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Motivations

Application of numerical methods in general relativity went through a rapid development in the past decades. Breakthroughs in the field made possible to study gravitational phenomena previously unavailable to us. My research interest lies in developing and applying numerical methods to study various aspects of general relativity.

Black hole perturbation theory

Teukolsky's master equation describes various kinds of perturbations around a rotating black hole. During my PhD, supervised by Prof. Dr. István Rácz, I developed a code, TeuSolver, solving the homogeneous Teukolsky equation in time domain using compactified hyperboloidal slices. In the angular sector we used spin-weighted spherical harmonic expansion, finite diffence methods in radial direction, and the method of lines with Kreiss-Oliger terms to integrate in time. Using this code we studied late time behaviour of perturbations and superradiant scattering.

- Rigorous proofs of Kerr spacetime's stability against perturbations rely on assumptions about how fast such perturbations decay over time. In [1] we studied the late time behaviour of scalar, electromagnetic, and gravitational perturbations confirming earlier results in axial symmetry and providing a detailed survey of nonaxisymmetric perturbations for the first time. We also used new canonically conserved currents constructed from two adjoint solutions to quantify the error in our computations.
- Superradiant scattering around rotating black holes is one possible channel of powering the most energetic astrophysical phenomena. Although this process is extensively studied in frequency domain, we used our time domain solver to circumvent the standard assumptions of the field [2]. Specifically we studied the effect of superradiance on wave packets initially not in contact with the horizon. We found that even optimally tuned wave packets might produce negligible effect because they get reflected before reaching the black hole. To quantify the effect we used the Teukolsky-Starobinsky identities to compute the previously mentioned currents from a single solution.

Alternative formulations of the constraint equations

Initial data in general relativity has to satisfy constraint equations, traditionally interpreted as an elliptic system. A new formulation devised by Prof. Dr. István Rácz, exploiting the underdetermined nature of the problem, starts by specifying the fields on a single 2-dimensional surface and constructs the 3-dimensional initial data by "evolving" them toward infinity. A major issue with the **evolutionary formulation of the constraints** was the inability to ensure the asymptotic behaviour corresponding to isolated systems.

- Extending the methods used in TeuSolver, I developed ConstraintSolver using spin-weighted spherical harmonic expansion on topological 2-spheres and integrating the equations using an adaptive Runge-Kutta method. The codebase has recently been made available to the public [3].
- Generating perturbed Schwarzschild initial data we showed that apart from a single mode of a single variable, $\mathbf{K}$, all the modes fall off according to asymptotic flatness [4]. Strengthening our claim we evolved the deviation separately from the background so the "anomalous" behaviour was more apparent.
- We devised a procedure which controls the behaviour of $\mathbf{K}$ by adjusting a single parameter. Depending on the set value of the parameter the method yields initial data that is either asymptotically flat in the strong sense or asymptotically flat in the weaker sense with a prescribed falloff rate [5]. To demonstrate the viability of the method we constructed perturbed Kerr initial data.

Although the problem of asymptotic flatness is resolved, there are still open questions to investigate.

- We are already studying asymptotically hyperboloidal initial data. Such a setup is better suited to read off gravitational wave content, but avoiding singular behaviour near infinity is an open issue. Our preliminary data is promising and expect to finish a manuscript in the next few months.
- Controlling the asymptotic behaviour of an isolated system is crucial, but it is less of an issue for cosmological initial data. Yet such a setup was not discussed in the literature until recently. The evolutionary formulation could yield initial data without enforcing the standard periodic boundary condition. Investigating the viability of such an initial data is among my plans for 2024.
- There are nonspinning binary black hole and antialigned spin binary black hole initial data in the literature produced by the evolutionary formulation. However working out a method that produces more generic setup, and in general devising algorithms that make this framework scalable to supercomputers is a middle to long term goal necessitating a few years of continuous effort.

Black holes in theories beyond general relativity

Testing the validity of general relativity requires predictions from **theories beyond general relativity**. Working with Dr. Leo Stein and his student, Subhayu Bagchi, we produce stationary and axially symmetric black hole solutions within dynamical Chern-Simons (dCS) theory.

To achieve our goal we familiarized ourselves with the fundamental concepts used in the Spectral Einstein Code (SpEC) and studied in detail its Spectral Elliptic Solver (SpEllS). We extended the functionalities of SpEllS to work with a symmetry reduced (r, ϑ) domain. This restriction makes our code efficient in exploring the parameter space of the solutions and we can study parameter dependence of the orbits of test particles, the structure of light ring, quasinormal modes, etc.

By mentoring a graduate student this project developed various soft skills and forced me to evaluate my work from a different perspective which benefitted my growth significantly.

Missing physics in binary black hole initial data

I have recently started working on a project which combines my expertise in **black hole perturbation theory** and understanding of the process of creating **binary black hole initial data**.

Numerical simulations of black hole binaries are essential in interpreting gravitational wave data. Yet, such simulations are contaminated by unphysical junk radiation. Although there are various methods to mitigate the effect of junk radiation on the waveforms, I argue that improving the quality of initial data is worth the time and effort.

Studies involving binaries of nonspinning black holes suggest that including more physics in the freely specifiable part of the initial data would significantly decrease the junk radiation. The current standard is to set the variables as sums of individual isolated black holes. Adding initial gravitational wave content from the post-Newtonian approximation should reduce the low frequency portion of junk radiation. Further adding tidal effects by matching black hole perturbation theory results to the post-Newtonian approximation should reduce the high frequency portion of the junk.

Gallouin *et al.* [6] already published the leading order contributions to nonprecessing binaries, but to my knowledge no studies evolved such initial data. My plan is to gradually implement their results starting with the tidal perturbations. This aligns with my expertise and expected to make computations cheaper, because trying to resolve high frequency junk is resource intensive. My plan is summarized by the following points:

- First, implementing only tidal perturbations for non-precessing binaries. Using the results of [6] this is doable in a year and would show if the benefits observed for non-spinning binaries translate to spinning ones.
- Second, working out the details for arbitrary spin alignment. The only complication in this step is to separate the factors that would be affected by rotating the black hole spin from the factors that are invariant. This step is conceptually simple and should not take more than a few months unless some implementation detail bogs me down.
- The harmonic coordinates used in [6] proved to be very efficient in slow spinning cases, however simulating faster spinning black holes requires the use of spherical Kerr-Schild coordinates. This

necessitates working out the matching between the post-Newtonian portion and the tidally perturbed black hole. The method is available, but the application looks tedious. Apart from redoing the calculations I need to reimplement all the perturbations in spherical Kerr-Schild coordinates, which makes this step time consuming, but more or less straightforward.

- Finally, adding the wave content in both coordinates, treating separately the near region and the asymptotic region, could be broken down into multiple points, but these steps are likely not being relevant until late 2025.

Additionally to my experience with SpEC, and specifically its initial data solver, I plan to continue working in the Simulating eXtreme Spacetimes collaboration and tap into their various expertise and knowledge.

Summary

Most of my work is related to using numerical methods to study certain aspects of gravity. My future plans, apart from concluding ongoing research, consist of some lower risk but valuable additions to the evolutionary interpretation of the constraints. As a member of the Simulating eXtreme Spacetimes collaboration I also aim to contribute to waveform generating efforts. The first manifestation of this effort is the project aiming to improve initial data quality. This project is riskier, but the first results within a year should show if the observed benefits translate to the spinning black hole case.

References

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