial\phi)^2 - \mu(\cosh b\phi - 1) \leftrightarrow S(\theta) = \frac{\sinh\theta - i\sin\pi p}{\sinh\theta + i\sin\pi p}$$

Infinite volume

$$E(\theta) = m\cosh\theta$$

$$p(\theta) = m\sinh\theta$$

$$E(\theta_1, ..., \theta_n) = \sum_i E(\theta_i)$$



Finite volume: free particles

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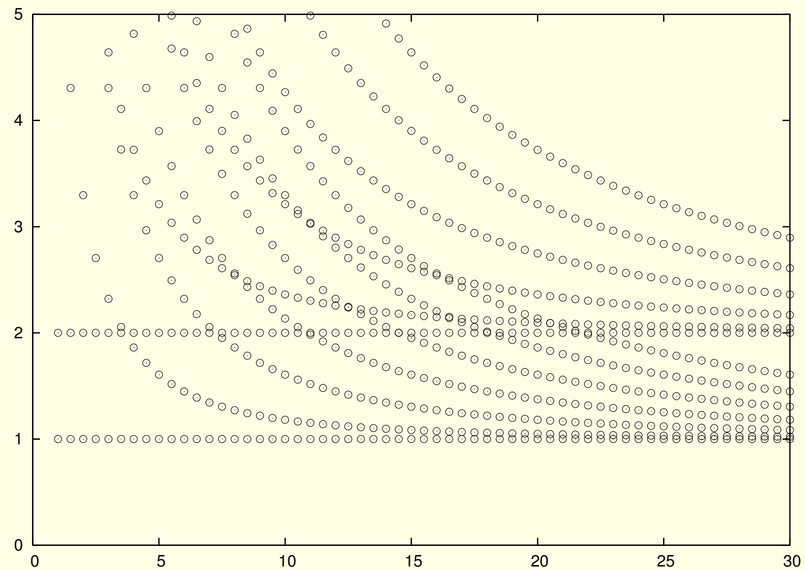

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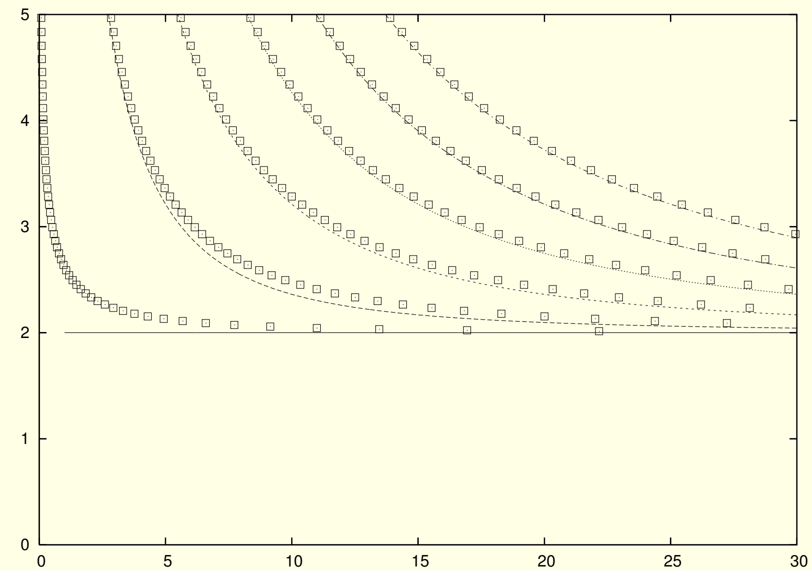
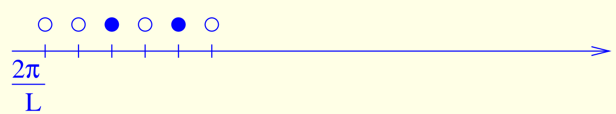
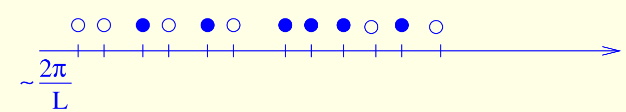


Momentum quantization $S(0) = -1$



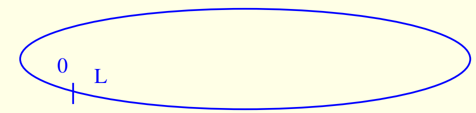


Groundstate energy in finite volume

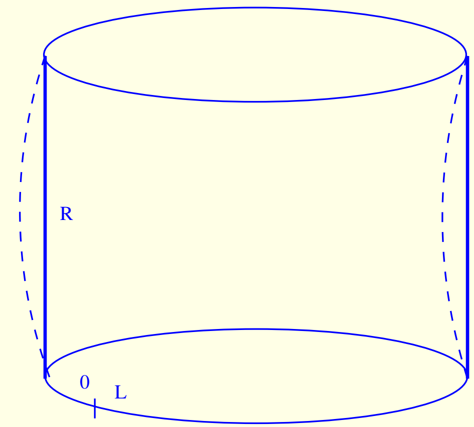
Groundstate energy $E_0(L) =$

Groundstate energy in finite volume

Groundstate energy $E_0(L) =$



Groundstate energy in finite volume



Groundstate energy $E_0(L) =$

$$-\lim_{R\to\infty} \frac{1}{R} \log(\mathrm{Tr}\,(e^{-H(L)R})) = -\lim_{R\to\infty} \frac{1}{R} \log(e^{-E_0(L)R})$$

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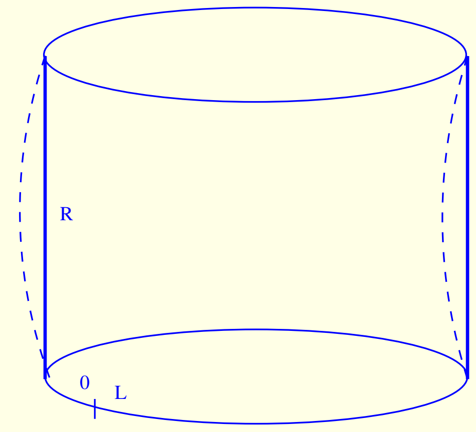

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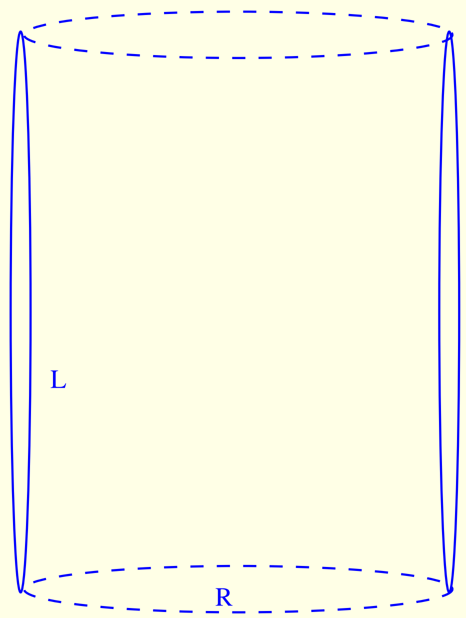
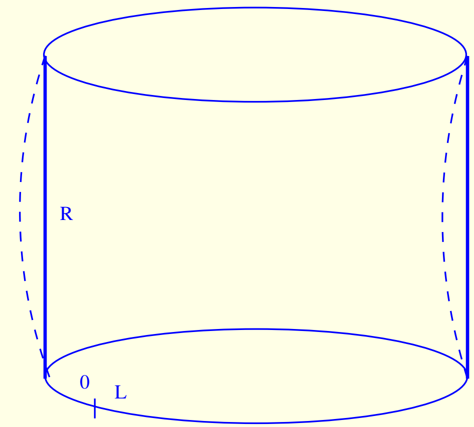


Groundstate energy in finite volume



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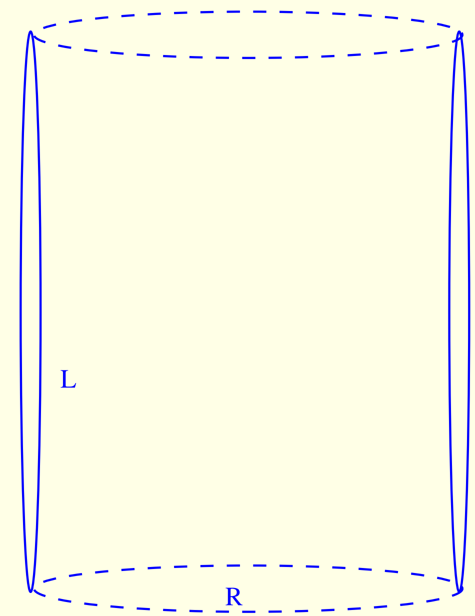
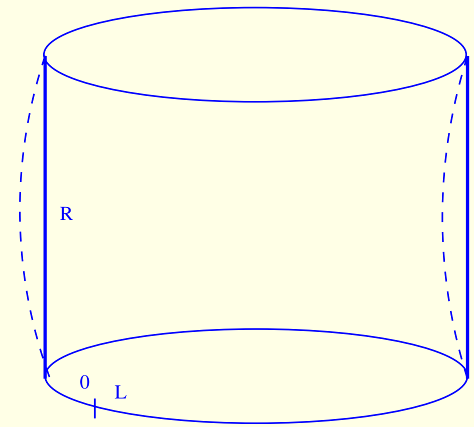


Dominant contribution for large L: one particle term

$$\mathrm{Tr}(e^{-H(R)L}) = 1 + \sum_k e^{-m\cosh\theta_k(R)L} + O(e^{-2mL})$$

Groundstate energy in finite volume



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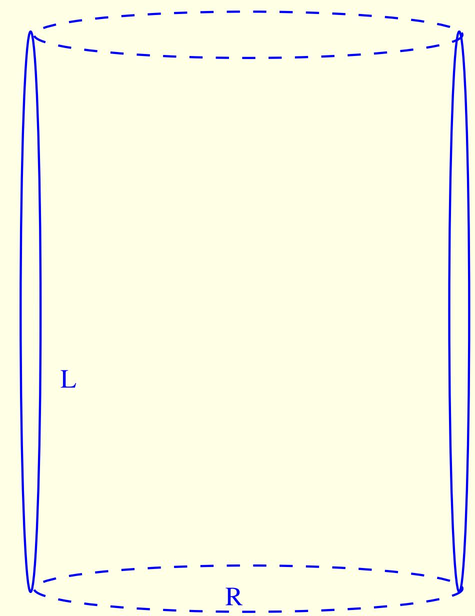

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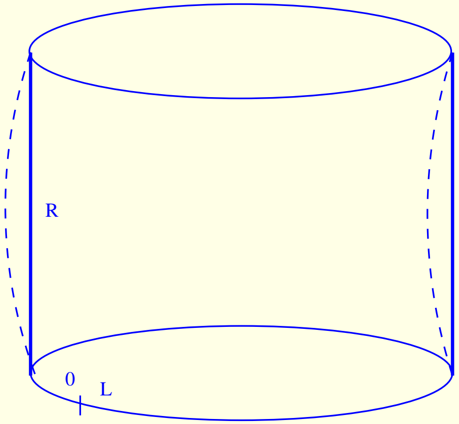
Groundstate energy in finite volume



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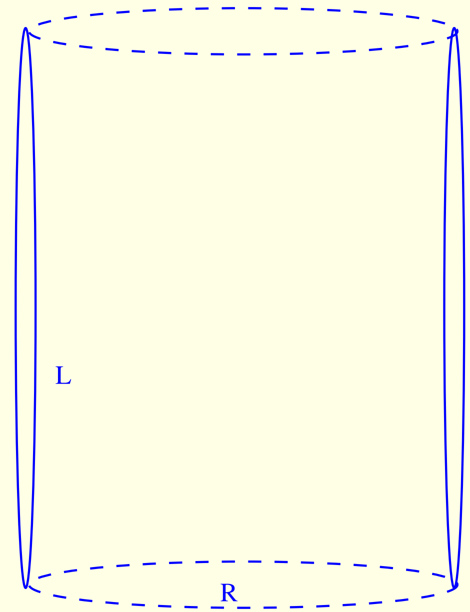

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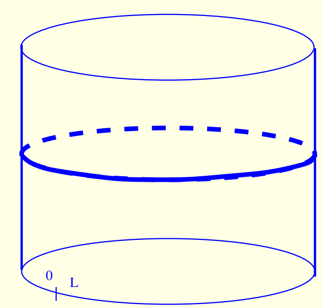
Ground state energy exactly: Al. Zamolodchikov '90

$$\boxed{E_0(L) = -m\int\frac{d\theta}{2\pi}\cosh(\theta)\log(1+e^{-\epsilon(\theta)})}$$

$$\epsilon(\theta) = mL\cosh\theta - \int\frac{d\theta'}{2\pi}\varphi'(\theta-\theta')\log(1+e^{-\epsilon(\theta')})$$



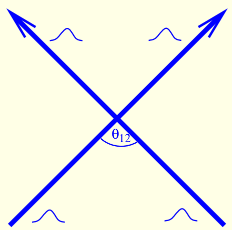
Plan of talk

Cylinder

Plan of talk

Cylinder

Infinite volume

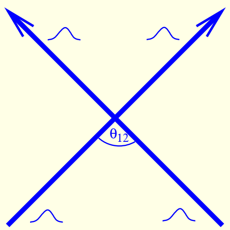


S-matrix

Plan of talk

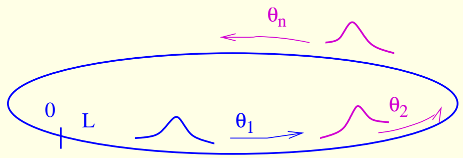
Cylinder

Infinite volume



S-matrix

Large volumes

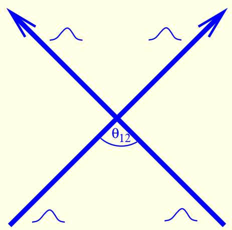


Bethe-Yang lines

Plan of talk

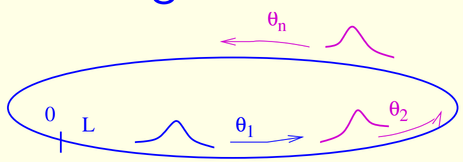
Cylinder

Infinite volume



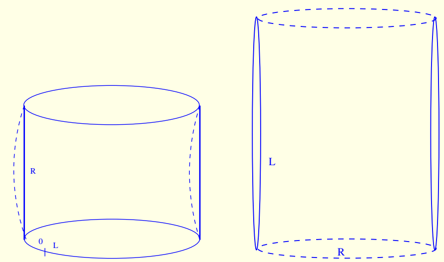
S-matrix

Large volumes



Bethe-Yang lines

Lüscher correction

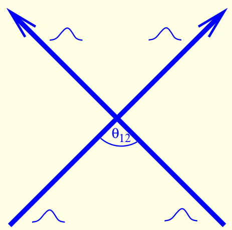


$$E_0(L) = -m \int \frac{d\theta}{2\pi} \cosh\theta \, e^{-m\cosh(\theta)L}$$

Plan of talk

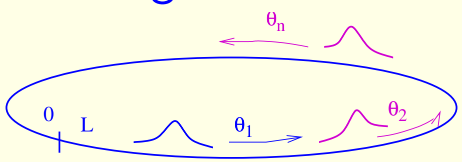
Cylinder

Infinite volume



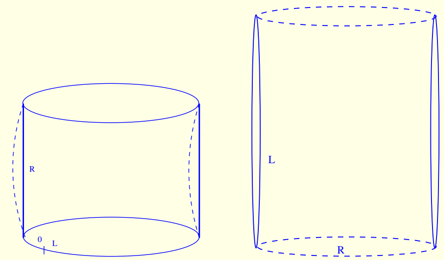
S-matrix

Large volumes



Bethe-Yang lines

Lüscher correction



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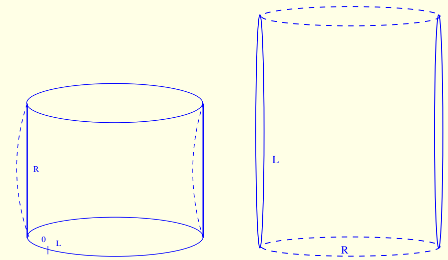
Exact groundstate

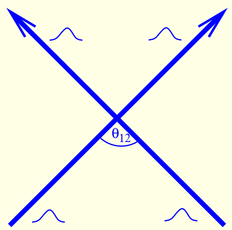


$$E_0(L) = -\frac{m}{2\pi}\int \cosh\theta \log(1+e^{-\epsilon(\theta)})d\theta$$

Plan of talk

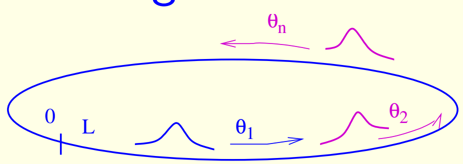
Cylinder

Infinite volume



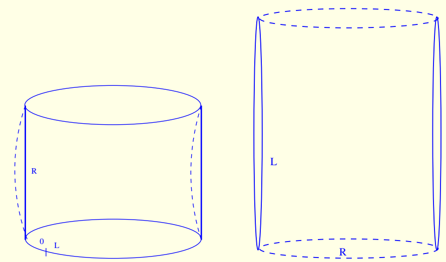
S-matrix

Large volumes



Bethe-Yang lines

Lüscher correction



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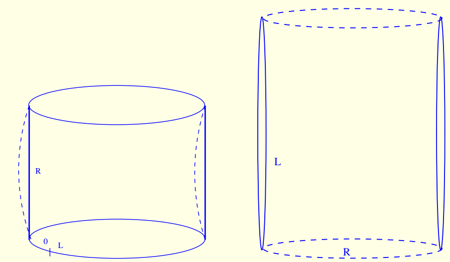


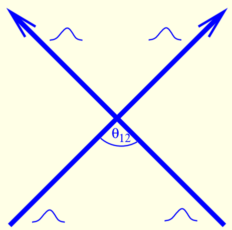
$$E_0(L) = -\frac{m}{2\pi} \int \cosh\theta \log(1 + e^{-\epsilon(\theta)}) d\theta$$

Strip

Plan of talk

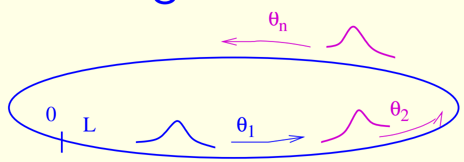
Cylinder

Infinite volume



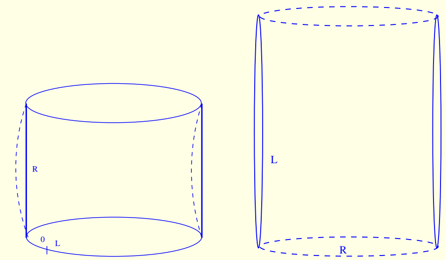
S-matrix

Large volumes



Bethe-Yang lines

Lüscher correction



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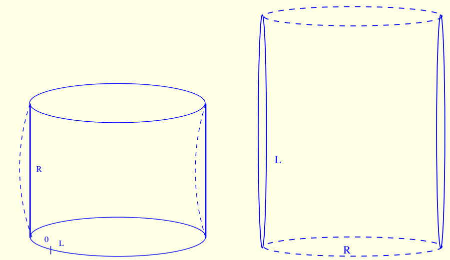
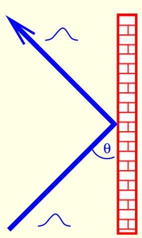


$$E_0(L) = -\frac{m}{2\pi}\int \cosh\theta \log(1+e^{-\epsilon(\theta)})d\theta$$

Strip

Semiinfinite volume

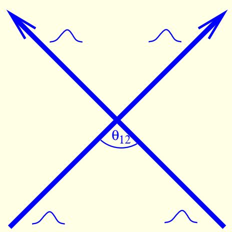


R-matrix

Plan of talk

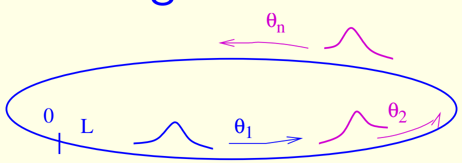
Cylinder

Infinite volume



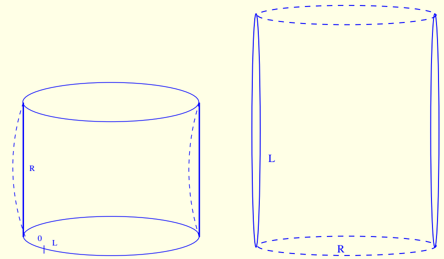
S-matrix

Large volumes



Bethe-Yang lines

Lüscher correction



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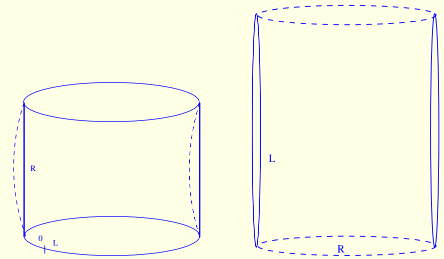
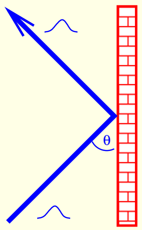
Exact groundstate



$$E_0(L) = -\frac{m}{2\pi}\int \cosh\theta \log(1+e^{-\epsilon(\theta)})d\theta$$

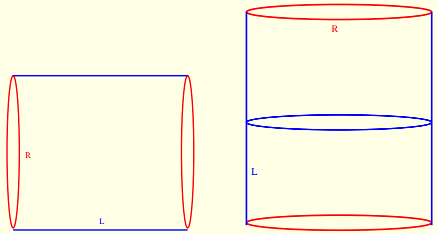
Strip

Semiinfinite volume



R-matrix

Boundary Lüscher correction



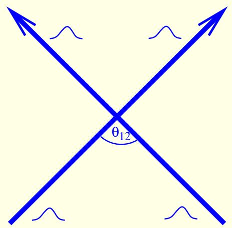
$$E_0(L) = -\int \frac{md\theta}{4\pi}\cosh\theta K(\theta)\bar{K}(\theta)e^{-2mL\cosh(\theta)}$$

$$K(\theta) = R(\frac{i\pi}{2}-\theta)$$

Plan of talk

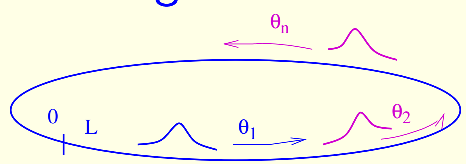
Cylinder

Infinite volume



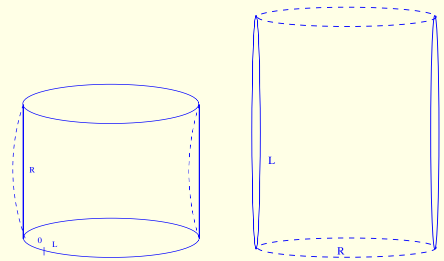
S-matrix

Large volumes



Bethe-Yang lines

Lüscher correction



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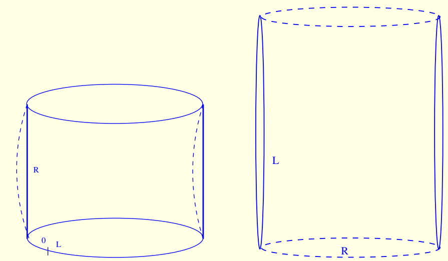
Exact groundstate



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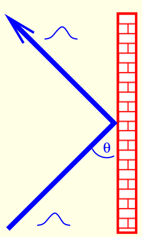
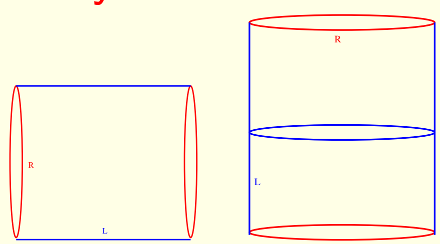
Strip

Semiinfinite volume



R-matrix

Boundary Lüscher correction



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Boundary TBA

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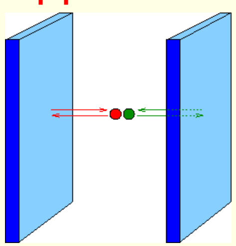
Application

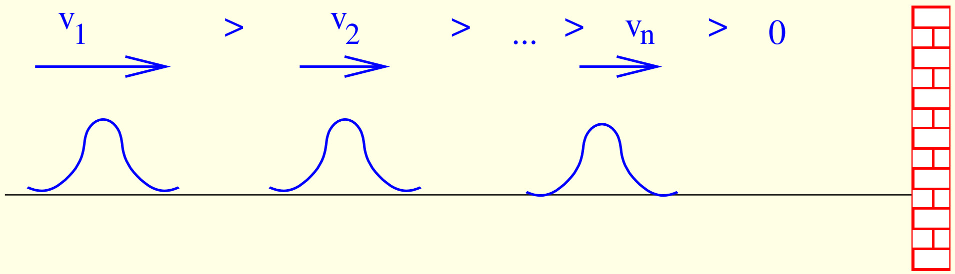


Casimir effect

Integrable boundary field theory: Bootstrap

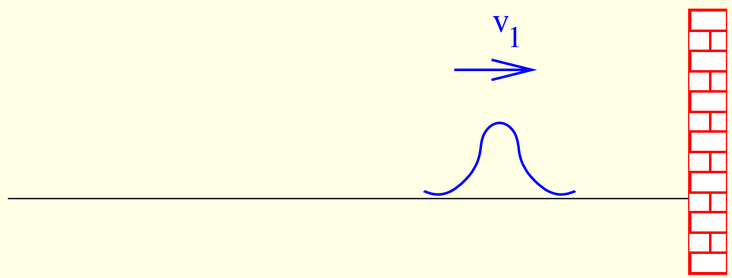
Integrable boundary field theory: Bootstrap

Boundary multiparticle state: with n particles



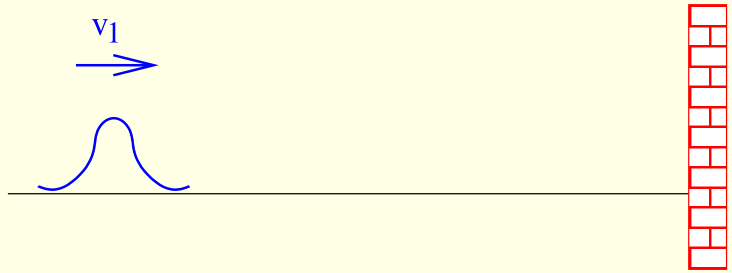
Integrable boundary field theory: Bootstrap

Boundary one particle state:



Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$



Integrable boundary field theory: Bootstrap

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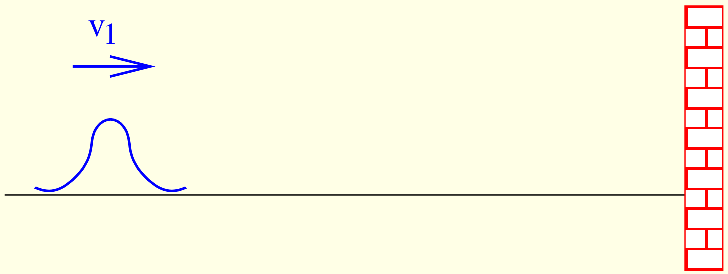
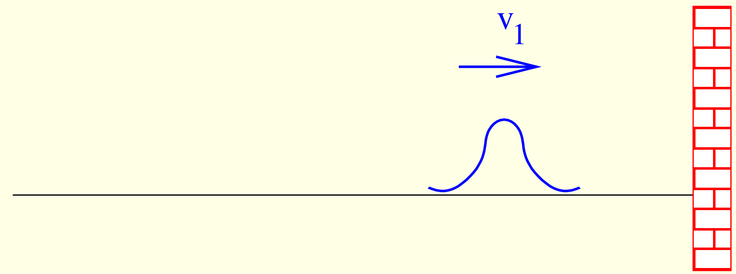
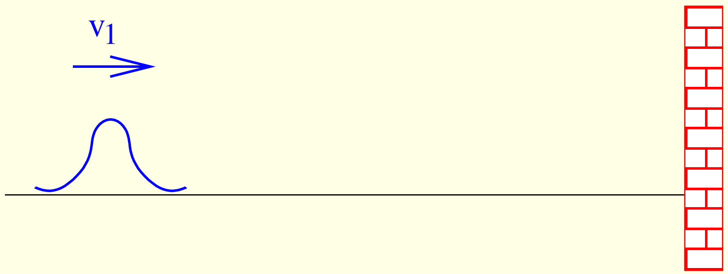


times develop

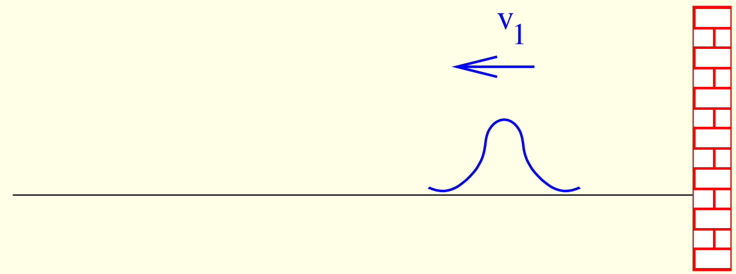


Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$



times develop further



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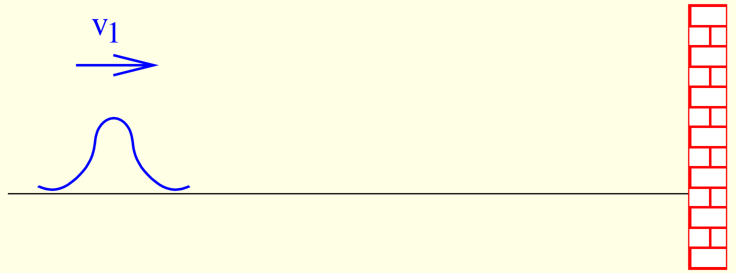
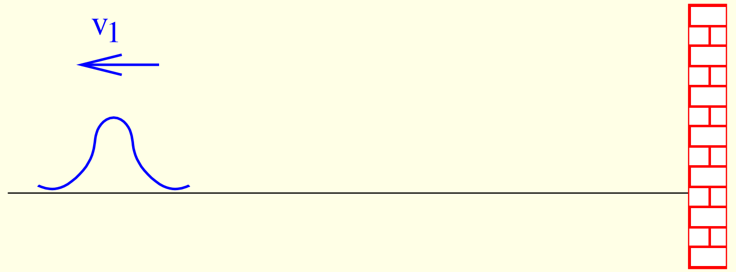
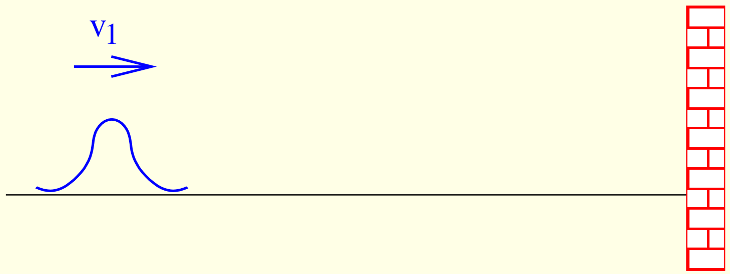


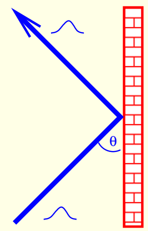
Boundary one pt out state: $t \to \infty$



Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$





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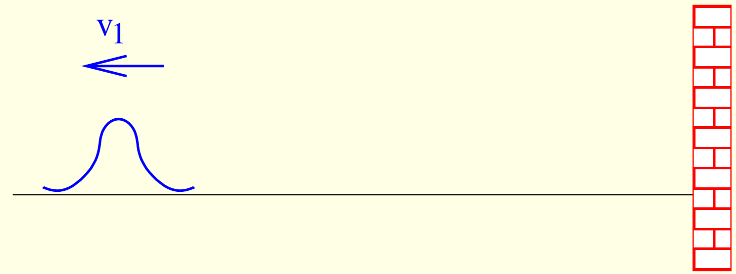
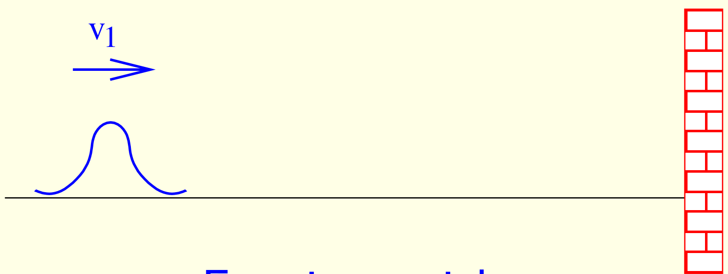
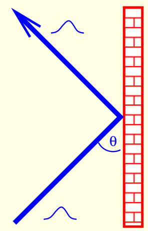


Integrable boundary field theory: Bootstrap

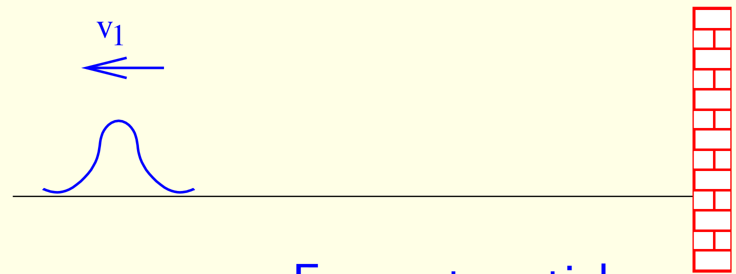
Boundary one particle in state: $t \to -\infty$



Free in particle



Boundary one pt out state: $t \to \infty$



Free out particle

Integrable boundary field theory: Bootstrap

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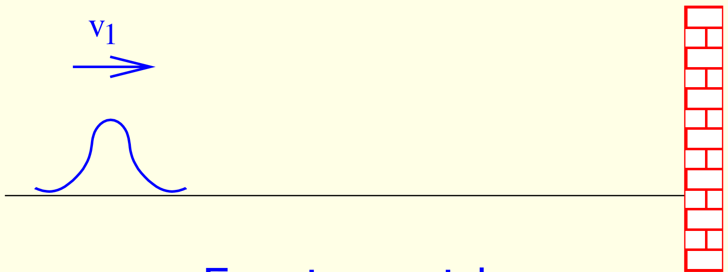
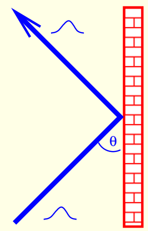
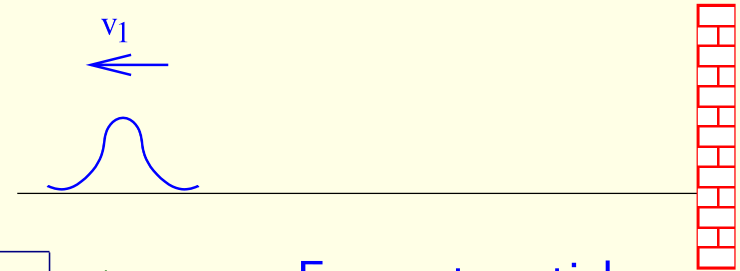
Boundary one pt out state: $t \to \infty$



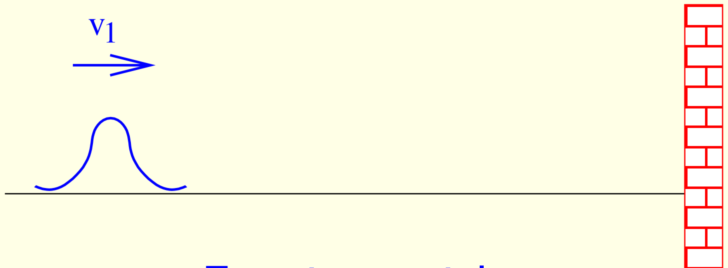


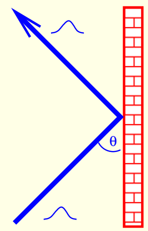


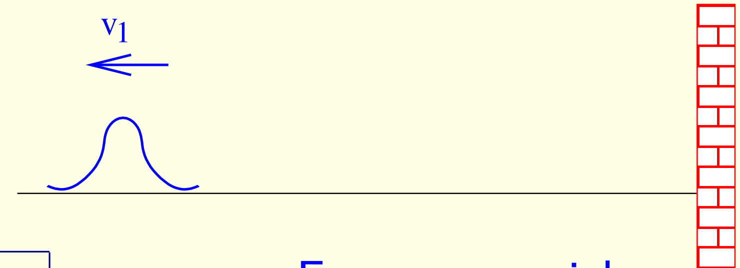
Free in particle $\leftarrow$ **R-matrix** $\rightarrow$ Free out particle

Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$ Boundary one pt out state: $t \to \infty$







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$$|\theta\rangle_B \quad = \quad R(\theta)|-\theta\rangle_B$$

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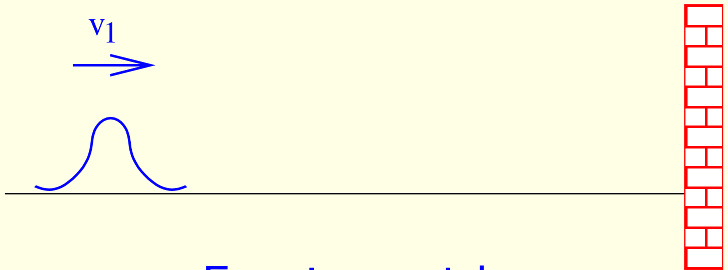

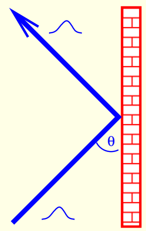

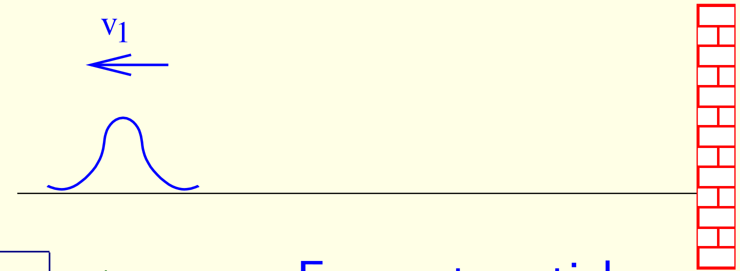

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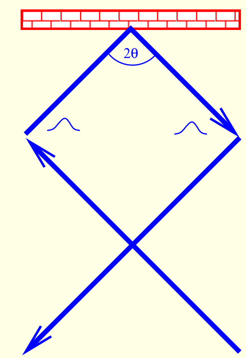
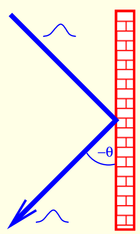


Unitarity

$$R^*(\theta) = R^{-1}(\theta) = R(-\theta)$$



Boundary crossing unitarity

$$R(\frac{i\pi}{2} + \theta) = S(2\theta)R(\frac{i\pi}{2} - \theta)$$

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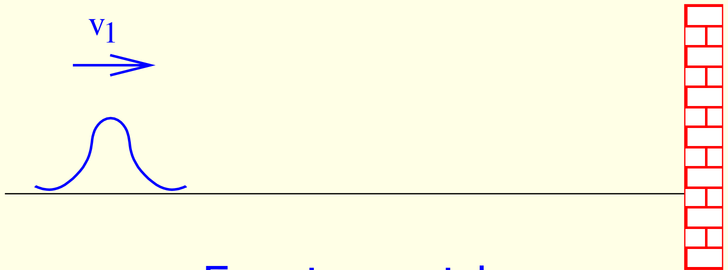

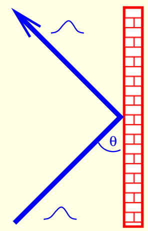

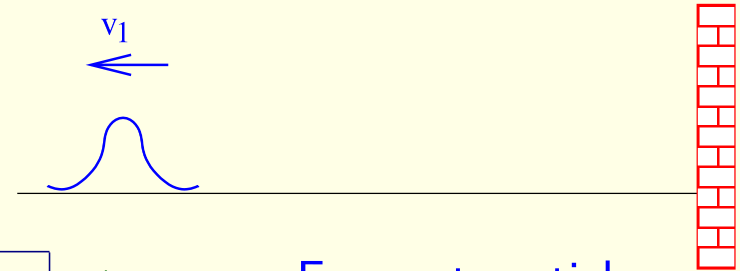

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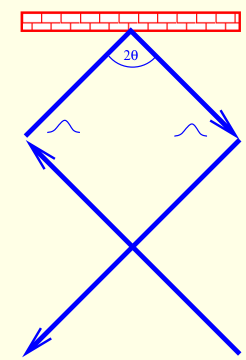

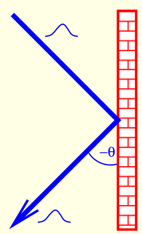

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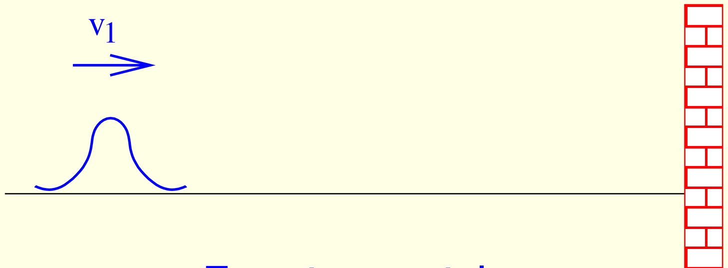

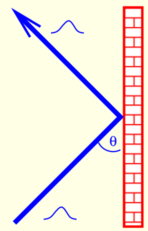

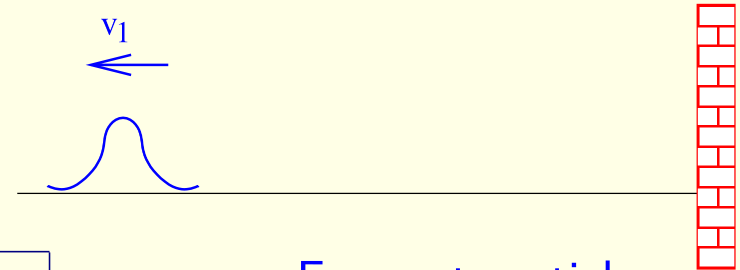

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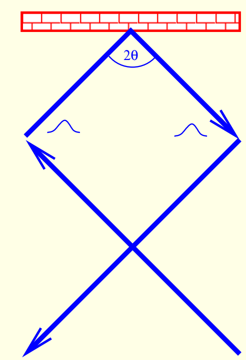

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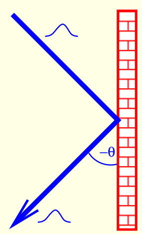

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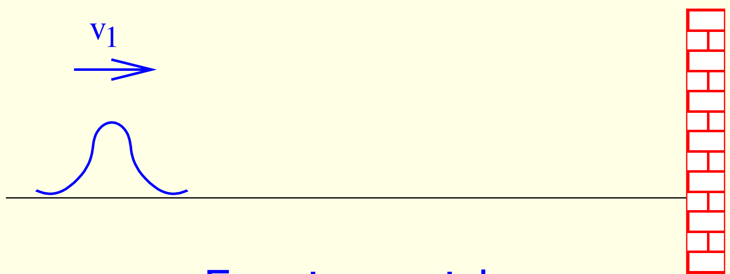

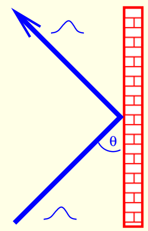

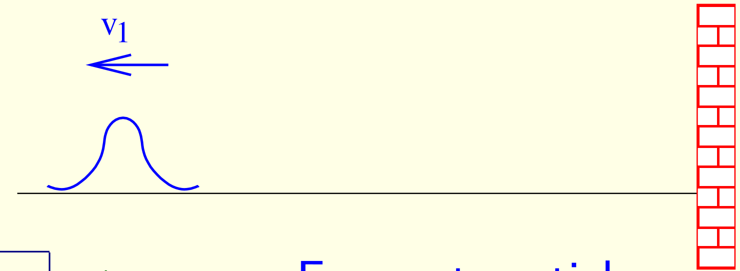

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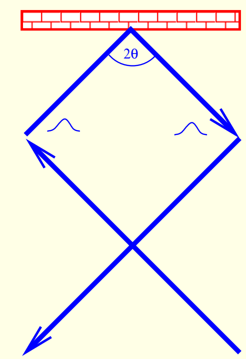

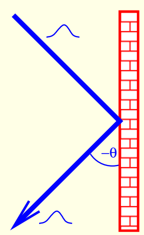

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Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \mu(\cosh b\phi - 1) - \delta\left[\mu_+^B e^{\frac{b}{2}\phi} + \mu_-^B e^{-\frac{b}{2}\phi}\right]$ $\sqrt{\frac{\mu}{\sin b^2}}\cosh b^2(\eta \pm \Theta) = \mu_\pm^B$

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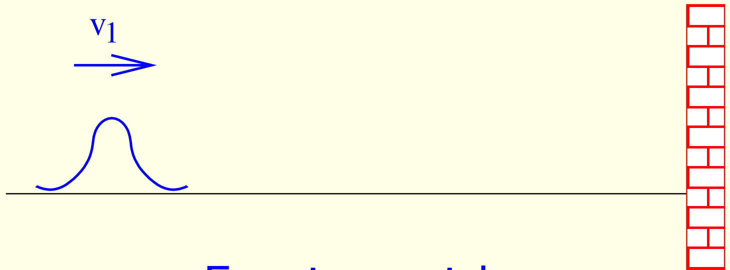
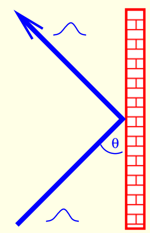
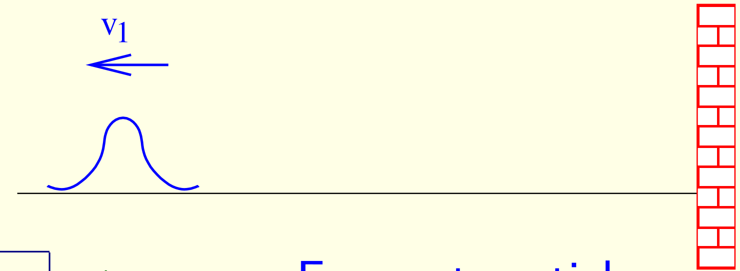






Free in particle

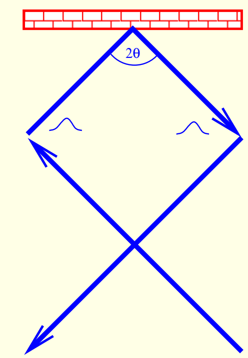
$|\theta\rangle_B$

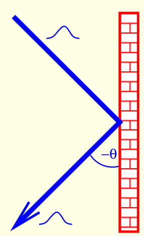
$\leftarrow$ **R-matrix** $\rightarrow$

$=$

Free out particle

$R(\theta)|-\theta\rangle_B$





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Ghoshal-Zamolodchikov '94

$$\sqrt{\frac{\mu}{\sin b^2}} \cosh b^2(\eta \pm \Theta) = \mu_\pm^B$$

Integrability $\rightarrow$factorizability: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B = \prod_{i<j} S(\theta_i - \theta_j) S(\theta_i + \theta_j) \prod_i R(\theta_i)|-\theta_1, -\theta_2, \ldots, -\theta_n\rangle_B$

Boundary Lüscher correction

Groundstate energy for large L from IR reflection: $E_0(L) =$

Boundary Lüscher correction

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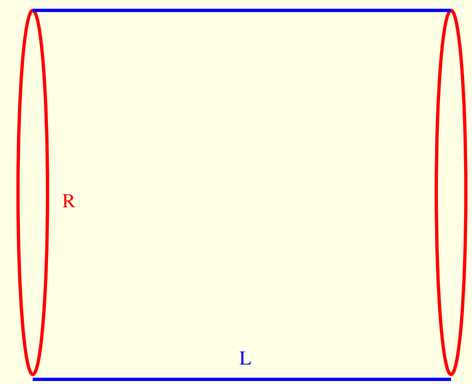
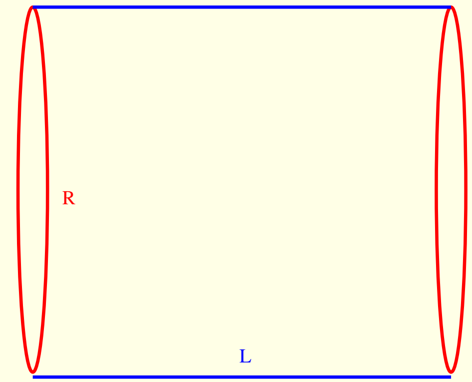
Groundstate energy for large L from IR reflection: $E_0(L) =$

$$-\lim_{R\to\infty}\frac{1}{R}\log(\mathrm{Tr}(e^{-H^B(L)R})) = -\lim_{R\to\infty}\frac{1}{R}\log(e^{-E_0(L)R})$$





Boundary Lüscher correction

Groundstate energy for large L from IR reflection: $E_0(L) =$

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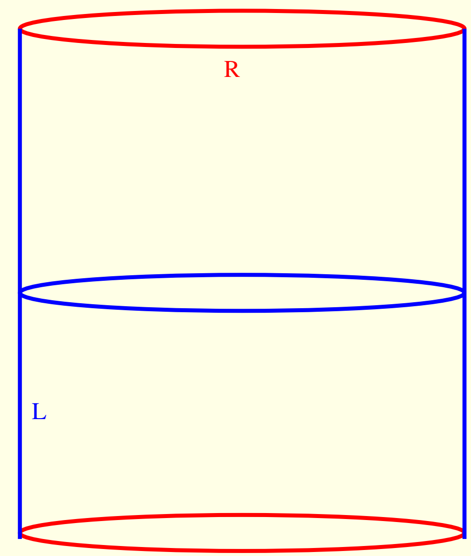

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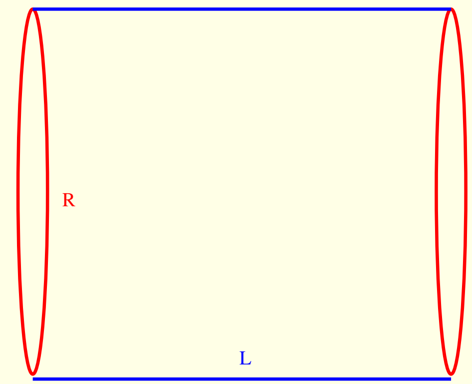

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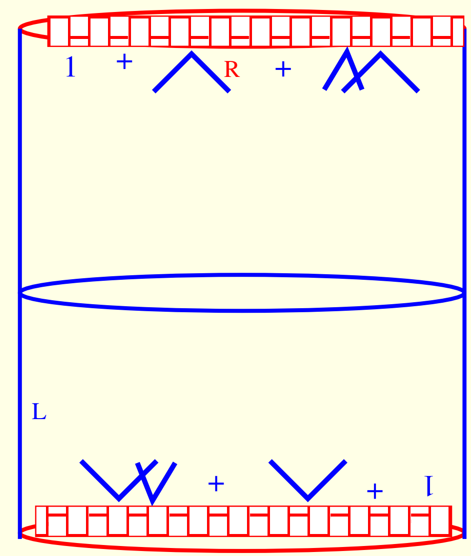
Boundary state $|B\rangle = \exp\left\{\int_{-\infty}^{\infty}\frac{d\theta}{4\pi}R(\frac{i\pi}{2}-\theta)A^+(-\theta)A^+(\theta)\right\}|0\rangle$



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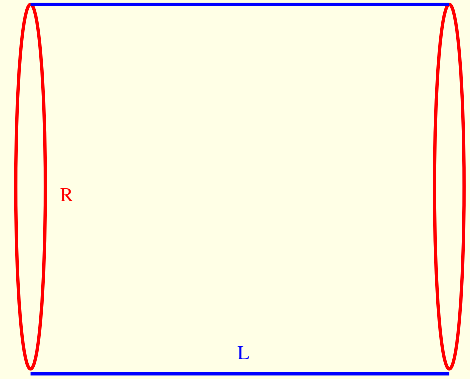

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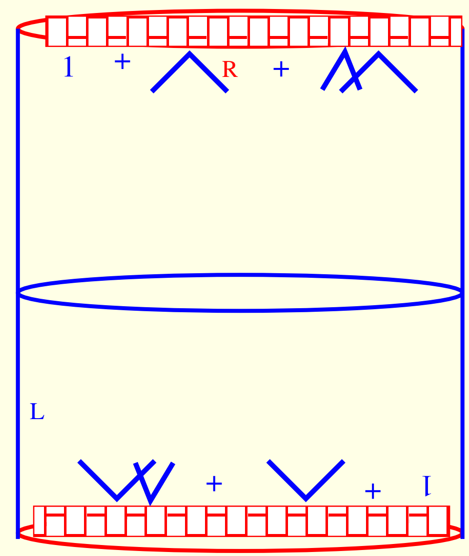
Dominant contribution for large L: two particle term

$$\langle B|e^{-H(R)L}|B\rangle = 1+\sum_k R(\frac{i\pi}{2}-\theta)R(\frac{i\pi}{2}+\theta)e^{-2m\cosh\theta_k L}+\dots$$



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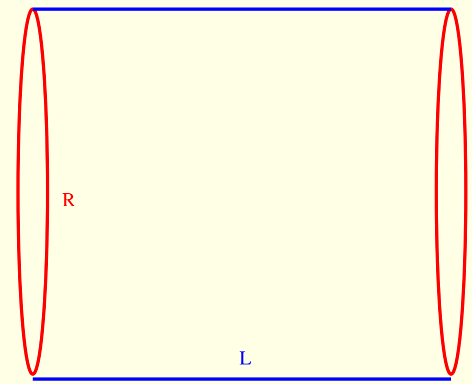

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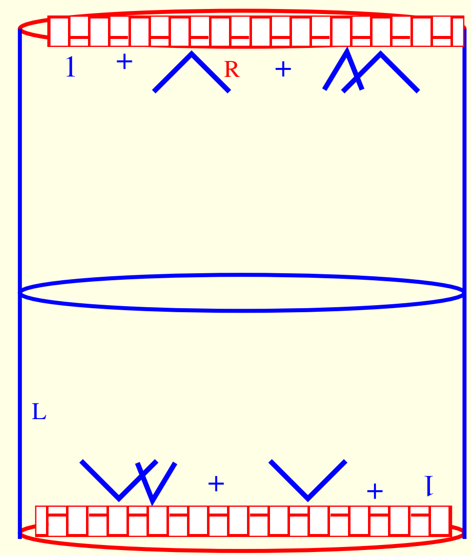

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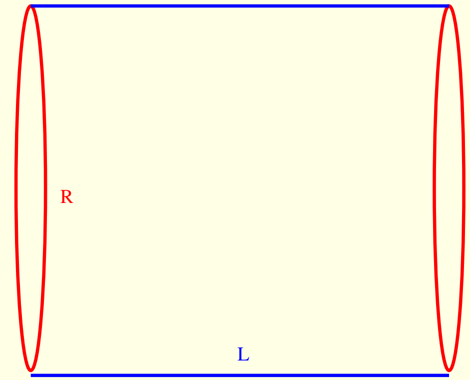
quantization condition: $m\sinh\theta_k = \frac{2\pi}{R}$ $\quad \sum_k \to \frac{Rm}{4\pi}\int d\theta\cosh\theta$

$E_0(L) = -\int\frac{m\cosh\theta d\theta}{4\pi}R(\frac{i\pi}{2}-\theta)R(\frac{i\pi}{2}+\theta)\,e^{-2mL\cosh\theta}$; Z.B, L. Palla, G. Takacs '04-'08

Boundary Lüscher correction



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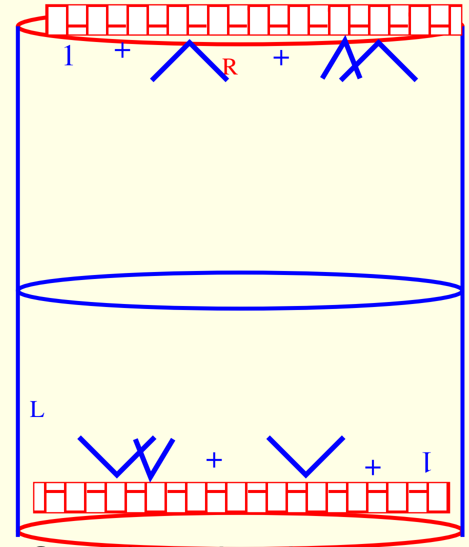

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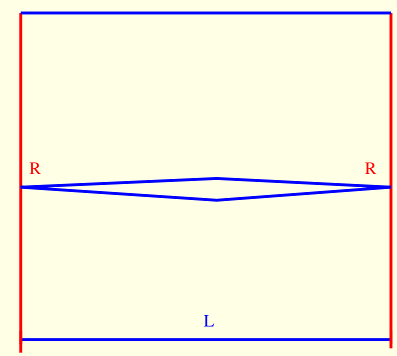
$$E_0(L) = -\int\frac{m\cosh\theta d\theta}{4\pi}R(\frac{i\pi}{2}-\theta)R(\frac{i\pi}{2}+\theta)\,e^{-2mL\cosh\theta}$$; Z.B, L. Palla, G. Takacs '04-'08



Ground state energy exactly: $\boxed{E_0(L) = -m\int\frac{d\theta}{4\pi}\cosh(\theta)\log(1+e^{-\epsilon(\theta)})}$

$$\epsilon(\theta) = 2mL\cosh\theta - \log(R(\frac{i\pi}{2}-\theta)R(\frac{i\pi}{2}+\theta))$$
$$-\int\frac{d\theta'}{2\pi}\varphi'(\theta-\theta')\log(1+e^{-\epsilon(\theta')})$$ LeClair, Mussardo, Saleur, Skorik

Casimir effect: Boundary finite size effect

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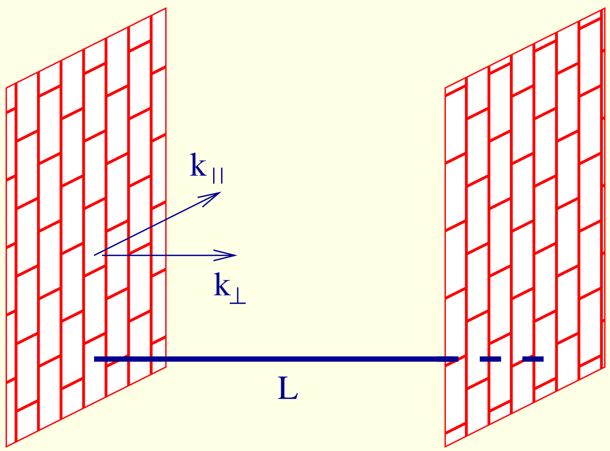


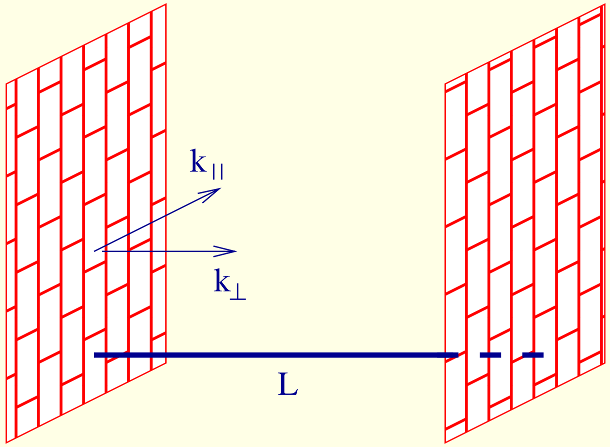
Extension to higher dimensions: $\vec{k}_{\|}$ label

Dispersion $\omega = \sqrt{m^2 + \vec{k}_{\|}^2 + k_{\perp}^2} = \sqrt{m_{\text{eff}}^2 + k_{\perp}^2}$

rapidity $\omega = m_{\text{eff}}(k_{\|}) \cosh\theta, \; k_{\perp} = m_{\text{eff}}(k_{\|}) \sinh\theta$

Reflection $R(\theta, m_{\text{eff}}(k_{\|}))$

Casimir effect: Boundary finite size effect



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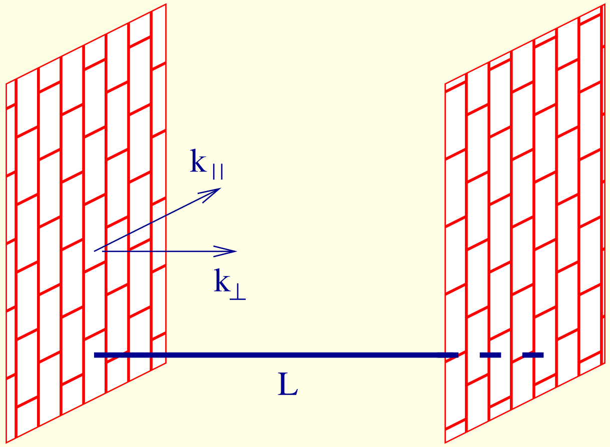
Bstate: $|B\rangle = \left\{ 1 + \int \frac{d^{D-1}k_{\|}}{(2\pi)^{D-1}} \frac{d\theta}{4\pi} R(\frac{i\pi}{2} - \theta, m_{\text{eff}}(k_{\|})) A^+(-\theta, -\vec{k}_{\|}) A^+(\theta, \vec{k}_{\|}) + \ldots \right\} |0\rangle$

Casimir effect: Boundary finite size effect



Extension to higher dimensions: $\vec{k}_{||}$ label

Dispersion $\omega = \sqrt{m^2 + \vec{k}_{||}^2 + k_\perp^2} = \sqrt{m_{\text{eff}}^2 + k_\perp^2}$

rapidity $\omega = m_{\text{eff}}(k_{||}) \cosh\theta$, $k_\perp = m_{\text{eff}}(k_{||}) \sinh\theta$

Reflection $R(\theta, m_{\text{eff}}(k_{||}))$

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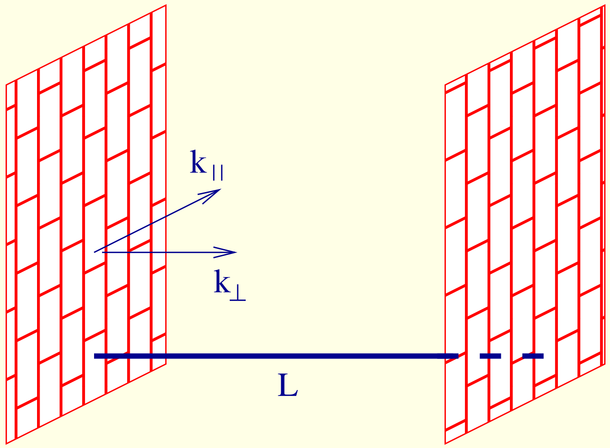
Ground state energy (for free bulk):

$$E_0(L) = \int \frac{d^{D-1}k_{||}}{(2\pi)^{D-1}} \frac{d\theta}{4\pi} \log(1 + R^1(\frac{i\pi}{2} + \theta, m_{\text{eff}}) R^2(\frac{i\pi}{2} - \theta, m_{\text{eff}}) e^{-2m_{\text{eff}} \cosh\theta L})$$

Casimir effect: Boundary finite size effect



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QED: Parallel dielectric slabs $(\epsilon_1, 1, \epsilon_2)$

reflections $E_{\|,\perp}, B_{\|,\perp} \longrightarrow R_{\|,\perp}$ look it up in Jackson:

$$R_{\perp}(\omega, k_{\|} = q) = \frac{\sqrt{\omega^2 - q^2} - \sqrt{\epsilon\omega^2 - q^2}}{\sqrt{\omega^2 - q^2} + \sqrt{\epsilon\omega^2 - q^2}} \quad R_{\|}(\omega, k_{\|} = q) = \frac{\epsilon\sqrt{\omega^2 - q^2} - \sqrt{\epsilon\omega^2 - q^2}}{\epsilon\sqrt{\omega^2 - q^2} + \sqrt{\epsilon\omega^2 - q^2}}$$

Casimir effect: Boundary finite size effect

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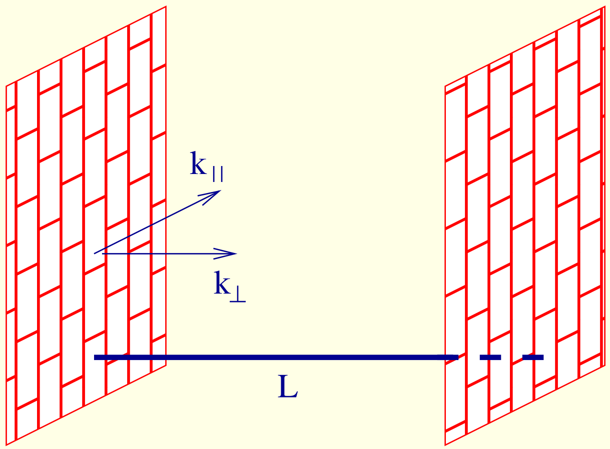

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Lifshitz formula

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Conclusion

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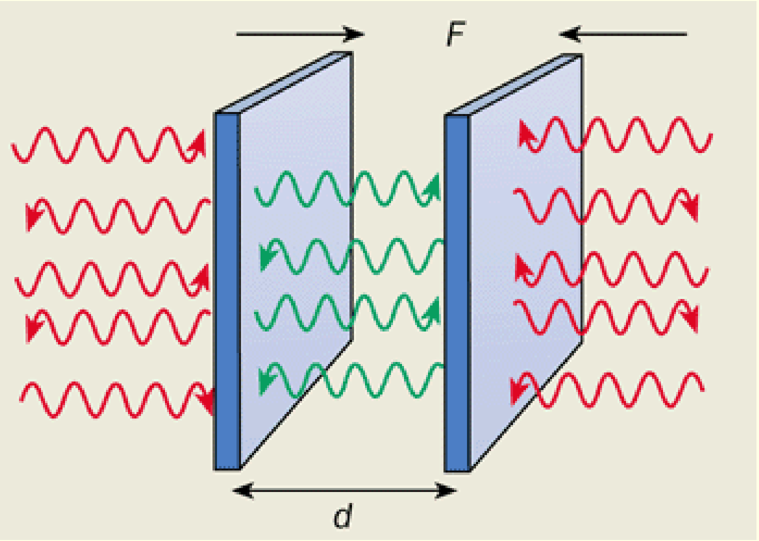

Usual derivation:

summing up zero freqencies

$E_0(L) = \frac{1}{2} \sum_{k(L)} \omega(k(L)) \propto \infty$

Complicated finite volume problem

$+$ divergencies

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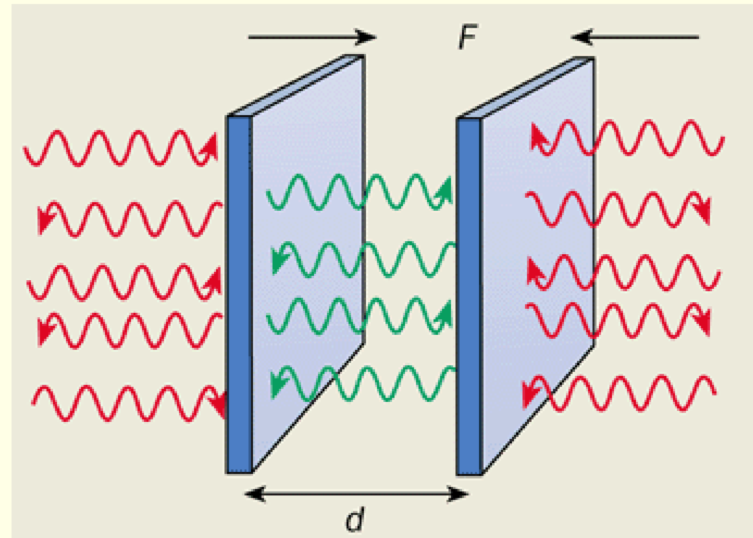
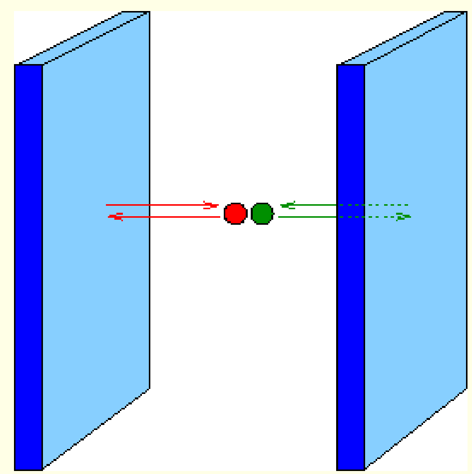


as a boundary finite size effect

$$\boxed{E_0(L) = -\int \frac{d\tilde{p}}{2\pi} \log(1 + R(-\tilde{p})R(\tilde{p})e^{-2E(\tilde{p})L})}$$



Reflection factor of the IR degrees of freedom:

semi infinite settings,

easier to calculate,

no divergences