Models for universal reduction of macroscopic quantum fluctuations

L. Diósi
Central Research Institute for Physics, H-1525 Budapest 114, P.O. Box 49, Hungary
(Received 17 October 1988; revised manuscript received 21 March 1989)

This paper adopts the hypothesis that the absence of macroscopic quantum fluctuations is due to a certain universal mechanism. Such a mechanism has recently been proposed by Ghirardi *et al.* [Phys. Rev. D **34**, 470 (1986)], and here we recapitulate a compact version of it. Károlyházy [Nuovo Cimento **52**, 390 (1966)] showed earlier the possible role of gravity and, along this line, we construct here a new parameter-free unification of micro- and macrodynamics. We apply gravitational measures for reducing macroscopic quantum fluctuations of the mass density. This model leads to classical trajectories in the macroscopic limit of translational motion. For massive objects, unwanted macroscopic superpositions of quantum states become destroyed in very short times. The relation between state-vector and density-operator formalisms has also been discussed. We only anticipate the need for elaborating characteristic predictions of the model in the region separating micro- and macroscopic properties.

I. INTRODUCTION

According to widespread views, quantum mechanics (QM) is in perfect agreement with all definite physics experiments. None of them forces us either to correct or to complete the theory. On the other hand, it has been recognized since the earliest time of understanding QM[1] that it contradicts our general macroscopic world view. Unfortunately, strict and detailed quantum-mechanical calculations cannot be performed for most macrosystems in question due to their complexity. Consequently, one cannot decide uniquely whether the formal contradictions with macroscopic experiences indicate relevant physical contradictions, or are merely illusory.

The literature on the above paradoxical situation with QM is extensive and here we mention only the most prominent classical issues, such as Schrödinger's cat paradox,[2] the Einstein-Podolsky-Rosen paradox,[3] and the measurement problem.[4,5] They are all related to each other and one hopes that an adequate solution to any of them will be applicable to the other ones as well. For the sake of definiteness, here we single out a specific aspect and expose the problem as follows: If quantum mechanics is universal, then macroscopic bodies would, in principle, possess macroscopic quantum fluctuations (MQF) in their positions, orientations, densities, etc.; such MQF are not seen in nature, however.

We classify the suggested solutions into two classes reflecting alternative explanations of the absence of MQF: either it follows purely from the inevitable interactions of macroscopic objects with their actual surrounding (class A) or, alternatively, the lack of MQF is universal and is due to a certain new mechanism which modifies the ordinary QM (class B).

Viewpoint A has been emphasized by Zeh.[6] Later, Wigner[7] also acknowledged it and estimated the effect of cosmic background radiation on simple macroscopic bodies.[8] Quite recently Joos and Zeh[9] discussed the quantum dynamics of dust particles in rare gaseous environments. All these calculations suggest that explanation A could probably work, but, in our opinion, other specific mechanisms involved by viewpoint B still cannot be excluded. Actually we need definite models in order to decide.

In favor of the hypothesis B, which this paper supports, we mention the most recent proposal[10] by Ghirardi, Rimini, and Weber, universally reducing MQF of position coordinates (see also Refs. 11–13). This theory will be paraphrased in Sec. III as *quantum mechanics with universal position localization* (QMUPL).

The other basic ingredient of our paper originates from an earlier model[14–16] by Károlyházy *et al.* (and especially from its variant[17,18] stressed by Diósi and Lukács), where universal gravitational fluctuations have been derived and claimed to suppress MQF of the matter density. In Sec. IV, a *quantum mechanics with universal density localizations* (QMUDL) is suggested. This new consistent modification of QM resembles the construction of the QMUPL in Sec. III but the position localizations are replaced by density localization processes whose strength is proportional to the gravitational constant. This choice makes QMUDL a *parameter-free unification of micro- and macrodynamics.*

On the technical part, one needs a certain stochastic modification of the ordinary Hamiltonian quantum dynamics. Such formalisms have been considered, e.g., in a great variety of continuous state reduction[19–23] and continuous measurement[24–27] models. Throughout our paper we apply the successful theory of *stochastic differential equations* (SDE's) which offers a compact and flexible language[27,28] to formulate our equations. The general theory of SDE's can be learned from Arnold's excellent book.[29] An introduction to this technique will be given automatically in Sec. II.

II. PHENOMENOLOGY OF MACROSCOPIC QUANTUM MECHANICS

A. Quantum-stochastic equations

Consider, for its simplicity, the center-of-mass motion of a certain free object of macroscopic mass M. The quantum state Ψ_Q satisfies the Schrödinger equation

$$\frac{d}{dt}\Psi_Q = -i(\hat{\mathbf{P}}^2/2M)\Psi_Q \ . \tag{2.1}$$

$\hat{\mathbf{P}}$ stands for the canonical momentum. Whatever viewpoint one prefers concerning the paradoxes of macroscopic QM, the simple Schrödinger equation must be modified due either to environmental interactions or to a certain new physical effect (cf. viewpoints *A* and *B*, respectively, in Sec. I) or to both. Anyhow, there exists a simple phenomenological modification[24–26] of Eq. (2.1) which we are going to recapitulate.

In order to reflect the reduction of MQF of the center of mass $\hat{\mathbf{Q}}$, one assumes that the quantum state Ψ_Q is subjected to instantaneous localization processes

$$\Psi_Q \to \exp[-\tfrac{1}{2}\alpha(\hat{\mathbf{Q}}-\overline{\mathbf{Q}})^2]\Psi_Q \ , \tag{2.2}$$

which occur with average frequency λ. Here α is the accuracy parameter of the localization process. In each localization process (2.2) the coordinate $\overline{\mathbf{Q}}$ is a random variable of the following probability distribution:

$$P(\overline{\mathbf{Q}}) = \text{const} \times \langle \exp[-\alpha(\hat{\mathbf{Q}}-\overline{\mathbf{Q}})^2]\rangle \ , \tag{2.3}$$

where $\langle\ \rangle$ stands for the quantum expectation value in the actual state Ψ_Q, while $\langle\ \rangle_{st}$ will be used later to denote the stochastic average according to the distribution (2.3) of $\overline{\mathbf{Q}}$. Because of the randomness of the localization processes (2.2), the quantum state Ψ_Q will be governed by a certain stochastic evolution equation instead of the deterministic Schrödinger equation (2.1). Let us now take the *continuous localization* limit

$$\alpha \to 0, \quad \lambda\alpha \equiv \Gamma \tag{2.4}$$

where Γ is the *strength* of the continuous localization and is kept constant.

In Ref. 27 we have proved that the construction (2.1)–(2.4) leads to an infinite-dimensional Gaussian process. For such processes, the proper mathematical technique seems to be the (quantum) stochastic differential equations[29] rather than the functional variant of the traditional Fokker-Planck-Kolmogorov equations. In our case, one can make use of a powerful theorem.

Theorem: Equations (2.1)–(2.4) are equivalent to the following SDE:

$$d\Psi_Q = \{[-i(\hat{\mathbf{P}}^2/2M) - \tfrac{1}{4}\Gamma(\hat{\mathbf{Q}}-\langle\hat{\mathbf{Q}}\rangle)^2]dt + (\hat{\mathbf{Q}}-\langle\hat{\mathbf{Q}}\rangle)d\xi\}\Psi_Q \ , \tag{2.5}$$

where ξ is a vectorial Wiener process whose Itô differential obeys the following multiplication rules:

$$\langle d\xi\rangle_{st} = 0 \ , \tag{2.6a}$$

$$d\xi \circ d\xi = \tfrac{1}{2}\Gamma \mathbb{1} dt \ , \tag{2.6b}$$

and higher than second powers of $d\xi$ vanish.

For many purposes it is more convenient to introduce the pure state projector $\hat{\rho}_Q \equiv \Psi_Q\Psi_Q^\dagger$ instead of Ψ_Q and to rewrite the SDE (2.5) in the following equivalent form:

$$d\hat{\rho}_Q = L[\hat{\rho}_Q]dt + \{\hat{\mathbf{Q}}-\langle\hat{\mathbf{Q}}\rangle, \hat{\rho}_Q\}d\xi \ , \tag{2.7}$$

$$L[\cdots] \equiv -i[\hat{\mathbf{P}}^2/2M, \cdots] - \tfrac{1}{4}\Gamma[\hat{\mathbf{Q}},[\hat{\mathbf{Q}},\cdots]] \ . \tag{2.8}$$

The equivalence between the SDE's (2.5) and (2.7) can be seen by inserting Eq. (2.5) into the identity $d\hat{\rho} \equiv d\Psi\Psi^\dagger + \Psi d\Psi^\dagger + d\Psi d\Psi^\dagger$ and by applying the rules (2.6). Recall that the product of two Itô differentials does not vanish but is always proportional to dt [cf. Eq. (2.6b)].

Notice that the SDE's (2.5) and (2.7) are both nonlinear. Nevertheless, the drift term of the SDE (2.7) has turned out to be linear, since the Liouville operator L (2.8) is obviously linear. Consequently, by taking the stochastic average of both sides of the SDE (2.7) and by considering Eq. (2.6a), one obtains a closed linear *master (or Liouville) equation for the density operator* $\langle\hat{\rho}_Q\rangle_{st}$ of the center-of-mass motion:

$$\frac{d}{dt}\langle\hat{\rho}_Q\rangle_{st} = L[\langle\hat{\rho}_Q\rangle_{st}] \ . \tag{2.9}$$

Using the notation $\rho(\mathbf{Q},\mathbf{Q}')$ for the coordinate representation of the density operator $\langle\hat{\rho}\rangle_{st}$, Eq. (2.9) takes the form

$$\frac{d}{dt}\rho(\mathbf{Q},\mathbf{Q}') = [\dot{\rho}(\mathbf{Q},\mathbf{Q}')]_{QM} - \tfrac{1}{4}\Gamma(\mathbf{Q}-\mathbf{Q}')^2\rho(\mathbf{Q},\mathbf{Q}') \ , \tag{2.10}$$

where $[\dot{\rho}(\mathbf{Q},\mathbf{Q}')]_{QM}$ stands for the contribution of the ordinary QM.

The existence of the linear evolution equation for the density operator is not accidental but a consequence of fundamental principles, as shown by Gisin[21,22] and Diósi.[23] In the present case the fulfillment of this constraint relies on the special choice (2.3) made for the probabilities of the various outcomes of localization processes (2.2).

B. Reduction of MQF

The first term on the right-hand side (RHS) of the SDE (2.5) represents the ordinary quantum-mechanical evolution with a certain characteristic time τ_{QM}. The second term violates QM; it definitely reduces the width $\Delta Q \equiv [\langle(\hat{\mathbf{Q}}-\langle\hat{\mathbf{Q}}\rangle)^2\rangle]^{1/2}$ of the wave function. The characteristic time scale τ_L on which the localization of ΔQ becomes effective can be estimated by a simple formula:

$$\tau_L \approx \Gamma^{-1}(\Delta Q)^{-2} \ . \tag{2.11}$$

If τ_L is much larger than τ_{QM}, then violation of QM will not be observed; it becomes effective when $\tau_L \ll \tau_{QM}$, which, via Eq. (2.11), means

$$\Delta Q \gg 1/\sqrt{\Gamma\tau_{QM}} \ . \tag{2.12}$$

One may notice that τ_{QM} also depends on ΔQ. Nevertheless, the above inequality implies a certain lower threshold (σ_∞, see later) for ΔQ. The strength Γ of continuous localization must be adjusted in such a way that the above threshold corresponds to a plausible condition for MQF of the position $\hat{\mathbf{Q}}$. Then, as expected, our phenomenological SDE (2.5) pushes down the unwanted MQF until the threshold (2.12) is reached and a stationary quantum fluctuation $\Delta Q \approx \sigma_\infty$ of the position $\hat{\mathbf{Q}}$ may be maintained. Exact results regarding this latter regime will be presented in Sec. II C.

C. Trajectories

In Ref. 28 we considered the solution of the SDE (2.5). We proved that, for large times, the wave function converges to the unique Gaussian shape

$$\text{const}\times\exp[-(1-i)(\mathbf{Q}/2\sigma_\infty)^2]$$

and that the motion of this wave packet will imitate classical trajectories.

The quantum fluctuations of $\hat{\mathbf{Q}}$ and $\hat{\mathbf{P}}$ tend to their stationary values

$$(\Delta Q)^2 \to 3\sigma_\infty^2\ , \tag{2.13a}$$

$$(\Delta P)^2 \to \tfrac{3}{2}\hbar^2/\sigma_\infty^2\ , \tag{2.13b}$$

with

$$\sigma_\infty^2 \equiv \sqrt{\hbar/2\Gamma M}\ , \tag{2.14}$$

where $\hbar$ has been restored. The quantum expectation values of the canonical coordinates satisfy the following stochastic differential equations:

$$d\langle\hat{\mathbf{P}}\rangle = \hbar d\xi\ , \tag{2.15a}$$

$$d\langle\hat{\mathbf{Q}}\rangle = (1/M)\langle\hat{\mathbf{P}}\rangle dt + 2\sigma_\infty^2 d\xi\ . \tag{2.15b}$$

Equations (2.15) show that the inertial motion of the wave packet will be modified by an *anomalous Brownian motion.* It can be shown (cf. Ref. 10) that the stochastic spreads of the trajectory coordinates increase with time as follows:

$$(\Delta Q)_{\text{st}}^2 = 3\hbar t/M + (3/2\sigma_\infty^2)(\hbar t/M)^2 + (1/4\sigma_\infty^4)(\hbar t/M)^3\ , \tag{2.16a}$$

$$(\Delta P)_{\text{st}}^2 = (3/2)\hbar^2\Gamma t\ . \tag{2.16b}$$

The full-squared uncertainties of the canonical coordinates $\hat{\mathbf{Q}},\hat{\mathbf{P}}$ are equal to the sum of the squared quantum [(2.13a) and (2.13b)] and stochastic [(2.16a) and (2.16b)] fluctuations, respectively. For example,

$$\begin{aligned}(\Delta Q)_{\text{full}}^2 &= (\Delta Q)^2 + (\Delta Q)_{\text{st}}^2 \\ &= 3\sigma_\infty^2[1+(t/t_0)+\tfrac{1}{2}(t/t_0)^2+(\tfrac{1}{12})(t/t_0)^3]\ ,\end{aligned} \tag{2.17}$$

where

$$t_0 = (2\Gamma\hbar/M)^{-1/2}\ . \tag{2.18}$$

If the strength of continuous localization Γ is large enough, then the stationary quantum fluctuation σ_∞ (2.14) of the position $\hat{\mathbf{Q}}$ becomes microscopic and, consequently, the MQF of the center-of-mass position $\hat{\mathbf{Q}}$ will be destroyed. Also, the classical trajectories will be fairly mimicked because the anomalous Brownian motion [see Eqs. (2.17) and (2.18)] becomes moderate enough or just unobservable. One has to add that, according to Károlyházy *et al.*, the tiny effect of the anomalous Brownian motion might possibly be observed in some sophisticated experiments.[15,16]

III. QUANTUM MECHANICS WITH UNIVERSAL POSITION LOCALIZATION

A. Construction of QMUPL

At this stage we possess a simple phenomenological model of translational motion of macroscopic objects (see Sec. II), reflecting the absence of MQF. According to hypothesis *B* of Sec. I, this model must take its origin in a certain universal (not environmental) mechanism.

Let Ψ denote now the quantum state of a closed macroscopic system composed on *N constituents* with corresponding Cartesian coordinates $\hat{\mathbf{q}}_n$ $(n=1,2,\ldots,N)$. It is crucial to emphasize that our treatment is nonrelativistic. We can then take electrons and nuclei as constituents, at least for ordinary macroscopic objects. Photons and bounded subnuclear particles in general must not be considered for nonrelativistic constituents. (Recall Ref. 10 where constituents have also been specified in a rather implicit way.)

In ordinary QM, Ψ satisfies the Schrödinger equation

$$\frac{d}{dt}\Psi = -i\hat{H}\Psi\ , \tag{3.1}$$

with the Hamiltonian $\hat{H}$ of the closed macroscopic system. Let us assume that the localization process (2.2) and (2.3) is to act *not* on the system as a whole but on each constituent separately:

$$\Psi \to \exp\left[-\tfrac{1}{2}\alpha\sum_n(\hat{\mathbf{q}}_n-\bar{\mathbf{q}}_n)^2\right]\Psi\ , \tag{3.2}$$

which differs slightly[13] from the original proposal of Ghirardi, Rimini, and Weber.[10] Consequently, the joint probability distribution of the random coordinates $\bar{\mathbf{q}}_n$ must be chosen as

$$P(\bar{\mathbf{q}}_1,\bar{\mathbf{q}}_2,\ldots,\bar{\mathbf{q}}_N) = \text{const}\times\left\langle\exp\left[-\alpha\sum_n(\hat{\mathbf{q}}_n-\bar{\mathbf{q}}_n)^2\right]\right\rangle\ . \tag{3.3}$$

We also take the continuous localization limit

$$\alpha\to 0,\quad \lambda\alpha\equiv\gamma\approx 10^{-7}\ \text{cm}^{-2}\,\text{s}^{-1}\ , \tag{3.4}$$

where we have applied the proposal of Ref. 10 for estimating the strength γ of the continuous localization of the constituents.

Equations (3.1)–(3.4) define the modified QM, which we call quantum mechanics with universal position localization. From these equations follows, by straightforward generalization of the *theorem* yielding the phenomenological SDE (2.5), the basic SDE of QMUPL:

$$d\Psi = \left[\left[-i\hat{H} - \tfrac{1}{4}\gamma \sum_n (\hat{\mathbf{q}}_n - \langle \hat{\mathbf{q}}_n \rangle)^2 \right] dt + \sum_n (\hat{\mathbf{q}}_n - \langle \hat{\mathbf{q}}_n \rangle) d\xi_n \right] \Psi \ . \tag{3.5}$$

Here N-independent vectorial Wiener processes $d\xi_n$ $(n=1,2,\ldots,N)$ have been introduced:

$$\langle d\xi_n \rangle_{\rm st} = 0, \quad n=1,2,\ldots,N \tag{3.6a}$$

$$d\xi_n \circ d\xi_m = \tfrac{1}{2}\gamma \mathbf{1}\delta_{nm} dt, \quad n,m=1,2,\ldots,N \tag{3.6b}$$

and higher than second-order products of $d\xi_n$ vanish. If one introduces the projector $\hat{\rho} \equiv \Psi\Psi^\dagger$, the SDE (3.5) can be rewritten in terms of $\hat{\rho}$ as follows:

$$d\hat{\rho} = L[\hat{\rho}]dt + \sum_n \{\hat{\mathbf{q}}_n - \langle \hat{\mathbf{q}}_n \rangle, \hat{\rho}\} d\xi_n \ , \tag{3.7}$$

where we have defined the linear Liouville operator by

$$L[\cdots] \equiv -i[\hat{H}, \cdots] - \tfrac{1}{4}\gamma \sum_n [\hat{\mathbf{q}}_n, [\hat{\mathbf{q}}_n, \cdots]] \ . \tag{3.8}$$

We note that the master equation of the form (2.9) is valid here, too. We can write it in coordinate representation as follows:

$$\frac{d}{dt}\rho(\mathbf{q},\mathbf{q}') = [\dot{\rho}(\mathbf{q},\mathbf{q}')]_{\rm QM} - \tfrac{1}{4}\gamma \sum_n (\mathbf{q}_n - \mathbf{q}'_n)^2 \rho(\mathbf{q},\mathbf{q}') \ , \tag{3.9}$$

where $\rho(\mathbf{q},\mathbf{q}')$ stands for the coordinate representation of the density operator $\langle \hat{\rho} \rangle_{\rm st}$.

Equations (3.5) [or (3.7) and (3.8)] together with (3.6) represent a consistent theory unifying micro- and macrodynamics. Our QMUPL (see also Ref. 13) is identical to the theory of Ghirardi, Rimini, and Weber in Ref. 10, apart from the continuous limit (3.4) which thereby was not taken.

B. Macroscopic phenomenology from QMUPL

QMUPL works in the following way. The extreme small strength γ (3.4) of constituent localizations assures that QMUPL reduces to the ordinary QM in case of microsystems. For example, if one considers a single microparticle whose wave-function width ΔQ is about 1 cm, the violation of the ordinary QM will enter only on time scales of the order $\tau_L \approx \gamma^{-1}$ cm$^{-2} \approx 10^7$ s [cf. Eq. (2.12)]. Controlling the above wave function for a year is quite illusory and one will not see any violation of QM. In the case of a macrosystem, however, the number N of constituents is of the order of 10^{23} and, as shown in Ref. 10 or by (3.11), the center-of-mass coordinate $\hat{\mathbf{Q}}$ will endure a strong localization.

The form (3.7)–(3.8) of QMUPL equations is very suitable to demonstrate the above effect. Introduce the relative-to-c.m.s coordinates $\mathbf{r}_n$ and substitute the expressions $\mathbf{q}_n = \mathbf{Q} + \mathbf{r}_n$ $(n=1,2,\ldots,N)$ into the SDE (3.7). Assume, furthermore, that the quantum state $\hat{\rho}$ has the separable form $\hat{\rho} = \hat{\rho}_Q \otimes \hat{\rho}_r$, where $\hat{\rho}_Q$ and $\hat{\rho}_r$ are pure states concerning the center of mass and the relative motions, respectively. Take then the partial trace over the state space of relative motion, on both sides of Eqs. (3.7), when one gets

$$d\hat{\rho}_Q = -i[\hat{\mathbf{P}}^2/2M, \hat{\rho}_Q] - \tfrac{1}{4}N\gamma[\hat{\mathbf{Q}},[\hat{\mathbf{Q}},\hat{\rho}_Q]] + \{\hat{\mathbf{Q}} - \langle \hat{\mathbf{Q}} \rangle, \hat{\rho}_Q\} d\xi \ , \tag{3.10}$$

where $\xi = \sum_{n=1}^N \xi_n$ and $\hat{\mathbf{P}}$ stands for the momentum conjugate to $\hat{\mathbf{Q}}$.

We have thus deduced the phenomenological SDE (2.7) and (2.8) of macroscopic objects from the universal SDE (3.7) and (3.8) of QMUPL. The macroscopic localization strength Γ has turned out to be N times larger than the localization strength γ of constituents. Invoking the estimation (3.4) of γ, one obtains the following order of magnitude for Γ (in cgs units):

$$\Gamma = N\gamma \approx 10^{17} M \ , \tag{3.11}$$

where we have taken $\approx 10^{24}$ constituents (electrons + nuclei) per unit mass.

In Sec. IV we shall point out that the above value of Γ is able to assure the proper asymptotics of the phenomenological SDE (2.7) and (2.8): for microsystems QM restores, and, opposingly, macroscopic objects move along classical trajectories without observable quantum or stochastic spreads. In Ref. 13 we have pointed out that QMUPL predicts classical behavior also for rotational motion of solids.

C. Remarks on QMUPL

One should inevitably notice certain *ad hoc* features of QMUPL, such as, e.g., the distinguishing role of the constituent coordinates. The parametrization of the theory seems to be practical but is elusive, as emphasized by Bell.[12] We hope to alleviate these troubles in the model that we will present in Sec. IV.

IV. QUANTUM MECHANICS WITH UNIVERSAL DENSITY LOCALIZATION

A. Construction of QMUDL

In this section we are going to repeat the construction (3.1)–(3.4) of QMUPL presented in Sec. III with one change. The continuously localized quantity will *not* be the constituent position but the mass distribution $\hat{f}(\mathbf{r})$ of the given system. This permits the localization effect to be parametrized by the Newton constant G.

The model does not work for pointlike mass distributions. In order to give an extension to constituents, one is compelled to rely on some approximations. For concreteness, we will consider the constituents as rigid spheres. Hence, the mass-density operator takes the form $\hat{f}(\mathbf{r}) = \sum_n^N f_n(\mathbf{r} - \hat{\mathbf{q}}_n)$, where f_n stands for the mass distribution of the nth constituent. The width of f_n may be chosen for about $r \approx 10^{-13}$ cm. (The classical electron radius and the typical nuclear size are both of that order of magnitude.) Nevertheless, such details of the mass-distribution operator $\hat{f}(\mathbf{r})$ turn out to be irrelevant in several important applications, as we shall see later.

For the completeness of this section, let us recall the Schrödinger equation of our system in ordinary QM:

$$\frac{d}{dt}\Psi=-i\hat{H}\Psi \ . \tag{4.1}$$

Similarly to the QMUPL model, the above deterministic evolution will be interrupted by instantaneous localization processes occurring with frequency λ:

$$\Psi \rightarrow \exp(-\tfrac{1}{2}\alpha\|\hat{f}-\bar{f}\|_G^2)\Psi \ , \tag{4.2a}$$

where $\hat{f}(\mathbf{r})$ is the Schrödinger operator of the mass density at point $\mathbf{r}$. The G norm of a given function $f(\mathbf{r})$ is defined by introducing the bilinear form U:

$$\|f\|_G^2=-U(f,f) \ , \tag{4.2b}$$

where

$$U(f_1,f_2)\equiv -G\int\int f_1(\mathbf{r}_1)f_2(\mathbf{r}_2)(1/r_{12})d\mathbf{r}_1 d\mathbf{r}_2 \ , \tag{4.2c}$$

i.e., so that U is formally equal to the gravitational interaction potential of the mass distributions f_1 and f_2. Obviously, the quadratic form U in Eq. (4.2b) is negative definite, as it must be.

The function $\bar{f}(\mathbf{r})$ in the Eq. (4.2a) is a random variable with the distribution functional

$$P[\bar{f}]=\text{const}\times\langle \exp(-\alpha\|\hat{f}-\bar{f}\|_G^2)\rangle \ . \tag{4.3}$$

Let us require the continuous localization limit

$$\alpha\rightarrow 0, \quad \lambda\alpha\equiv\kappa \ , \tag{4.4}$$

where κ is a certain number. Fortunately, it is dimensionless and we shall assume its order is of unity.

The construction of *quantum mechanics with universal density localization* has thus become completed. Now we anticipate the compact mathematical form of QMUDL. From Eqs. (4.1)–(4.4) it follows that the quantum state satisfies the basic SDE:

$$d\Psi=\left[(-i\hat{H}-\tfrac{1}{4}\kappa\|\hat{f}-\langle\hat{f}\rangle\|_G^2)dt + \int[\hat{f}(\mathbf{r})-\langle\hat{f}(\mathbf{r})\rangle]d\xi(\mathbf{r})d\mathbf{r}\right]\Psi \ , \tag{4.5}$$

where the continuous set $\xi(\mathbf{r})$ of the scalar Wiener processes has been introduced:

$$\langle d\xi(\mathbf{r})\rangle_{\text{st}}\equiv 0 \ , \tag{4.6a}$$

$$d\xi(\mathbf{r}_1)d\xi(\mathbf{r}_2)=\tfrac{1}{2}\kappa G(1/r_{12})dt \ . \tag{4.6b}$$

Higher-order products of $d\xi$ vanish.

It is convenient to introduce the operator representation of the quantum state. If we introduce the projector $\hat{\rho}\equiv\Psi\Psi^\dagger$ and then substitute Eqs. (4.5) and (4.6) into the identity $d\hat{\rho}\equiv d\Psi\Psi^\dagger+\Psi d\Psi^\dagger+d\Psi d\Psi^\dagger$ we obtain the following SDE:

$$d\hat{\rho}=L[\hat{\rho}]dt+\int[\hat{f}(\mathbf{r})-\langle\hat{f}(\mathbf{r})\rangle,\hat{\rho}]d\xi(\mathbf{r})d\mathbf{r} \ , \tag{4.7}$$

with the linear Liouville operator

$$L[\cdots]\equiv -i[\hat{H},\cdots]-\tfrac{1}{4}\kappa U([\hat{f},[\hat{f},\cdots]]) \ . \tag{4.8}$$

The shorthand notation $U([\hat{f},[\hat{f},\cdots]])$ should be read as

$$-G\int\int[\hat{f}(\mathbf{r}_1),[\hat{f}(\mathbf{r}_2),\ldots]](1/r_{12})d\mathbf{r}_1 d\mathbf{r}_2 \ .$$

For later discussions, let us write down the master equation which governs the evolution of the density operator $\langle\hat{\rho}\rangle_{\text{st}}$. The stochastic average of both sides of the SDE (4.7) yields the master equation proposed by Diósi:[18]

$$\begin{aligned}\frac{d}{dt}\langle\hat{\rho}\rangle_{\text{st}}&=L[\langle\hat{\rho}\rangle_{\text{st}}]\\ &\equiv -i[\hat{H},\langle\hat{\rho}\rangle_{\text{st}}]\\ &\quad -\tfrac{1}{4}\kappa G\int\int[\hat{f}(\mathbf{r}_1),[\hat{f}(\mathbf{r}_2),\langle\hat{\rho}\rangle_{\text{st}}]]\\ &\quad\times(1/r_{12})d\mathbf{r}_1 d\mathbf{r}_2 \ .\end{aligned} \tag{4.9}$$

Let us verify briefly that the SDE's (4.5) or, equivalently, (4.7) follow from the construction (4.1)–(4.4) of QMUDL. To see this connection, we introduce the Fourier representation of the mass density:

$$f(\mathbf{r})=\sum_{\mathbf{k}} f_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}}\equiv\sum_{\mathbf{k}}(q_{\mathbf{k}1}+iq_{\mathbf{k}2})e^{i\mathbf{k}\cdot\mathbf{r}} \ . \tag{4.10}$$

The normalization volume has been set, by convention, to 1; $q_{\mathbf{k}1}$ and $q_{\mathbf{k}2}$ stand for the real and imaginary parts of the Fourier component $f_{\mathbf{k}}$, respectively. Using the discrete series of variables $q_{\mathbf{k}\mu}$, instead of the distribution function $f(\mathbf{r})$, Eqs. (4.2) of density localization can be rewritten as

$$\Psi\rightarrow\exp\left[-\tfrac{1}{2}\alpha\sum_{\mathbf{k},\mu}4\pi Gk^{-2}(\hat{q}_{\mathbf{k}\mu}-\bar{q}_{\mathbf{k}\mu})^2\right]\Psi \ , \tag{4.11}$$

and the random variables $\bar{q}_{\mathbf{k}\mu}$ possess the joint probability distribution

$$P(\bar{q})=\text{const}\times\left\langle\exp\left[-\alpha\sum_{\mathbf{k},\mu}4\pi Gk^{-2}(\hat{q}_{\mathbf{k}\mu}-\bar{q}_{\mathbf{k}\mu})^2\right]\right\rangle \ . \tag{4.12}$$

Equations (4.1), (4.11), (4.12), and (4.4) reformulate QMUDL in terms of the localization of the Hermitian quantities $\hat{q}_{\mathbf{k}\mu}$. By comparing these equations to Eqs. (3.1)–(3.4), one recognizes their mathematical similarity. One is led, therefore, to the SDE's (3.5) and (3.6), which now, *mutatis mutandis,* read as

$$d\Psi=\left[\left[-i\hat{H}-\tfrac{1}{4}\kappa G\sum_{\mathbf{k},\mu}4\pi k^{-2}(\hat{q}_{\mathbf{k}\mu}-\langle\hat{q}_{\mathbf{k}\mu}\rangle)^2\right]dt + \sum_{\mathbf{k},\mu}(\hat{q}_{\mathbf{k}\mu}-\langle\hat{q}_{\mathbf{k}\mu}\rangle)d\zeta_{\mathbf{k}\mu}\right]\Psi \ . \tag{4.13}$$

The scalar Wiener processes $\zeta_{\mathbf{k}\mu}$ satisfy the following Itô algebra:

$$\langle d\zeta_{\mathbf{k}\mu}\rangle_{\text{st}}\equiv 0 \ , \tag{4.14a}$$

$$d\zeta_{\mathbf{k}\mu}d\zeta_{\mathbf{k}'\mu'}=\tfrac{1}{2}\kappa G4\pi k^{-2}\delta_{\mathbf{k}\mathbf{k}'}\delta_{\mu\mu'}dt \ . \tag{4.14b}$$

Let us return now to the spatial representation. Define the scalar Wiener process $\xi(\mathbf{r})$ by its Fourier coefficients as follows:

$$\xi(\mathbf{r})=\tfrac{1}{2}\sum_{\mathbf{k}}(\zeta_{\mathbf{k}1}+i\zeta_{\mathbf{k}2})e^{i\mathbf{k}\cdot\mathbf{r}}+\text{c.c.} \tag{4.15}$$

In the SDE (4.13), as well as in Eqs. (4.14), perform the transformation (4.10) and (4.15). The results will be just identical to the SDE (4.5) and to the Itô algebra (4.6), respectively.

B. Macroscopic phenomenology from QMUDL

It is obvious from the construction of QMUDL that the violation of ordinary QM depends, first of all, on the gravitational energies represented by the given state of the system. In the case of microsystems we thus expect that the violation of QM can be neglected, while we hope for explicit classical behavior in the macroscopic properties, instead of QM.

The second RHS term of the basic SDE (4.5) of QMUDL yields an estimation for the time scale τ_L at which localization breaks QM:

$$\tau_L\approx\hbar/\langle\|\hat f-\langle\hat f\rangle\|_G^2\rangle\equiv\hbar/(\Delta_G f)^2\ , \tag{4.16}$$

where the Planck constant has been restored. It is important to observe that the *G-norm-squared density fluctuation* $(\Delta_G f)^2$ is the quantity that QMUDL cuts down, in contrast to the ordinary quantum dynamics. Formally, $(\Delta_G f)^2$ is measured by the expectation value of the difference between the gravitational interaction Hamiltonian and of its semiclassical (or mean-field) approximation, i.e.,

$$(\Delta_G f)^2\equiv\langle\|\hat f-\langle\hat f\rangle\|_G^2\rangle=\langle -U(\hat f,\hat f)+U(\hat f,\langle\hat f\rangle)\rangle \tag{4.17}$$

[cf. the definition (4.2b) and (4.2c) of the G norm].

For a system of several constituents (e.g., electrons, nuclei) of masses about $m\approx10^{-27}$–10^{-24} g and size $r\approx10^{-13}$ cm, a safe upper estimate of the RHS of Eq. (4.17) can be given by the quantity $Gm^2/r\approx10^{-49}$–10^{-43} erg. Via Eq. (4.16) we obtain $\tau_L\approx10^{16}$–10^{22} s. This huge result means that in the microscopic properties QM will not be violated at all and QMUDL therefore reduces to the ordinary QM. In particular, the microscopic degrees of freedom of a macroscopic object will obey pure QM.

We should deal with more massive objects when we hunt special QMUDL effects. Consider, e.g., a macroscopic object of size $R\approx1$ cm, of mass $M\approx1$ g, and of position spread ΔQ also about 1 cm. From Eq. (4.17), the corresponding density fluctuation $(\Delta_G f)^2$ can be estimated by G g^2/cm$\approx10^{-8}$ erg which yields, via Eq. (4.16), $\tau_L\approx10^{-19}$ s. This result shows that typical QMUDL effects may strongly modify the usual quantum dynamics of macroscopic degrees of freedom, while microscopic degrees of freedom remain unaffected.

This latter fact enables us to integrate the QMUDL equations over the microscopic degrees of freedom, which we are going to exploit in the case of rigid macroscopic objects.

We assume the state vector of a given solid in the separable form $\Psi\equiv\Psi_Q\otimes\Psi_\Theta\otimes\Psi_i$, where Ψ_Q, Ψ_Θ, and Ψ_i are state vectors concerning the center-of-mass, rotational, and internal motions, respectively. (The vector Θ parametrizes the rotation. The solid is meant to be rotated by an angle $|\Theta|$ around the axis parallel to Θ.)

As we have noticed above, QMUDL will practically not alter Ψ_i. This feature allows one to approximate the mass density operator $\hat f(\mathbf{r})$ by its partial expectation value

$$\hat f(\mathbf{r})\approx\text{tr}_i[\hat\rho_i\hat f(\mathbf{r})]\ , \tag{4.18}$$

where $\hat\rho_i\equiv\Psi_i\Psi_i^\dagger$ and tr_i stands for tracing over the internal degrees of freedom. In the approximation (4.18), the operator $\hat f(\mathbf{r})$ depends only on $\hat{\mathbf{Q}}$ and $\hat{\mathbf{\Theta}}$ but not on the internal variables. Hence, the basic QMUDL equation (4.5) splits into two separate equations for $\Psi_{Q\Theta}\equiv\Psi_Q\otimes\Psi_\Theta$ and for Ψ_i, respectively.

The first equation governs the translational and rotational motion

$$d\Psi_{Q\Theta}=\Big\{[-\tfrac{1}{2}i\hat{\mathbf{P}}^2/M-\tfrac{1}{2}i\hat{\mathbf{J}}(I_{\hat\Theta})^{-1}\hat{\mathbf{J}}-\tfrac{1}{4}\kappa\|\hat f-\langle\hat f\rangle\|_G^2]dt+\int[\hat f(\mathbf{r})-\langle\hat f(\mathbf{r})\rangle]d\xi(\mathbf{r})d\mathbf{r}\Big\}\Psi_{Q\Theta}\ , \tag{4.19}$$

where $\hat{\mathbf{J}}$ stands for the angular momentum canonically conjugated to $\hat{\mathbf{\Theta}}$, $I_{\hat\Theta}\equiv R_{\hat\Theta}I_0R_{-\hat\Theta}$ is the tensor of inertia in the c.m.s, and R_Θ denotes the 3×3 matrix of rotation by $\mathbf{\Theta}$.

The second equation, governing Ψ_i, turns out to be identical to the ordinary Schrödinger equation with a Hamiltonian of the internal degrees of freedom. Let the state Ψ_i be the ground state which is stationary; hence, the RHS of Eq. (4.18) takes a simple form

$$\hat f(\mathbf{r})\approx F(R_{\hat\Theta}(\mathbf{r}-\hat{\mathbf{Q}}))\ , \tag{4.20}$$

where $F(\mathbf{r})$ is the "gravitational" form factor (i.e., the mass density) of the ground state.

The form factor $F(\mathbf{r})$ is a definite characteristic of the given solid, calculable by means of solid-state theory. It is built up mainly from the contributions of the nuclei separated from each other by $\approx10^{-8}$ cm. In the ground state (i.e., at zero temperature) the position spread of a given nucleus inside the solid is typically about 10^{-10} cm (cf. Ref. 30). Recall that the extension of a nucleus is less by two to three orders of magnitude. The details of nuclear structure thus become irrelevant.

Now we may ask what happens to the SDE (4.19) if we wash out the microstructure of $F(\mathbf{r})$: we replace $F(\mathbf{r})$ by the macroscopic mass distribution of the solid, which is much easier to calculate. By heuristical arguments, we expect this macroscopic $F(\mathbf{r})$ to work correctly, provided microscopic structures (of scales 10^{-8} cm or less) were not present in the wave function $\Psi_{Q\Theta}$, either.

Consequently, the SDE (4.19) with the smoothed mass distribution, instead of $F(\mathbf{r})$ in (4.20), is the general QMUDL equation for the *macroscopic* translational and rotational motion of rigid objects. One expects that the above equation describes the suppression of MQF of the coordinates $\hat{\mathbf{Q}}$ and $\hat{\mathbf{\Theta}}$ so the classical motion is always restored. Since the rotational motion is more difficult to discuss,[13] we choose the translation for further study.

C. Ballistics and trajectories

Take a rigid spherical ball of radius R and of mass M, and assume it is macroscopically homogeneous. Let us approximate its form factor by that of a completely homogeneous sphere:

$$F(\mathbf{r})=(M/V_R)\Theta(R-|\mathbf{r}|)\ , \tag{4.21}$$

where $V_R=\frac{4}{3}\pi R^3$ and Θ is the step function. Thus, in approximation (4.20), we obtain

$$\hat{f}(\mathbf{r})=(M/V_R)\Theta(R-|\mathbf{r}-\hat{\mathbf{Q}}|)\ . \tag{4.22}$$

Since $\hat{f}(\mathbf{r})$ is now independent of $\hat{\mathbf{\Theta}}$, the basic SDE (4.19) yields a closed SDE for the macroscopic translation of the ball

$$d\Psi_Q=\left[[-i(\hat{\mathbf{P}}^2/2M)-\tfrac{1}{4}\kappa\|\hat{f}-\langle\hat{f}\rangle\|_G^2]dt + \int[\hat{f}(\mathbf{r})-\langle\hat{f}(\mathbf{r})\rangle]d\xi(\mathbf{r})d\mathbf{r}\right]\Psi_Q\ . \tag{4.23}$$

In what follows, we assume that the quantum uncertainty ΔQ of the position is much smaller than the size of the ball. This allows one to use the Taylor expansion of the density (4.22) around $\hat{\mathbf{Q}}=\langle\hat{\mathbf{Q}}\rangle$, yielding

$$\hat{f}(\mathbf{r})-\langle\hat{f}(\mathbf{r})\rangle =(M/V_R)\delta(R-|\mathbf{r}-\langle\hat{\mathbf{Q}}\rangle|)(\mathbf{r}-\langle\hat{\mathbf{Q}}\rangle)\cdot(\mathbf{Q}-\langle\hat{\mathbf{Q}}\rangle)/R\ , \tag{4.24}$$

where terms of the order of $(\Delta Q/R)^2$ have been neglected.

The coefficient of the specific nonlinear term of the SDE (4.23) can be approximated by

$$\|\hat{f}-\langle\hat{f}\rangle\|_G^2=(GM^2/R^3)(\hat{\mathbf{Q}}-\langle\hat{\mathbf{Q}}\rangle)^2\ , \tag{4.25}$$

which follows from Eq. (4.2b), (4.2c), and (4.24). In the same approximation, the stochastic term of the SDE (4.23) can be rewritten as well:

$$\int[\hat{f}(\mathbf{r})-\langle\hat{f}(\mathbf{r})\rangle]d\xi(\mathbf{r})d\mathbf{r}\Psi_Q=(\hat{\mathbf{Q}}-\langle\hat{\mathbf{Q}}\rangle)d\xi\Psi_Q\ , \tag{4.26}$$

where we have defined a vectorial Wiener process by

$$\xi=(M/V_R)R^{-1}\int\delta(R-|\mathbf{r}|)\mathbf{r}\xi(\mathbf{r})d\mathbf{r}\ . \tag{4.27}$$

Inserting Eqs. (4.25) and (4.26) into the SDE (4.23) one obtains the phenomenological SDE (2.5). Of course, the validity of the Itô algebra (2.6) also has to be checked by using Eqs. (4.6) and (4.27). The effective localization strength of the ball position turns out to be

$$\Gamma=(\kappa G/\hbar)M^2/R^3=(4\pi\kappa G\rho/3\hbar)M\approx 10^{19}M \tag{4.28}$$

(in cgs units), provided the ball density ρ is about 1 $\mathrm{g\,cm}^{-3}$.

Remember that the QMUPL theory also led to the effective macroscopic SDE (2.5); moreover, the order of magnitude of the QMUPL value (3.11) of Γ almost coincides numerically with Γ (4.28) in the QMUDL theory. Hence we see that, at least for the macroscopic translation of rigid (or rigid enough) macroscopic objects, our parameter-free QMUDL theory becomes similar to the QMUPL theory, including its parametrization suggested by Ghirardi *et al.*,[10] after careful phenomenological arguments.

However, the validity of the SDE (2.5) is restricted due to the approximations we applied before. For example, the width ΔQ of the wave function Ψ_Q must fulfill the conditions

$$10^{-8}\ \mathrm{cm}\ll\Delta Q\ll R\ . \tag{4.29}$$

The lower bound is necessary because the choice (4.21) is not suitable when the wave function contains microstructure. The higher bound comes from the Taylor expansion (4.24).

Let us now turn to the trajectorylike solutions (see Sec. II C) of the SDE (2.5) with localization strength (4.28). Equation (2.14) yields the stationary width of the center-of-mass wave function:

$$\sigma_\infty=(8\pi\kappa G\rho/3\hbar^2)^{-1/4}M^{-1/2}\approx 10^{-11}M^{-1/2}\ . \tag{4.30}$$

The anomalous Brownian motion, as shown by Eqs. (2.17) and (2.18) leads to the following spread of the coordinate (in cgs units);

$$(\Delta Q)_{\mathrm{full}}\approx 10^{-11}M^{-1/2}[1+(t/t_0)+\tfrac{1}{2}(t/t_0)^2 +\tfrac{1}{12}(t/t_0)^3]^{1/2}\ , \tag{4.31}$$

where $t_0=(8\pi\kappa G\rho/3)^{-1/2}\approx 10^2$–$10^3$ s. It is interesting to note that the time scale t_0 of the anomalous Brownian motion depends only on the Newton constant and on the mass density of the ball; the Planck constant has canceled.

Inserting σ_∞ (4.30) in place of ΔQ, the conditions (4.29) leads to unpleasant restrictions, i.e., $10^{-4}\ll R\ll 10^{-2}$ cm, which, unfortunately, excludes all sizes but the $R\approx 10^{-3}$ cm range. For such small balls, Eqs. (4.30) yields $\sigma_\infty\approx 10^{-7}$–$10^{-6}$ cm. The anomalous Brownian motion (4.31) would produce further statistical elongation of position (e.g., $10^{+3}\sigma_\infty$ in about a day) if the isolation of the ball were not unattainable.

It would be desirable, of course, if we were able to derive classical trajectories from QMUDL for larger balls, too. For larger balls ($R\gg 10^{-2}$ cm), Eq. (4.30) predicts σ_∞ of subatomic scale. We know that the SDE (4.23) pushes the wave-function width ΔQ down towards its stationary value σ_∞. When ΔQ has already become smaller than 10^{-8} cm, our SDE is not reliable anymore. One can, nevertheless, conclude that the stationary width of the wave function (if it exists) as well as the scales of the anomalous Brownian motion are strongly microscopic and, presumably, unobservable in practice. For a more detailed description, which is a future task, one should calculate the complete microscopic form factor $F(\mathbf{r})$ of the given ball.

D. Reduction of MQF

In this section we wish to concentrate on the mechanism reducing the MQF of the density. Therefore we will completely neglect the Hamiltonian term in the SDE (4.5); this choice will be verified later.

Let us consider a macroscopic (or, maybe, mesoscopic) system which is assumed to be in a superposition

$$\Psi=\sum_A c_A\phi_A,\quad \sum_A c_A^2=1 \tag{4.32}$$

of a certain number of normalized states ϕ_A ($A=1,2,3,\ldots$). Exploting gauge freedom, each amplitude c_A can be made real. Introducing the notation $\phi_A^\dagger\hat f\phi_A\equiv\langle\hat f\rangle_A$, we assume that the expectation values $\langle\hat f(\mathbf r)\rangle_1$, $\langle\hat f(\mathbf r)\rangle_2$, $\langle\hat f(\mathbf r)\rangle_3,\ldots$, are much different from each other as compared to the density fluctuations $\Delta_G f$ belonging to the component states ϕ_A. Consequently, the superposition Ψ (4.32) may represent MQF of the density (i.e., large $\Delta_G f$), while the fluctuations in the component states ϕ_A will be neglected. Heuristically, one is then allowed to consider the set $\{\phi_A\}$ as the orthonormal set of the approximate eigenstates of the mass density operator

$$\hat f(\mathbf r)\phi_A=\langle\hat f(\mathbf r)\rangle_A\phi_A\ , \tag{4.33a}$$

$$\phi_A^\dagger\phi_B=\delta_{AB}\ . \tag{4.33b}$$

[A rigorous formulation of the above approximation would be based on the G norm (4.2b) and (4.2c) as the measure of distances in the space of mass density functions.]

Using the approximation (4.33), the substitution of the ansatz (4.32) into the SDE (4.5) (recall $\hat H=0$) yields the closed SDE for the amplitudes c_A:

$$dc_A=\left[-\tfrac14\kappa\|\langle\hat f\rangle_A-\langle\hat f\rangle\|_G^2dt+\int[\langle\hat f(\mathbf r)\rangle_A-\langle\hat f(\mathbf r)\rangle]d\xi(\mathbf r)d\mathbf r\right]c_A\ . \tag{4.34}$$

Remember that $\langle\hat f\rangle=\sum_A c_A^2\langle\hat f\rangle_A$, provided the approximation (4.33) holds.

From the SDE (4.34) one can obtain an equivalent closed SDE introducing the normalized probability distribution p_A instead of the amplitudes c_A:

$$p_A\equiv c_A^2,\quad \sum_A p_A=1\ . \tag{4.35}$$

Let us substitute Eq. (4.34) into the Itô identity $dp_A\equiv 2c_A dc_A+(dc_A)^2$. The Itô equations (4.6) will lead to the following equations:

$$\langle dp_A\rangle_{\rm st}=0 \tag{4.36a}$$

$$dp_A dp_B=-2\kappa p_A p_B\times\sum_{R,S}p_R p_S U(\langle\hat f\rangle_A-\langle\hat f\rangle_R,\langle\hat f\rangle_B-\langle\hat f\rangle_S)dt\ , \tag{4.36}$$

where we applied the definition (4.2b) and (4.2c) of the G norm.

Such probabilistic equations, with phenomenological constant coefficients in place of $U(\langle\hat f\rangle_A-\langle\hat f\rangle_R,\langle\hat f\rangle_B-\langle\hat f\rangle_S)$, are well known from the theory of *continuous state reduction*.

As a special application, we are going to show that Eqs. (4.36) describe the reduction of *distant macroscopic superpositions*. For simplicity's sake, let the components ϕ_A of the superposition (4.32) represent the same quantum state ϕ translated into various distant positions in space. For example, ϕ_A belongs to the Ath position of a pointer of a given measuring device. If the spatial separations between different pointer positions are much larger than the thickness of the pointer, then

$$U(\langle\hat f\rangle_A,\langle\hat f\rangle_B)=\delta_{AB}U(\langle\hat f\rangle_\phi,\langle\hat f\rangle_\phi) \tag{4.37}$$

is a good approximation and Eqs. (4.36) take the form

$$\langle dp_A\rangle_{\rm st}=0\ , \tag{4.38a}$$

$$dp_A dp_B=2\kappa U(\langle\hat f\rangle_\phi,\langle\hat f\rangle_\phi)\times p_A p_B\left[\delta_{AB}-p_A-p_B+\sum_R p_R^2\right]dt\ . \tag{4.38b}$$

These two equations are mathematically equivalent to Gisin's continuous reduction model.[19,21] As can be shown, for times

$$t\gg\hbar/U(\langle\hat f\rangle_\phi,\langle\hat f\rangle_\phi)\ , \tag{4.39}$$

each probability $p_A(t)$ will approach zero, except for a single one (e.g., the Kth) which becomes unity. In continuous reduction models the probability of the Kth outcome must be equal to the initial probability $p_K(0)$. Recalling Eq. (4.35), this would mean that the superposition (4.32) will be reduced to one of the component states, e.g., to ϕ_K, with the proper quantum-mechanical probability c_K^2.

The semiclassical energy $U(\langle\hat f\rangle_\phi,\langle\hat f\rangle_\phi)$ of a typical macroscopic object (e.g., a pointer) of mass $M\approx1$ g, of size $R\approx1$ cm and of quantum uncertainty $\Delta Q\ll1$ cm, is about $GM^2/R\approx10^{-8}$ erg. The distant superposition of such states would be reduced after a period of the order of $\hbar/10^{-8}$ erg$\approx10^{-19}$ s [see Eq. (4.39)]. This period is much shorter than the time scale of any nonrelativistic quantum evolution and, consequently, the above distant macroscopic superposition could not even come into existence. Causality forbids formation of a configuration of 1 cm within 10^{-19} s. Needless to say, the neglect of the Hamiltonian motion during the reduction period has been justified as well.

For lighter objects, the reduction time becomes more realistic. If, e.g., $R\approx10^{-2}$ cm, $M\approx10^{-6}$ g, then $GM^2/R\approx10^{-18}$ erg and, consequently, the reduction time is of the order of $\hbar/10^{-18}$ erg$\approx10^{-9}$ s.

E. Quantum SDE versus master equation

It is well known in the ordinary QM that, in pure quantum states $\hat\rho\equiv\Psi\Psi^\dagger$, the measurable quantities are of the form ${\rm tr}(\hat O\hat\rho)=\Psi^\dagger\hat O\Psi=\langle\hat O\rangle$, where $\hat O$ is an arbitrary Hermitian operator. According to the notations of our paper, the density operator has been denoted by $\langle\hat\rho\rangle_{\rm st}$, where $\hat\rho$ stands for the pure-state projector $\Psi\Psi^\dagger$. The general form of a measurable quantity in a mixed quantum state takes the form ${\rm tr}(\hat O\langle\hat\rho\rangle_{\rm st})=\langle\langle\hat O\rangle\rangle_{\rm st}$. If we declare the above set of measurable quantities for QMUDL, too (it is not necessary, see later), then the state vector Ψ becomes redundant, since the density operator

$\langle \hat{\rho} \rangle_{st} \equiv \langle \Psi\Psi^\dagger \rangle_{st}$ will account for all measurable quantities. Furthermore, the density operator will satisfy the closed master (Liouville) equation (4.9). Hence the surprising result comes: the master equation (4.9) is completely equivalent to the SDE (4.5), if only measurable predictions are concerned.[23] Of course, analogous master equations exist in cases in QMUPL found in Sec. III, and of Ghirardi's theory,[10] too.

It is natural to ask why people prefer the complicated nonlinear SDE instead of the linear and deterministic master equation (cf. Joos's criticism[31] on Ghirardi's theory[10]). To demonstrate the reasons, consider the reduction process shown in Sec. IV D. In the final state, the density operator is given by $\sum_A p_A \phi_A \phi_A^\dagger$ and contains the full measurable physical information on the system. For example, the double expectation value

$$\langle \langle \hat{f}(\mathbf{r}) \rangle \rangle_{st} = \sum_A p_A \langle \hat{f}(\mathbf{r}) \rangle_A \tag{4.40}$$

is measurable, but the terms $\langle \hat{f}(\mathbf{r}) \rangle_A$ of its decomposition are not. Now recall that the mass distributions $\langle \hat{f}(\mathbf{r}) \rangle_A$ were chosen to be macroscopically different, and there may be a certain request for a formalism that automatically reflects the obvious decomposition (4.40).

Still, this request is completely subjective, since, as a matter of fact, $\langle \hat{f} \rangle_A$ is not measurable in itself. The only objective reason that may support the SDE formalism is a guess[27] that a certain (still unknown) physical mechanism will modify the set of measurable quantities and, by means of this mechanism, the terms of the decomposition (4.40) will be distinguished.

Let us present a simple example. In this paper we did not consider the back reaction of the mass density $\hat{f}(\mathbf{r})$ on the Newtonian gravitational potential $\Phi(\mathbf{r})$. For the sake of the example assume that Φ is classical and the *mean-field equation*[32] is exact. There are, however, two possibilities:

$$\Delta\Phi(\mathbf{r}) = 4\pi G \langle \langle \hat{f}(\mathbf{r}) \rangle \rangle_{st} \tag{4.41a}$$

or

$$\Delta\Phi(\mathbf{r}) = 4\pi G \langle \hat{f}(\mathbf{r}) \rangle \ . \tag{4.41b}$$

The source term of Eq. (4.41a) has been assumed to be a measurable quantity, i.e., it is calculable from the density operator. It is, however, not sensitive to the reduction process described in Sec. IV D, as we can seen from Eq. (4.40). We should therefore choose the second version (4.41b) of the mean-field equations. Since now the measurable Newton potential couples to the quantum expectation value $\langle \hat{f} \rangle$ of the mass distribution, this latter has become measurable, too. Of course, the equivalence of the master equation with the SDE disappears, since $\langle \hat{f} \rangle$ is not calculable from the density operator but from the state vector.

Of course, the basic SDE (4.5) of the QMUDL has to be added by a Hamiltonian term $\int \Phi(\mathbf{r})\hat{f}(\mathbf{r})d\mathbf{r}$ corresponding to gravitational interactions. Such a theory may be considered as the naive prototype of a "quantum field theory without observers" suggested by Bell.[33]

V. DISCUSSION

We have presented two models, QMUPL and QMUDL, both of which tend, by construction, to localize unwanted MQF. QMUPL is a slightly modified version of the theory of Ghirardi *et al.*; it contains one free physical parameter. In the case of QMUDL, a definite gravitational measure has been postulated for the reduction of the MQF of mass densities and only a single dimensionless number has to be fixed by hand. This kind of modification of the ordinary dynamics seems to eliminate certain paradoxical features of QM and a unified description of the micro- and macroscopic properties becomes possible. In particular, we have pointed out that QMUDL eliminates the "monstrous quantum states[34] of macroscopic objects" so rapidly (e.g., in 10^{-19} s) that even the formation of such states becomes practically forbidden.

Despite these *conceptual successes,* one should look for nontrivial applications of such unified theories. Consequently, one has to investigate the transient region between the quantum and the classical world. In this region the localization effects, postulated in QMUPL or QMUDL, will compete with the ordinary Hamiltonian dynamics, especially with the interaction due to the environment.

We think that, first of all, the QMUPL or QMUDL models have to be tested against supercurrent and superfluid effects. Special attention should be paid to Leggett's experiments[35] with macroscopic quantum superposition of supercurrents. Károlyházy *et al.*[15,16] have carefully discussed the possibility of observing the anomalous Brownian motion, in the context of their reduction theory, which is similar to QMUDL. Their proposal is an experiment aboard a satellite. Its adaption to QMUDL, if possible, would be desirable.

As an outlook, we notice the following possibility. At the present stage of its elaboration, QMUDL may have no characteristic experimental predictions other than marginal effects. Even though, this model suggests a certain deep connection between nonrelativistic quantum and gravity theories. If this connection exists in some way that is similar to QMUDL, then, in a more developed form, QMUDL will shed light on characteristic new effects.

After the completion of this work, two related papers have been brought to my attention. In Pearle's work[36] a certain SDE is proposed which incorporates the main points of the theory[10] of Ghirardi *et al.* This SDE contains two independent parameters α and λ. It would be interesting to see if Pearle's SDE is equivalent to our QMUPL in the limit (3.4), in some proper way. In a new paper,[37] Ghirardi *et al.* extends their theory[10] to systems of identical particles in order to preserve the symmetry or antisymmetry properties of the wave function. We note that this problem is inherently solved in our QMUDL.

ACKNOWLEDGMENTS

The author thanks Dr. A. Frenkel and, especially, Dr. B. Lukács for valuable discussions and for the careful reading of the manuscript.

[1]W. Heisenberg, Z. Phys. **33**, 879 (1925); M. Born and P. Jordan, *ibid.* **34**, 858 (1925); M. Born, W. Heisenberg, and P. Jordan, *ibid.* **35**, 557 (1926).
[2]E. Schrödinger, Naturwissenschaften **23**, 844 (1935).
[3]A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. B **47**, 777 (1935).
[4]J. von Neumann, *Matematische Grundlagen der Quantenmechanik* (Springer, Berlin, 1932).
[5]E. Wigner, J. Am. Phys. **31**, 1 (1963).
[6]H. D. Zeh, Found. Phys. **1**, 69 (1970).
[7]E. Wigner, in *Foundation of Quantum Mechanics,* edited by B. D'Espagnat (Academic, New York, 1971).
[8]E. Wigner, in *Quantum Optics, Experimental Gravity and Measurement Theory,* edited by P. Meystre and M. O. Scully (Plenum, New York, 1983).
[9]E. Joos and H. D. Zeh, Z. Phys. B **59**, 223 (1985).
[10]G. C. Ghirardi, A. Rimini, and T. Weber, Phys. Rev. D **34**, 470 (1986).
[11]F. Benatti, G. C. Ghirardi, A. Rimini, and T. Weber, Nuovo Cimento **100B**, 27 (1987).
[12]J. S. Bell, in *Schrödinger—Centenary Celebration of a Polymath,* edited by C. W. Kilmister (Cambridge University Press, Cambridge, England, 1987).
[13]L. Diósi, Europhys. Lett. **6**, 285 (1988).
[14]F. Károlyházy, Nuovo Cimento **52**, 390 (1966).
[15]F. Károlyházy, A. Frenkel, and B. Lukács, in *Physics as Natural Philosophy,* edited by A. Shimonyand H. Feschbach (MIT Press, Cambridge, MA, 1982).
[16]F. Károlyházy, A. Frenkel, and B. Lukács, in *Quantum Concepts in Space and Time,* edited by R. Penrose and C. J. Isham (Clarendon, Oxford, 1986).
[17]L. Diósi and B. Lukács, Ann. Phys. **44**, 488 (1987).
[18]L. Diósi, Phys. Lett. **120A**,377 (1987).
[19]P. Pearle, Phys. Rev. D **33**, 2240 (1986).
[20]P. Pearle, in *Quantum Concepts in Space and Time,* edited by R. Penrose and C. J. Isham (Clarendon, Oxford, 1986).
[21]N. Gisin, Phys. Rev. Lett. **52**, 1657 (1984).
[22]N. Gisin, Phys. Rev. Lett. **53**, 1776 (1984).
[23]L. Diósi, J. Phys. A: Math. Gen. **21**, 2885 (1988).
[24]A. Barchielli, L. Lanz, and G. M. Prosperi, Nuovo Cimento **B72**, 79 (1982).
[25]A. Barchielli, Phys. Rev. A **34**, 1642 (1986).
[26]C. M. Caves and G. J. Milburn, Phys. Rev. A **36**, 5543 (1987).
[27]L. Diósi, Phys. Lett. A **129**, 419 (1988).
[28]L. Diósi, Phys. Lett. A **132**, 233 (1988).
[29]L. Arnold, *Stochastic Differential Equations: Theory and Application* (Wiley, New York, 1974).
[30]C. Kittel, *Introduction to Solid State Physics* (Wiley, New York, 1976).
[31]E. Joos, Phys. Rev. D **36**, 3285 (1987).
[32]L. Diósi, Phys. Lett. **105A**, 199 (1984).
[33]J. S. Bell, Phys. Rev. **137**, 49 (1986).
[34]A. Shimony, in *Philosophical Consequences of Quantum Theory,* edited by J. Cushing and E. McMullin (Notre Dame Press, Notre Dame, in press).
[35]A. J. Leggett, Contemp. Phys. **25**, 583 (1984).
[36]P. Pearle, Phys. Rev. A **39**, 2277 (1989).
[37]G. C. Ghirardi, O. Nicrosini, A. Rimini, and T. Weber, Nuovo Cimento **102B**, 383 (1988).