SYMMETRY BREAKING GUT PHASE TRANSITIONS WITH IRREVERSIBILITIES

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For some values of the GUT energy scale parameter, irreversibilities essentially modify the cosmological expansion and may drive inflation even without supercooling. Since in this case the GUT phase transition is affected too, a proper dynamical treatment of the evolution of the early universe is given.

Grand unified theories (GUTs) are very successful in unifying all the known elementary interactions; however, when applying them to the very early universe, they tend to give some disturbing predictions. One of them is the dominance of monopoles produced in the symmetry breaking GUT phase transition; calculating the monopole/entropy ratio in the usual way, it is definitely not below 10^{-8} [1]. This ratio may decrease to 10^{-10} by subsequent pair annihilation [2], but thereafter it remains constant in the standard model. In view of the high monopole mass, such a ratio is definitely incompatible with astronomical facts [3]. Evidently, this contradiction could be solved by a sufficiently strong entropy producing process occurring after the monopole creation [4]. Various candidate mechanisms have been proposed for realizing this idea, as, e.g. the old and new inflation [4–6]. The common feature of these models is some delay in GUT phase transition, leading to an inequilibrium transition, which obviously can generate extra entropy [7]. Unfortunately, in order to obtain long enough delay one has to accept specially adjusted Higgs self-interaction potentials [8]. However, entropy production can occur without supercooling too, e.g. via irreversible momentum transfer processes, which exist in any real continuum [9]. In fact, an estimation indicates that after the phase transition this process in itself may be sufficient to produce the required entropy if the scale parameter μ of the spontaneous symmetry breaking is as high as several times 10^{15} GeV [10].

Some near-equilibrium irreversibilities are treatable even for two phases, and if the momentum transfer is sufficiently strong, then at the last stage of the transition the irreversibilities become essential, but detailed calculations for the proper dynamics of a first order phase transition have been performed only without taking the momentum transfer into account [11]. Therefore here we are going to extend these studies to include irreversibilities too.

Of course, the proper dynamical treatment of such a complex system as the GUT continuum at the phase transition is possible only with some technical simplifications. The details of the model can be found in ref. [11]. It contains a single Higgs field with a general quartic potential, and with self-consistently evaluated one-loop thermal corrections [12]; the contributions of the other particle degrees of freedom being calculated only in the high and low temperature limits. The quartic potential contains four essential parameters, one of them is the scale parameter μ, the others are dimensionless constants:

$$V_0(\phi) = C - \tfrac{1}{2}\mu^2 \mathrm{Tr}\,\phi^2 + \tfrac{1}{3}\epsilon\mu\,\mathrm{Tr}\,\phi^3 + \tfrac{1}{4}[\lambda_1(\mathrm{Tr}\,\phi^2)^2 + \lambda_2\,\mathrm{Tr}\,\varphi^4]\ . \qquad (1)$$

In this approximation the model equations of state (for the symmetric and the deeper asymmetric states, respectively) take the forms [11]

$$p_{\mathrm{sym}}(T) = -V_0(0) + \tfrac{5}{24} T^2\mu^2 + [\tfrac{142}{90}\pi^2 - \tfrac{5}{288}(7\lambda_1 + 3\lambda_2)]\, T^4 ,$$

$$p_{\mathrm{asym}}(T) = \tfrac{113}{90}\pi^2 T^4 , \qquad (2)$$

for the pressures, and for the energy density

$$\rho = T p_T - p . \qquad (3)$$

Here we are interested mainly in the dependence of the behaviour of the model universe on the GUT scale parameter μ, thus we accept some reasonable values for the numerical constants ϵ, λ_1 and λ_2. Since ϵ is a measure of the first order character of the phase transition, we take two values for it, 0.1 and 1.0, corresponding to a weakly and an essentially first order transition, respectively. It seems as if the λ values were to have less direct dynamical consequences. Thus we choose such λ values that the self-consistency of the model be maximal. According to ref. [11], this seems to happen near $\lambda_1 = \lambda_2 = \frac{1}{9}$, in conformity with some independent expectations that λ would be in the order of g^4 [13,14], where $g^2 = 4\pi\alpha \simeq 0.28$ is the GUT coupling constant [1].

For the universe we accept the usual Robertson–Walker metric [15]

$$\mathrm{d}s^2 = \mathrm{d}t^2 - R^2(t)[\mathrm{d}r^2 + S^2(r)(\mathrm{d}\vartheta^2 + \sin^2\vartheta\, \mathrm{d}\varphi^2)] , \qquad (4)$$

where $k = +1, 0, -1$, for $S(r) = \sin r, r, \operatorname{sh} r$, respectively, for the symmetry SO(4), E(3), SO(3, 1), respectively. $R(t)$ is the scale factor of the geometry. The dynamics of the evolution is governed by the Einstein equations, whose only non-trivial components are now

$$\dot{R}^2 = -k + (8\pi/3M^2)\rho R^2, \quad \ddot{R} = -(4\pi/3M^2)(\rho + 3P)R , \qquad (5)$$

with such units that $\hbar = c = 1$; $M = 1.22 \times 10^{19}$ GeV is the Planck mass. P stands for the spatial stresses, while ρ is the energy density. This form of eqs. (5) is unique if the space curvature constant k of eq. (4) is fixed by the specific symmetry.

Because of the possible irreversible processes in the continuum, P is generally not equal to the thermodynamic pressure p. In a homogeneous, isotropically expanding continuum it can be written as [9]

$$P = p - \xi u^r{}_{;r} + (\text{higher terms in } u^r{}_{;r}) , \qquad (6)$$

where u^i is the four-velocity. This correction leads to an entropy production proportional to ξ [10]:

$$(sR^3)^{\cdot} = 9(\xi/T)\dot{R}^2 R . \qquad (7)$$

According to ref. [9], in a gas of point particles the coefficient ξ takes the value (up to numerical constants depending on the details of the differential cross section)

$$\xi = \alpha^{-2} m^3 f(T/m) ,$$

$$f(z) = z^{9/2} \quad \text{if } z \ll 1 , \qquad = z^{-1} \quad \text{if } z \gg 1 , \qquad (8)$$

where T is the temperature. One can see that the correction to p is maximal somewhere at $T \simeq m$; in the GUT continuum there are particles with mass of the order of the phase transition temperature, e.g. the X bosons for which, after the symmetry breaking [16],

$$m_X = (\tfrac{25}{2}\pi\alpha)^{1/2} [\mu/(30\lambda_1 + 7\lambda_2)] \times \{\epsilon + [\epsilon^2 + 4(30\lambda_1 + 7\lambda_2)]^{1/2}\} . \qquad (9)$$

Now, substituting the right-hand side of eqs. (5) from eqs. (2), (3), (6) and (8), (9), one obtains two equations for the two unknown quantities R and T.

Nevertheless, during a first-order phase transition the temperature is constant, while the matter is a mixture of phase domains. If these domains are too large, the large scale homogeneity breaks down, and line element (4) is no more valid. In such cases the usual cosmological equations (5) are not applicable [11]. If the domains are small enough, then – because of the long range character of gravity – the energy–momentum tensor is to be substituted by its volume average [7],

$$P = xP_1 + (1 - x)P_2 , \quad \rho = x\rho_1 + (1 - x)\rho_2 , \qquad (10)$$

where x is the volume ratio of the first phase.

The temperature of the phase transition, T_{tr}, is determined by the Gibbs condition

$$p_{\mathrm{sym}}(T_{\mathrm{tr}}) = p_{\mathrm{asym}}(T_{\mathrm{tr}}) . \qquad (11)$$

Therefore the temperature is constant during the transition, the two variables governed by eqs. (5) are R and x there.

Then the evolution of our model universe consists of three subsequent stages: $T > T_{\mathrm{tr}}$, $T = T_{\mathrm{tr}}$ and $T < T_{\mathrm{tr}}$, respectively [11]. Since $m_X = 0$ in the symmetric phase, there $\xi = 0$, the equations of state are given by eqs. (2). Although analytic solutions cannot be given for the second and third stages, it is convenient to use again the dimensionless variables of ref. [11]:

$$\tau = (\mu^2/M)t, \quad y = T^2/\mu^2 . \tag{12}$$

Then all but the irreversible terms (containing ξ) show scaling with $\beta = \mu/M$. Therefore the entropy production is a function of β.

For early stages of the evolution $k = 0$ can be used [1,4,15]. Then there is a scaling in R, so the specific initial condition for R is immaterial. Choosing a convenient initial value for T the equations of motion can be numerically integrated. For $\beta = \mu/M$ we took various values starting from 4.0×10^{-5} suggested by the low energy coupling constants [17]. The main results are displayed in figs. 1–3.

Fig. 1 shows the monopole/entropy ratio at the final step of the calculation (at $T = 0.1 T_{tr}$) as a function of β, for the two chosen values of ϵ. The initial monopole density just after the transition has been taken as the endproduct of the annihilation [2] calculated in the analytic adiabatic model of the evolution [11]. One can observe a critical value of β (somewhere just above 6.0504×10^{-4} when $\epsilon = 0.1$ and 5.655×10^{-4} when $\epsilon = 1.0$), at which $n_m/s \to 0$ because of the strong entropy production just after the transition.

Fig. 2 is R/R_0 versus τ for various combinations of ϵ and β. Near the critical value of β an almost exponential inflation can be seen.

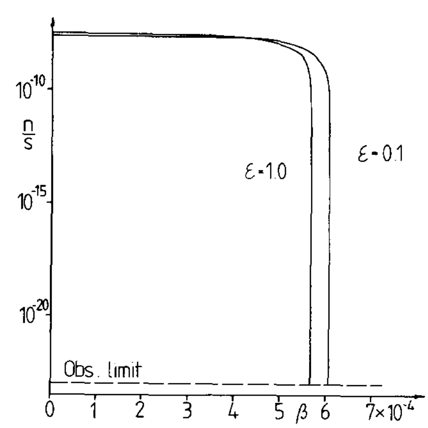

Fig. 1. The monopole/entropy ratio, n_m/s, at $T = 0.1 T_{tr}$ for two different values of ϵ as a function of $\beta = \mu/M$.

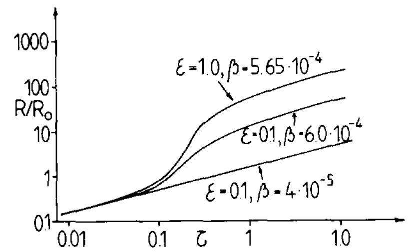

Fig. 2. The cosmological scale factor R (in arbitrary units) versus the dimensionless time $\tau = \mu^2 t/M$, for three characteristic combinations of the parameters ϵ and β.

The thermal history is displayed on fig. 3. Near β_{cr} a plateau can be observed just after the phase transition; this plateau is in direct connection with the inflation [10].

The curves near β_{cr} can be regarded as realizations of the idea formulated in ref. [10]; for some special values of the symmetry breaking scale parameter μ the irreversibilities (represented here by the momentum transfer) can drive an almost exponential inflation even without any supercooling; if $\beta < \beta_{cr}$, after some time the universe automatically escapes from the inflation. Some fine tuning is needed for a final ratio $n_m/s \sim 10^{-23}$ [1], the necessary tuning was estimated to be $\approx 5\%$ [10]. The critical value of μ is very near to our order of magnitude prediction in ref. [10], note

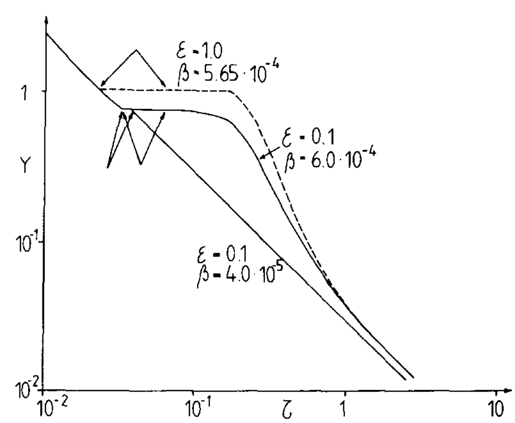

Fig. 3. Dimensionless temperature square $y = T^2/\mu^2$ as a function of the dimensionless time τ for three combinations of ϵ and β. The arrows indicate the beginning and end of the phase transitions. Observe the plateau after the transition for high values of β, corresponding to an isothermal inflation.

also the numerical coincidence with ref. [18].

Thus one may conclude that this result seems to be promising to partially or fully eliminate the monopole dominance. Nevertheless, two major problems still remain.

(a) The present calculation is merely a linear approximation in the irreversibilities [cf. the neglected terms in eq. (6)], it may become insufficient near β_{cr} where the irreversibilities are strong; and

(b) the critical value of μ, where the irreversibilities are strong enough is in the range $(6-7) \times 10^{15}$ GeV, which seems to be uncomfortably high from a particle physicist's viewpoint.

The final version of this text was written in the absence of one of the authors (B.K.); therefore she is not responsible for the formulation of the paper.

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