VISCOSITY AND THE MONOPOLE DENSITY OF THE UNIVERSE

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The influence of viscosity is investigated on the monopole density of the Universe. The result is that if the bulk viscosity coefficient in the GUT continuum after symmetry breaking exceeds a critical value, then viscosity can drive an exponential expansion diluting monopoles below the observational limit. This critical value of the viscosity coefficient means a lower bound foi the energy scale paiameter of the GUT, somewhere above 10^{15} GeV. Nevertheless, since the behaviour of the GUT continuum near the phase transition is not yet reliably known, the exact value of this lower bound cannot be precisely calculated.

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1. *Introduction*

Although GUT-type theories are very promising from several viewpoints as e.g. unification of interactions or explanations for the baryon-antibaryon asymmetry of the present Universe, they do make a prediction which definitely cannot be correct: the estimated monopole density in the present Universe tends to be too high. While astronomical observations seem to indicate the existence of some non-luminous matter with density up to 30 times that of the observed matter (Faber and Gallagher, 1979), this mass ratio cannot be as high as 10^{15}, predicted by decent and conservative monopole calculations

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(Kibble, 1982). In spite of the fact that there are some ways to diminish the predicted mass ratio by several orders of magnitude, generally there remains a serious discrepancy.

Guth has mentioned a mechanism to decrease the present monopole density (and to solve both the flatness and the horizon problems at the same time) (Guth, 1981). He argues that if the expansion of the Universe were nonadiabatic, then the present monopole/entropy ratio would be lower than in the standard models, because some part of the entropy would emerge after monopole creation. Since in this case the expansion rate would be higher, the flatness and horizon problems might also be simultaneously solved.

To carry out this idea, Guth has constructed the so-called inflationary scenario. There, first the initial phase transition ends in a "false vacuum" (a local but not global minimum of the Higgs potential). Then the Universe cools down adiabatically for a long time, and finally there is a non-equilibrium phase transition into the true vacuum, producing a great amount of entropy. He estimates that the temperature should decrease by a factor 10^{28} (i.e. until some 0.1 K) in the false vacuum to explain both the flatness and the horizon problems, but, obviously, such a tremendous supercooling is not necessary to solve the monopole problem itself; probably during the second, violent nonequilibrium phase transition the domain (and thus the monopole) structure is disarranged, and a new one emerges. Thus if the temperature after reheating is sufficiently low (there are some estimations that $T_r < 10^{11}$ GeV would be sufficient), then the new monopole density will be low enough. On the other hand, there are some arguments suggesting that supercooling probably cannot continue below 10^{11} GeV (Kibble, 1982; Hawking and Moss, 1982). Since reheating may produce a temperature T_r comparable to the phase transition temperature $T_{tr} \simeq 10^{14}-10^{15}$ GeV, the situation is not clear, because supercooling helps in solving the monopole problem only if $T_r < T_{tr}$ (Kibble, 1982).

In the so-called new inflationary scenario the transition is more continuous, so not the whole expansion is adiabatic (Linde, 1982; Albrecht and Steinhardt, 1982). Thus the necessary temperature decrease during supercooling is less prominent, nevertheless it remains quite substantial.

Independently of the mechanism producing the extra entropy, Guth's original idea was that after monopole creation the expansion should be nonadiabatic. Thus various irreversible processes may lead to extra expansion and dilution of monopoles. Viscosity is a phenomenological description of some irreversible processes. In the presence of viscosity the expansion is nonadiabatic from the beginning and the local stable thermodynamic equilibrium can be maintained. So it is worthwhile to find out if viscosity can help to solve the monopole problem, as it can remove the initial singularity for the suitable values of the bulk viscosity coefficient (Heller et al., 1973; Murphy, 1973; Heller and Suszycki, 1974; Lukács, 1976; Lukács, 1981), or can substitute the C-field of the steady-state cosmology (Hoyle, 1958) without nonvanishing divergences of the energy-momentum tensor (Heller et al., 1973).

The inclusion of viscosity is a more or less natural idea in the study of this problem, because the sources of the bulk viscosity are expected to be present in the continuum just after the phase transition, when the leptoquarks acquire masses approximately equal to

T_{tr}, which is ideal for the bulk viscosity of point particles. Because there is only one energy scale in the theory, nothing is obviously negligible at T_{tr}. However, the question is twofold. What is the necessary viscosity for diluting monopoles below the observational limit? Can the GUT continuum just below T_{tr} produce this viscosity?

In principle both questions can be answered and the result is a limit for the energy scale parameter of the specific GUT, since these theories contain only one arbitrary parameter (which is bound from below because the proton is stable in observations). Nevertheless, the fact that all the predictions of a specific GUT are functions of a single parameter does not mean that in the present state of understanding the theory one can in fact evaluate these predictions. So, while accepting a near-equilibrium (or linear) transport formalism we can answer the first question, for the necessary scale parameter we can give only an estimation, which seems to be an overestimation.

2. *The hydrodynamic description*

We are interested in Universe solutions, so here the Robertson–Walker metrics of six parameter spatial symmetry

$$ds^2 = dt^2 - R^2(t)\,(dx^2 + f^2(x)d\Omega^2)$$

$$k = +1 \qquad 0 \qquad -1$$

$$f(x) = \sin x \qquad x \qquad \mathrm{sh}\, x \tag{2.1}$$

are used. (The six symmetries correspond to the observed isotropy + homogeneity.) Since for early stages of evolution the k terms are generally negligible, in this paper we restrict ourselves to the subcase $k = 0$.

If a timelike unit vector field u^i is defined, then the energy-momentum tensor T_{ik} of the matter can be decomposed as

$$T_{ik} = \varrho u_i u_k + q_i u_k + q_k u_i - P(g_{ik} - u_i u_k) + \tau_{ik},$$

$$q^r u_r = \tau^{ir} u_r = \tau^r{}_r = 0. \tag{2.2}$$

For an observer of velocity u^i, ϱ is the energy density, q_i is the energy flux, while the remaining terms represent the stresses. If a timelike unit vector field is uniquely defined by the motion of the matter elements, this can be chosen as u^i.

Requiring the same symmetries for the matter fields as for the metric, one gets

$$u^i = (u^0, 0, 0, 0), \qquad q_i = 0, \qquad \tau_{ik} = 0 \tag{2.3}$$

and the nonvanishing terms depend only on t.

Imposing the Einstein equation on the metric, two ordinary differential equations are obtained:

$$\ddot{R} = -\frac{4\pi}{3m_P^2}(\varrho + 3P)R, \qquad \dot{R}^2 = \frac{8\pi}{3m_P^2}\varrho R^2, \tag{2.4}$$

where $m_P = 1.22 \cdot 10^{19}$ GeV is the Planck mass. (Throughout this paper $\hbar = c = 1$.)

Even without specifying the behaviour of the matter it can be seen that an accelerating expansion is possible only if

$$\varrho + 3P < 0, \tag{2.5}$$

while an exponential expansion is equivalent with

$$\varrho + P = 0. \tag{2.6}$$

Such relations are unusual for familiar types of matter. Generally the energy density is not negative, while the positivity of P can be proven for thin gases (Ehlers, 1971). Nevertheless, there are cases when $P < 0$. If in this case P is purely the thermodynamic pressure, then this state is hydrodynamically not stable against droplet formation, so it is regarded as metastable (Landau and Lifshic, 1976; Danielewicz, 1979). The thermodynamic pressure is negative for some overheated fluids (Landau and Lifshic, 1976), and there is a tendency for negative pressure in the QCD continuum at low temperatures and densities, because of the negative contribution of the "bag constant". Similarly, the pressure is negative in a cold nuclear matter below normal nuclear density. A discussion of the dynamics of such states is given by Danielewicz (1979).

The GUT continuum at the symmetry breaking phase transition is a quite exotic state of matter, and our present knowledge about this state is limited (to some extent). At most, general physical principles can be imposed on the continuum, as e.g. the energy positivity conditions. The weak energy condition requires that

$$T_{rs}v^r v^s \geqslant 0, \quad \text{if} \quad v_r v^r = 1, \tag{2.7}$$

while the dominant energy condition contains the additive restriction

$$T_{ru}v^u T^{rs} v_s \geqslant 0. \tag{2.8}$$

The physical meaning of these conditions is immediate: the first requires that the energy density of the continuum be nonnegative for any possible observer, while the second means that the energy density dominates the energy flux (Hawking and Ellis, 1973). The so-called strong energy condition is not a direct energy condition but rather some restriction for the Ricci tensor, and there are physical examples violating it (Bekenstein, 1975), so we do not discuss this condition here. The dominant energy condition is something summarizing our common sense knowledge about energy-momentum tensors, and there is no reasonable counterexample to it. Consequently, we will require Conds. (2.7–8), although they, as principles, cannot be generally proven.

For an energy-momentum tensor of form (2.2–3) Conds. (2.7–8) give

$$\varrho \geqslant 0, \quad \varrho + P \geqslant 0 \tag{2.9}$$

(in fact, Cond. (2.8) is automatically fulfilled). So inflationary Universes do not trespass the dominant energy condition. In different inflationary scenarios some mechanisms have been shown leading approximately to Eq. (2.6). Thus one can conclude that $\varrho + P \simeq 0$ is not incompatible with the GUT continuum. In the present state of knowledge it is more decent to stop here.

For a given system ϱ and P are functions of known form of some characteristic data of the system. Here we want to use the transport approximation of irreversible thermodynamics, and, specifically, to investigate the role of viscosity.

There are some calculated viscous model Universes in the literature (see e.g. Heller et al., 1973; Murphy, 1973; Heller and Suszycki, 1974; Weinberg, 1972; Lukács, 1976). The original motivation was mainly to avoid the initial singularity, or to produce the observed entropy/baryon ratio. In fact, for $k = 0$ it has been possible to get solutions without any singularity either for ξ = const., or for $\xi \sim n^{1/3}$, or for $\xi \sim \varrho$, where ξ stands for the bulk viscosity coefficient. For ξ = const. Heller et al., (1973) noted that the evolution equations are identical with those of Hoyle's steady-state cosmology, and it was possible to obtain one true steady-state Universe (with constant local data and exponential expansion, see Sol. BIII in their paper). This fact shows the close relationship of viscous models with the inflationary scenario.

Viscosity is a phenomenological description of some irreversible processes, thus the viscosity coefficients should be calculated from these processes. If the deformation velocities are not too great, one can stop at the linear terms in the stresses, and then there are two viscosity coefficients (Maugin, 1973; Heller et al., 1973):

$$P = p - \xi u^r{}_{;r},$$
$$\tau_{ik} = \eta(u_{r;s} + u_{s;r} - \tfrac{2}{3}\, g_{rs} u^t{}_{;t}) h_i^r h_k^s,$$
$$h_{ik} = g_{ik} - u_i u_k. \tag{2.10}$$

In this linear approximation ϱ, p, ξ and η depend on the local thermodynamical quantities, p is the thermodynamic pressure, while η and ξ stand for the coefficients of the shear and bulk viscosities, respectively. Of course, it is difficult to decide if this linear approximation is sufficient or not in the early Universe, in order to decide it a well established second order approximation would be needed.

From the full spatial symmetry $\tau_{ik} = 0$, so only the bulk viscosity works in Robertson-Walker Universes; clearly this is the case when the influence of irreversibilities is minimal. The contracted Bianchi identity leads to an energy balance equation

$$\varrho_{,r} u^r + (\varrho + p - \xi u^r{}_{;r}) u^s{}_{;s} = 0. \tag{2.11}$$

Since ϱ, p and ξ depend only on the local thermodynamical data, the Second Law of Thermodynamics requires that

$$\varrho_s(s_{,r} u^r + s u^r{}_{;r}) = \xi (u^r{}_{;r})^2, \quad \operatorname{sgn} \varrho_s = \operatorname{sgn} \xi, \tag{2.12}$$

otherwise the entropy production would not be positive semidefinite. With $\varrho_s = T \geqslant 0$, $\xi \geqslant 0$ is necessary.

Using Eq. (2.12), Eq. (2.11) can be recognized as the differential form of the First Law (Heller et al., 1973)

$$dE = TdS - pdV. \tag{2.13}$$

Thus there is a spontaneous (local) expansion if $p > 0$, i.e. the matter remains hydrodynamically stable even for $P < 0$ if $p > 0$.

Eqs. (2.10), (2.12) show that for expansion viscosity imitates a negative pressure, so the expansion becomes more expressed. Thus viscosity may play both roles necessary in explaining the scarcity of monopoles: the expansion becomes more pronounced, and extra entropy is generated. Nevertheless, it has to be investigated if the irreversibility can be sufficiently strong.

Consider now Eqs. (2.4), governing the expansion of the Universe. With the energy--momentum tensor (2.2–3), (2.10) they get the form (Heller et al., 1973):

$$\ddot{R} = -\frac{4\pi}{3m_P^2}\left(\varrho+3p-9\xi\frac{\dot{R}}{R}\right)R, \qquad \dot{R}^2 = \frac{8\pi}{3m_P^2}\varrho R^2. \tag{2.14}$$

It is possible to eliminate the second derivative of R, obtaining a balance equation for the entropy. In order to see it, assume first that there is no conserved particle number in the system (as it is the case in a photon-Universe or in a radiation-dominated model). Then the only independent intensive is the temperature, and the characteristic quantities fulfil the relations

$$p = p(T), \qquad \varrho = Tp_T - p, \qquad s = p_T. \tag{2.15}$$

Combining Eqs. (2.14) and (2.15) one then gets:

$$(sR^3)^{\bullet} = 9\frac{\xi}{T}\dot{R}^2 R. \tag{2.16}$$

There we have assumed that there is only one phase in the system. During a phase transition the situation is more complicated, and the result depends on the equilibrium nature of the transition (Lukács, 1983; Csernai and Lukács, 1983).

If there are some particles in the system obeying balance equations, then the thermodynamical relations are:

$$p = \sum_r n_r f_{n_r}(n_i, T) - f, \qquad \varrho = f - Tf_T, \qquad s = -f_T, \tag{2.17}$$

where f is the free energy density. If there are no source terms for the particles, then

$$(n_i R^3)^{\bullet} = 0 \tag{2.18}$$

and then Eq. (2.16) again holds. With source terms there may be some extra entropy production (Biró, Barz et al., 1983).

Having fixed the form of the proper thermodynamic potential $p(T)$ or $f(n_i, T)$, and the viscosity coefficient ξ, Eqs. (2.14), (2.15) and (2.18) completely determine the thermal history of the Universe.

3. *The condition for exponential expansion*

Assume that there is no conserved particle in the system, which seems plausible in the early hot Universe. Then the only local thermodynamic characteristic quantity is T. Now the equations governing the system are the second of (2.14), and (2.15–17), together with

$p = p(T)$ and $\xi = \xi(T)$. R and $\dot{R}$ can be eliminated from the dynamical equations, when an equation is obtained purely for the temperature:

$$p_{TT}\dot{T} = \frac{24\pi}{m_P^2 T}(Tp_T - p)\left[\xi - \sqrt{\frac{1}{24\pi}}\, m_P T \frac{p_T}{\sqrt{Tp_T - p}}\right]. \tag{3.1}$$

If the bracketed term vanishes, then $\dot{T} = 0$, and we have arrived at a steady-state Universe (Heller et al., 1973). This steady-state Universe is necessarily exponentially expanding since both k and the cosmological constant is assumed to be 0 (cf. Eq. (2.4)). The condition for exponential expansion is then, in more familiar terms

$$\xi = \sqrt{\frac{1}{24\pi}}\, m_P \frac{\varrho + p}{\sqrt{\varrho}}. \tag{3.2}$$

Both sides are functions of T alone. If this equation has roots, then the Universe shows steady-state behaviour.

Now let us assume that there exists a root. Then, if both sides are continuous, two possibilities exist. If the left hand side dominates at lower temperatures, then the cooling Universe reaches the steady state at some T_0, and remains there. This definitely is not the situation in the present Universe. In the opposite case, when ξ dominates for temperatures higher than T_0, the Universe cannot cool. However, at a first order phase transition neither the left nor the right hand sides are continuous. If the jumps are so arranged that above T_{tr} the right hand side dominates, while after the transition Eq. (3.2) approximately holds, but the difference in the bracketed term of Eq. (3.1) is a small negative quantity, then the Universe cools down until T_{tr}. There the temperature remains almost constant for a long time, with an almost exponential expansion, but finally the Universe escapes the steady state. This case is not incompatible with any obvious fact. Of course, one should see if the thermodynamical quantities have the necessary behaviour.

4. *The investigated scenario*

In GUTs the self-interactions of the Higgs fields lead to spontaneous symmetry breaking. Generally the symmetry is unbroken at high temperatures, and there only the Higgs bosons are massive, while at some $T_{tr} \simeq 10^{14} - 10^{15}$ GeV the system undergoes a phase transition, the Higgs fields acquire some nonvanishing expectation values $\langle\phi\rangle \sim T_{tr}$, and some of the particles obtain masses of order T_{tr}. The numerical factors connecting m_H, T_{tr} and m_i to $\langle\phi\rangle$ are model-dependent and practically unknown.

Our task would be to evaluate both sides of the relation (3.2) at T_{tr}. Nevertheless, it would be premature to seriously try this. The left hand side needs the analysis of momentum transfer processes in a nonlinear quantum system of many degrees of freedom at the phase transition point. The right hand side seems to be simpler, containing only equilibrium data. Nevertheless, even the correct value of the phase transition temperature is not yet known. The calculations extrapolate the high temperature expansion of the equation of state, and obviously there must be a structural change at T_{tr}. Now, the only known example when both sides of a first order phase transition can be calculated by means of the same approach is the phenomenological van der Waals equation of state. Thus one simply cannot expect that the high temperature expansion is correct for calculating T_{tr}.

But then $p(T_{tr})$ and $p_T(T_{tr})$ are even less known. Thus one can conclude that only cautious order of magnitude estimates can be given at present for evaluating Eq. (3.2).

Now, for ξ, note that its value strongly depends both on the temperature and on the internal degrees of freedom of the particles. There is an essential bulk viscosity (comparable to the shear one) if other than translational degrees of freedom are excited (Kohler, 1948; Waldmann, 1958). For point particles, the bulk viscosity vanishes both in the classical and in the ultrarelativistic limit of a thin gas (Lifshic and Pitaevskii, 1979), and starts quadratically with the number density when using the rigid sphere model (Reed and Gubbins, 1973). Nevertheless, there is a bulk viscosity even for a thin gas of point particles when $m \simeq T$. Its value is model-dependent, some numerical factors are sensitive to the dependence of the differential cross section on the angles and momenta. Neglecting these factors, in the limiting cases the result is (Stewart, 1973):

$$\xi = \begin{cases} T^{5/2} m^{-3/2} \sigma^{-1} & \text{if} \quad T \ll m, \\ T^{-1} m^4 \sigma^{-1} & \text{if} \quad T \gg m, \end{cases} \tag{4.1}$$

where σ is the cross section. Continuing both curves until the crossing point, where the maximum of ξ is expected, one gets

$$\xi = T/\sigma \quad \text{if} \quad m = T. \tag{4.2}$$

Now consider the leptoquarks. Their rest masses are circa T_{tr} after the phase transition (and 0 before), so they get the maximum viscosity at T_{tr}. For the cross section one can use the approximation $\sigma \sim \alpha^2/T^2$ (Kibble, 1982), wnere $\alpha \simeq 1/45$, so

$$\xi_{lg}(T_{tr}) \simeq \frac{1}{\alpha^2} T_{tr}^{\,3}, \tag{4.3}$$

wnile decreasing as $T^{9/2}$ for low temperatures. Since this bulk viscosity comes from simple kinetic considerations, it seems to be a lower limit for ξ. Of course, Eq. (4.3) is only an order of magnitude estimate for the bulk viscosity coefficient, because the numerical factor between m_{lq} and T_{tr} is not known.

For the right hand side of Eq. (3.2) the radiation-dominated limit yields a rough estimate. Then (Guth, 1981)

$$p = \frac{\pi^2}{90} N T^4, \tag{4.4}$$

where N is the number of particle degrees of freedom, about 160 in the simplest GUT. Then the physical picture is similar to that considered by Heller and Suszycki (1974) and by Lukcás (1981), but the viscosity coefficient is different. In this limit the right hand side of Eq. (3.2) is $\sqrt{N\pi/405}\, m_P T^2$. Using this value, and Eq. (4.3) for ξ, the result is that Eq. (3.2) approximately holds if $T_{tr} \simeq 6 \cdot 10^{15}$ GeV.

This seems to be a rather high value for T_{tr}. Nevertheless, first, this was only an order of magnitude estimate Second, we estimated the bulk viscosity from the simplest momentum transfer process; such a viscosity must exist (Stewart, 1973), but there may be other sources of viscosity too. Third, it can be seen that the radiation-dominated approximation cannot be valid at the phase transition, and it very probably yields an upper bound for the right hand side of Eq. (3.2). Namely, if Eq. (4.4) were valid then there would not be any phase transition. Generally in first order transitions ϱ jumps in such a way that it is greater for the high temperature phase. Since the right hand side of Eq. (3.2) contains only $\sqrt{\varrho}$ in the denominator, one can expect that the whole expression has such a jump that it is smaller in the asymmetric phase. Combining this with the fact that even the high temperature expansion tends to deviate from the radiation-dominated values in $(p+\varrho)/\sqrt{\varrho}$ to lower values, because of the negative sign of the T^2 term in p, it seems, indeed, that the right hand side of Eq. (3.2) is smaller than the estimated $\sqrt{N\pi/405}\, m_P T^2$. However, today it is not possible to calculate its correct value.

5. *Conclusion*

The conclusion is the Scottish verdict. Namely, the viscosity itself can generate an exponential expansion, and so dilute the monopoles, if

a) matter can carry the sufficient negative spatial stresses; for this there are some positive indications, because such stresses are not unfamiliar in GUT continua, but the answer is model-dependent; and if

b) Eq. (3.2) is approximately valid at T_{tr}, while the right hand side becomes dominating at low temperatures.

Obviously Cond. b) is an equation for T_{tr}, but the correct evaluation is not yet possible. Using the simplest approximations $T_{tr} \simeq 6.10^{15}$ GeV would seem necessary, which is rather high. However, this value becomes lower if ξ is higher than estimated for point particles via Eq. (4.3) or if $(p+\varrho)/\sqrt{\varrho}$ is lower than in the radiation-dominated limit. While there are good arguments that $(p+\varrho)/\sqrt{\varrho}$ is indeed lower because of the phase transition, the decrease cannot be calculated at present.

Similarly, one can imagine various mechanisms for increasing the bulk viscosity. For example, ξ can increase if there are objects of finite size and internal structure in the system, provided that their cross section is not too great. Obviously, the continuum does contain complex structures just below T_{tr}. Similarly, the bulk viscosity is higher if the leptoquark component is dense in the sense that $nv \simeq 1$, where n is a characteristic particle number density and v is a characteristic specific volume (Reed and Gubbins, 1973). Now, this is definitely not the case for an ordinary Bose gas, because then, estimating v as $\sigma^{3/2}$, nv is approximately $N^*\alpha^3 \ll 1$ (where N^* stands for the leptoquark degrees of freedom). Nevertheless, the leptoquarks could be densely packed if some condensation occurred. Finally, for technical reasons we have completely ignored the roles of the Higgs sector and the internal forces in the momentum transfer.

Thus, while it has not been proven that the viscosity itself could realize the inflationary scenario, and thus solve the monopole problem, counterevidence has not been found either, and the question obviously would need some futher investigations by more advanced methods partially not yet existing.

In any case, one can see that a not too fine a tuning would be necessary between T_{tr} and m_P. Even if we cannot calculate the necessary value for T_{tr}, Eq. (3.2) has to hold with a precision of 5% or better in order to maintain the almost isothermal expansion for a time sufficient for 10^5 linear expansion, reducing the monopole density to an unobservable level, while 1% tuning is necessary for the 10^{28} expansion solving the horizon and flatness problems (Guth, 1981). Although such a tuning is not too fine, and cannot be excluded even by probability considerations, in GUT type theories m_H and m_P are unrelated quantities. In supergravity such a tuning may be a consequence, but today one cannot say anything more about this.

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