On the KNO Affinity of a Certain Class of Multiplicity Distribution Functions.

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Summary. – We prove that KNO-scaled solutions derived from a recently proposed multiplicity distribution function ansatz and from two KNO-type momentum constraints do not change (at least locally), if we use only one momentum constraint.

In a recent paper (1), Krasznovszky and Wagner introduced a successful modification of the semi-fenomenological method proposed by Novero and Predazzi (2) and they computed quite realistic hadronic production cross-sections $\sigma_n(s)$. Here we make a comment on the necessity of the assumptions in ref. (1).

Since the discovery of KNO-scaling (3), we believe that the energy-dependent multiplicity distribution function $p_n(S)$ must obey the set of constraints

$$\langle n^k\rangle_s = c_k\langle n\rangle_s^k\,, \tag{1}$$

for $k = 1, 2, 3, \ldots$ and for any high enough value of the energy s.

From ref. (1), it follows that adopting the ansatz

$$p_n(s) \sim e(n)\exp[-\varphi(s)n^2] \tag{2}$$

and requiring that two KNO constraints (1) for $k = 2$ and $k = 3$ be satisfied, one can derive the function

$$e(n) \sim n^{2A-1}\exp[\gamma n^2]\,, \qquad A > 0\,, \tag{3}$$

where A and γ are constants. The resulting distribution function $p_n(s)$ will then satisfy

(1) S. Krasznovszky and I. Wagner: *Nuovo Cimento A*, **76**, 539 (1983).
(2) C. Novero and E. Predazzi: *Nuovo Cimento A*, **63**, 129 (1981).
(3) Z. Koba, H. B. Nielsen and P. Olesen: *Nucl. Phys. B*, **40**, 317 (1972).

the KNO constraints (1) for all positive integer values of k. As we are concerned with $p_n(s)$ and not with $\sigma_n(s)$, we can omit the constraint $\langle n\rangle_s = \alpha\sigma(s)$ used in ref. [1,2].

Here we show that with only one KNO-type constraint satisfied, the set of solutions for $p_n(s)$ will not change, at least locally.

Let us take the $k = 2$ constraint from (1) and introduce the notations $t = n^2$ and $E(t) = e(n)$. Since eq. (1), $E(t)$ must obey the following integral equation for $k = 2$ (we replace summation over n by integration over $t^{\frac{1}{2}}$:

$$\text{(4)}\qquad \int_0^\infty t^{\frac{1}{2}} E(t)\exp[-\varphi(s)t]\,\mathrm{d}t\int_0^\infty t^{-\frac{1}{2}}E(t)\exp[-\varphi(s)t]\,\mathrm{d}t - c_2\left[\int_0^\infty E(t)\exp[-\varphi(s)t]\,\mathrm{d}t\right]^2 = 0\,.$$

Trying to generalize the family of solutions (3), we introduce one more continuous parameter B, in the following manner:

$$\text{(5)}\qquad E(t) = t^{A-\frac{1}{2}}\exp[\gamma t]\exp[B\,\varepsilon(t, B)]\,.$$

Here $\varepsilon(t, B)$ is an arbitrary function.

Applying (4, 5) at $B = 0$, we get the relation

$$\text{(6)}\qquad \Gamma(A+1)\Gamma(A) - c_2[\Gamma(A+\tfrac{1}{2})]^2 = 0$$

between the constant c_2 and the parameter A [1]. Differentiating the l.h.s. of eq. (4) by B at $B = 0$ and substituting t by $t/\lambda(s)$, where $\lambda(s) = \varphi(s) - \gamma$, one gets the equation for ε

$$\text{(7)}\qquad \int_0^\infty \mathrm{d}t\exp[-t]\,\varepsilon(\lambda(s)t, 0)\,[t^A\Gamma(A) + t^{A-1}\Gamma(A+1) - 2c_2t^{A-\frac{1}{2}}\Gamma(A+\tfrac{1}{2})] = 0\,.$$

We try to find the function ε in the form of its Taylor series:

$$\text{(8)}\qquad \varepsilon(t, 0) = \sum_{r=0}^{\infty} a_r t^r\,.$$

Substituting (8) into eq. (7) and considering that this latter has to be satisfied for different positive values of λ, we get an infinite number of algebraic equations for the a_r's:

$$\text{(9)}\qquad a_r G_r = 0\,, \qquad r = 0, 1, \ldots;$$

where

$$\text{(10)}\qquad G_r = \Gamma(A+r+1)\Gamma(A) + \Gamma(A+r)\Gamma(A+1) - 2c_2\Gamma(A+r+\tfrac{1}{2})\Gamma(A+\tfrac{1}{2})\,.$$

Using (6), we can eliminate c_2 and write G_r in the following form:

$$\text{(11)}\qquad G_r = \Gamma(A+1)\Gamma(A)\left[\prod_{i=1}^{r}(A_i+\tfrac{1}{2}) + \prod_{i=1}^{r}(A_i-\tfrac{1}{2}) - 2\prod_{i=1}^{r}A_i\right]$$

with $A_i = A - \frac{1}{2} + i > \frac{1}{2}$.

It is easy to see that all the G_r's are definitely positive with the exception of G_0 and G_1 which vanish. Therefore, combining eqs. (8) and (9), we can write the most general form of the function $\varepsilon(t, 0)$ as

$$\varepsilon(t, 0) = a_0 + a_1 t \tag{12}$$

and the general solution (5) for $E(t)$ up to terms of the order of B^2 will be

$$E(t) = t^{A-1} \exp[\gamma t] \exp[a_0 + a_1 t] \sim t^{A-1} \exp[\gamma' t] , \qquad k' = \gamma + a_1 . \tag{13}$$

One can see that with the continuous parameter B moving out from zero, the family of solutions (3) remains constant.

Conclusion. The number of KNO-type constraints (1) for ansatz (2) can be reduced to one without changing— at least in local sense—the KNO-invariant family of solutions for $p_n(s)$ derived in [1]. In other words, the family of solutions cannot be extended continuously, *i.e.* no relevant continuous parameter can be introduced in addition to the parameters A and $\varphi(s)$ in the multiplicity distribution formulae (2), (3).