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Károly Zoltán Csukás

Motivations

Due to the non-linearity of Einstein's equations analytic considerations alone do not provide us completely satisfactory tools to describe highly dynamical processes. In the last decades numerical relativity—by modelling sources of gravitational waves, studying perturbations of known backgrounds, investigating relativistic stars etc.—became an essential tool on its own way to discover physically interesting aspects of strong gravitational fields. My work is centered around developing new tools to investigate the dynamics of strong gravitational fields numerically.

Methods

The methods I use are inspired by the numerical setup covered by GridRipper, developed years before by P. Csizmadia, A. László and I. Rácz in Wigner RCP [1]. The basic assumption is that the spacetime can be foliated by a two-parameter family of topological 2-spheres. This approach enables us to handle the angular-dependent part of physical and geometric quantities by spherical harmonics expansion and evaluate the tangential derivatives analytically instead of using less precise finite differencing methods. A unique feature of our implementation is the fully spectral evaluation of the equations, i.e. every computation is done by referring only the expansion coefficients of the fields.

To improve the capabilities of this construction I have introduced the systematic use of spin-weighted spherical harmonics into the framework. The transformation properties of spin-weighted spherical harmonics enable us to represent and solve equations valid not just for scalar but for vectorial and tensorial quantities.

Achievements

So far we have applied these new methods to study two disjunct set of problems with fundamentally different theoretic backgrounds.

The first one is linear perturbations of Kerr black holes. Teukolsky's master equation, which represent the radiative linear perturbations, are in the center of various investigations.

- Based on the study of the simpler Minkowski and Schwarzschild spacetimes its solutions are a key part in the study of the full, non-linear stability of Kerr. Our code was stable for long enough to observe the asymptotic behaviour of the perturbations and read the decay rates directly at the horizon, at future null infinity and at some locations in between. Using our numerical setup we could—as far as we know, for the first time—carry out a systematic investigation of the decay rates of general (non-axially symmetric) electromagnetic and gravitational perturbations [2]. Our results passed various checks based on analytic arguments, most notably the conservation of a novel canonically conserved current was monitored.
- Studies on superradiant scattering of linear radiative perturbations are usually focused on frequency-domain analyses. We investigated (initially) pure mode perturbations of various compact support by solving Teukolsky's equation in time-domain. The conclusion of our study is that for compactly supported initial data the separation between the black hole and the initial data plays crucial part in superradiant amplification. Initial data with large separation tends to be reflected before even interacting with the ergoregion therefore widely separated perturbations produce significantly less amplification if any [3].

The second area of application is producing deformed black hole initial data. These type of initial data always contain gravitational waves hence their importance is indisputable. In determining the corresponding initial data one has to keep in mind that the constraint equations are underdetermined. The widely used method to solve them is based on the conformal method of Lichnerowicz. Until recently it was customary to apply a set of simplifications to the constraints most notably to assume conformal flatness of the induced metric. It is clear now that solutions of these simplified equations do not represent well the physics of binary black hole systems which results in a non-physical junk radiation. Developing alternative methods for generating initial data therefore is of great interest. Such a method was found by I. Rácz. By solving the constraints for different sets of variables they obtain evolutionary form [4, 5], which is an alternative to the whole conformal approach.

- As this is a radically new approach it is a natural question to ask how to produce asymptotically flat and thus physically meaningful configurations. Our work motivated by previous results in the literature pointed out that controlling the monopole part of the exterior curvature's tensorial projection's trace is a necessity in order to achieve this goal [6]. We concluded that the previously used definition of "near Schwarzschild" solutions is too strict to allow any other asymptotically flat initial data than the Schwarzschild one, however, by loosening this definition one may find new families of asymptotically flat initial data.

Future plans

The works presented in the previous paragraphs show that the new tools are useful in a wide range of applications. Also both of these studies have natural continuations. As for the constraints I am already studying perturbed Kerr initial data wich is technically more complex than the Schwarzschild case and we have plans on generating binary black hole initial data. Since solving the constraints in their evolutionary form is much cheaper we aim to implement a fully constrained evolutionary scheme sketched out in [4]. There is a hope this last step will be conclusive whether by omitting boundary conditions in the strong field regime we may produce gravitational wave signals with significantly less junk radiation.

References

[1] Péter Csizmadia, András László, and István Rácz. "On the Use of Multipole Expansion in Time Evolution of Non-linear Dynamical Systems and Some Surprises Related to Superradiance". In: *Class. Quant. Grav.* 30 (2013), p. 015010. DOI: 10.1088/0264-9381/30/1/015010. arXiv: 1207.5837 [gr-qc].

[2] Károly Csukás, István Rácz, and Gábor Zsolt Tóth. "Numerical investigation of the dynamics of linear spin s fields on a Kerr background: Late-time tails of spin $s = \pm 1, \pm 2$ fields". In: *Phys. Rev. D* 100.10 (2019), p. 104025. DOI: 10.1103/PhysRevD.100.104025. arXiv: 1905.09082 [gr-qc].

[3] Károly Zoltán Csukás and István Rácz. "Numerical investigation of the dynamics of linear spin s fields on a Kerr background II: Superradiant scattering". In: (Jan. 2021). arXiv: 2101.05530 [gr-qc].

[4] István Rácz. "Cauchy problem as a two-surface based 'geometrodynamics'". In: *Class. Quant. Grav.* 32 (2015), p. 015006. DOI: 10.1088/0264-9381/32/1/015006. arXiv: 1409.4914 [gr-qc].

[5] István Rácz. "Constraints as evolutionary systems". In: *Class. Quant. Grav.* 33.1 (2016), p. 015014. DOI: 10.1088/0264-9381/33/1/015014. arXiv: 1508.01810 [gr-qc].

[6] Károly Csukás and István Rácz. "Numerical investigations of the asymptotics of solutions to the evolutionary form of the constraints". In: *Class. Quant. Grav.* 37.15 (2020), p. 155006. DOI: 10.1088/1361-6382/ab8fce. arXiv: 1911.02900 [gr-qc].