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Notes on certain Newton gravity mechanisms of wavefunction localization and decoherence

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Abstract
Both an additional nonlinear term in the Schrödinger equation and an additional non-Hamiltonian term in the von Neumann equation, proposed to ensure localization and decoherence of macro-objects, resp., contain the same Newtonian interaction potential formally. We discuss certain aspects that are common for both equations. In particular, we calculate the enhancement of the proposed localization and/or decoherence effects, which would take place if one could lower the conventional length-cutoff and resolve the mass density on the interatomic scale.

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1. Introduction

Experts in the history of science may perhaps know what von Neumann's approach would be to the concept of a fully quantized universe. His measurement theory yields perfect statistical interpretation of the quantum state as long as there exists a classical—non-quantized—sector of the universe. The challenge of a fully quantized universe has been attracting many theorists even in the lack of pressing experimental evidence. Where might this evidence—or at least an indication—come from? That must be the combination of extreme high energies and extreme high gravitating mass densities. As a consequence, the mainstream concept of a quantized universe targets a quantized cosmology through the quantization of the Einstein theory of spacetime. Despite theoretical efforts through the past decades, that big step has not been done so far. Experts do not agree what the bottleneck is. It may be our concept of the quantum or our concept of the spacetime. Both, certainly. I used to emphasize one: the bottleneck is the quantum. The von Neumann theory of measurement becomes useless if the whole universe is quantized. To make a shortcut to our subject, we cite a figure from [1], with the Schrödinger equation of the universe written in the middle, see figure 1. A similar figure had earlier been published by Kuhař [2]. Our failure to interpret the universal wavefunction Ψ may not be related to relativity. The formal argument of the figure is almost categoric: one of the three partially unified theories is missing. Then, why not, the bottleneck may be the

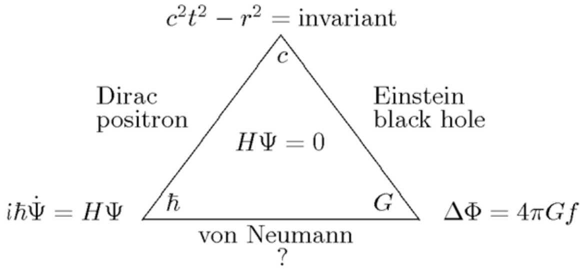

Figure 1. Scheme of Physics' Building. $c =$ velocity of light, $G =$ Newton's gravitational constant, $\hbar =$ Planck constant. The corners of the triangle represent the three fundamental theories, the sides correspond to partially unified theories while the middle symbolizes the fully unified theory.

missing unified theory for quantum mechanics ($\hbar$) and Newtonian gravity (G)—once called Newtonian quantum gravity. We may assume that the path up to a relativistic theory of a quantized universe goes through the non-relativistic theory of Newtonian quantum gravity explaining the quantized motion of common macroscopic objects. In particular, we can make a small step towards the theory of quantized universe if we establish a theory of 'spontaneous' measurement of quantized non-relativistic macro-objects.

For the past 20 years, many authors have investigated the possible role of Newtonian gravity in resolving the apparent controversy between the common classical motion of macro-objects and their quantum mechanical description [1–16]. Among various proposals, the works of Penrose and myself show a certain likeness [17]. Accordingly, the standard Schrödinger–von-Neumann equations of quantum mechanics can be modified by concrete gravitational terms of simple and transparent mathematical structure. In the present notes, these modifications will be shown and analysed using only the simplest wavefunction $\psi(X)$ and density matrix $\rho(X, Y)$ formalisms. The notes contribute to the discussion of the notorious divergence problem and I calculate a possible enhancement of the proposed gravitational effects. In the simple formalism chosen by the notes, I formulate the structural relationship between the presented gravitational decoherence and the mass-proportional decoherence used in the CSL theory of Gian-Carlo Ghirardi and his co-workers [18]. The present notes tend to support the idea that the mechanism of universal localization and decoherence is gravity related and not merely mass proportional. However, for the time being we have no ultimate evidence. The notes remain open ended, formulating some alternatives and perspectives.

2. Two mechanisms, two models, one Newtonian structure

The studies of our interest concentrated on two inter-related elements of classical behaviour of a rigid macro-object: precise centre-of-mass *localization* and *decoherence* (decay) of superposition between separate positions. To guarantee the first, the attractive Newtonian self-consistent gravitational field was introduced into the Schrödinger equation [3, 12]. To guarantee the second, a universal decay mechanism was postulated for superpositions between separate positions, scaled by the difference between the corresponding Newtonian field strengths [5, 6, 9]. In both *localization* and *decoherence* mechanisms, resp., the relevant quantity is the Newtonian interaction

$$U(X, X') = -G \int \frac{f(\mathbf{r}|X) f(\mathbf{r}'|X')}{|\mathbf{r}' - \mathbf{r}|} \mathrm{d}\mathbf{r}\, \mathrm{d}\mathbf{r}' \qquad (1)$$

between two mass densities corresponding to two configurations X, X' of the macro-objects that form our quantum system. For each configuration X, the function $f(\mathbf{r}|X)$ provides the mass density at location $\mathbf{r}$. Typically for rigid objects, configuration X contains the centre-of-mass coordinates $\mathbf{x}_1, \mathbf{x}_2, \ldots$, and the rotation angles $\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots$. For simplicity, we shall consider spherically symmetric or point-like objects, to discuss their translational degrees of freedom. Hence, X stands for $\mathbf{x}_1, \mathbf{x}_2, \ldots$, only.

With the help of the interaction potential (1), we construct the Schrödinger–Newton equation for the wavefunction $\psi(X)$ of the massive objects [3, 12]:

$$i\hbar\frac{d\psi(X)}{dt} = \text{standard q.m. terms} + \int U(X, X')|\psi(X')|^2\, dX'\, \psi(X). \tag{2}$$

The second term on the rhs leads to stationary solitary solutions. The Schrödinger-Newton equation ensures the stationary localization of the objects. Yet, the equation cannot account for the expected decoherence of macroscopic superpositions like $|X\rangle + |Y\rangle$.

An alternative, irreversible, equation serves this latter purpose. We start from the von Neumann equation which is equivalent with the standard Schrödinger equation. It evolves the density matrix $\rho(X, Y)$ rather than the wavefunction $\psi(X)$. The construction of the von-Neumann–Newton equation[1] reads [5, 6]

$$\frac{d\rho(X, Y)}{dt} = \text{standard q.m. terms} + \frac{U(X, X) + U(Y, Y) - 2U(X, Y)}{2\hbar}\rho(X, Y). \tag{3}$$

The second term on the rhs contributes to an exponential decay of the superposition $|X\rangle + |Y\rangle$, with the following decoherence time [5, 6, 9]:

$$\frac{2\hbar}{2U(X, Y) - U(X, X) - U(Y, Y)}. \tag{4}$$

This expression[2] is $+\infty$ for $X = Y$. It decreases with the distance between the two configurations X, Y so that the second term on the rhs of equation (3) suppresses the interference term $\rho(X, Y)$ for large separations between X and Y.

To avoid misunderstandings, we emphasize that the Schrödinger–Newton equation (2) and the von-Neumann–Newton equation (3) are two *alternative* equations to modify the standard quantum mechanics for macro-objects. In our notes, we shall treat these two separate equations parallel to each other because the gravitational terms depend on the same Newton interaction (1) in both equations. Equation (2) evolves the wavefunction and realizes its localization, equation (3) evolves the density matrix and realizes its decoherence. The desired two effects, localization plus decoherence, have been realized in [6] for individual wavefunctions through a single nonlinear stochastic Schrödinger equation based invariably on the structure $U(X, X')$. The present notes are restricted on the ensemble decoherence and do not extend their scope on the stochastic Schrödinger equations of individual collapse. (Various models of individual collapse should be discussed and compared first at the ensemble level, for two reasons. The ensemble equations are simpler than the stochastic equations. Current concepts of experimental tests [20] all remain in the realm of ensemble decoherence.)

3. Case study of a rigid ball

Following tradition, we are going to characterize the above modifications of quantum mechanics on the centre-of-mass motion of a single macroscopic (or at least mesoscopic)

[1] It is a mathematically correct master equation of the Lindblad class, as it is explicitly seen from its operator representation [5, 6].

[2] Regarding (4), [5] contains a trivial algebraic error, while it presents the correct form of (3), cf [17] about the emerged misinterpretations.

rigid ball of mass M and radius R. Due to the simple structure of the corresponding equations, we can determine the typical scales of gravitational self-localization and decoherence time.

Let us begin with the mass density of the ball which depends on the distance from the centre of mass $\mathbf{x}$:

$$f(\mathbf{r}|\mathbf{x}) = f(\mathbf{r}-\mathbf{x}), \tag{5}$$

where f is spherically invariant function. The Newtonian interaction (1) depends on the distance $\mathbf{x}'-\mathbf{x}$:

$$U(\mathbf{x}'-\mathbf{x}) = -G\int\int \frac{f(\mathbf{r}-\mathbf{x})f(\mathbf{r}'-\mathbf{x}')}{|\mathbf{r}'-\mathbf{r}|}\,\mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{r}'. \tag{6}$$

Through this section, we assume that the characteristic distances $|\mathbf{x}'-\mathbf{x}|$ are small compared to any other relevant length scales of the problem. Then we expand the interaction potential up to the first nontrivial order in $\mathbf{x}'-\mathbf{x}$ [3]:

$$U(\mathbf{x}'-\mathbf{x}) = U_0 + \tfrac{1}{2}M\omega_G^2|\mathbf{x}'-\mathbf{x}|^2, \tag{7}$$

where ω_G is a certain gravitational frequency of self-interacting bulk matter [15]. We can write it into the following simple form:

$$\omega_G^2 = \frac{4\pi}{3M}G\int f^2(\mathbf{r})\,\mathrm{d}\mathbf{r}. \tag{8}$$

At constant mass density $\bar{f} = 3M/4\pi R^3$, we obtain

$$\omega_G^2 = \frac{4\pi}{3}G\bar{f} = \frac{GM}{R^3}. \tag{9}$$

As we mentioned in section 1, the Newtonian interaction potential (1) plays the key role in the proposed mechanisms of localization (2) or decoherence (3) of macro-objects. Using the approximation (7) for $U(X, X')$, we obtain the nonlinear Schrödinger–Newton equation (2) for the wavefunction of the centre of mass of our ball:

$$\mathrm{i}\hbar\frac{\mathrm{d}\psi(\mathbf{x})}{\mathrm{d}t} = -\frac{\hbar^2}{2M}\Delta\psi(\mathbf{x}) + \frac{1}{2}M\omega_G^2|\mathbf{x}-\langle\mathbf{x}\rangle|^2\psi(\mathbf{x}). \tag{10}$$

For simplicity, we assumed the absence of external potentials. This nonlinear equation has exactly calculable solitary solutions. In the co-moving system, the quantum mechanical mean value $\langle\mathbf{x}\rangle$ is constant and the system becomes isomorphic with a harmonic oscillator of frequency ω_G. The width of its localized ground state is the following [3]:

$$\sqrt{\frac{\hbar}{M\omega_G}} = \left(\frac{\hbar^2}{GM^3}\right)^{1/4} R^{3/4}. \tag{11}$$

This could be the natural quantum mechanical localization of the ball. As it is obvious from equation (10), nothing prevents the ball from getting into and then remaining in the superposition of two localized ground states. (Yet more spectacular 'cat' solutions exist in the general case of two distant superposed wave packets: they can perform an eternal Kepler motion around each other, cf [3].)

These 'cat' states of macro-objects can be excluded from the theory via the decoherence mechanism modelled by the von-Neumann–Newton master equation (3). Applying again the approximation (7) for $U(X, X')$, the master equation reduces to the following form [5, 6]:

$$\frac{\mathrm{d}\rho(\mathbf{x},\mathbf{y})}{\mathrm{d}t} = \frac{\mathrm{i}\hbar}{2M}(\Delta_x - \Delta_y)\rho(\mathbf{x},\mathbf{y}) - \frac{1}{2\hbar}M\omega_G^2|\mathbf{x}-\mathbf{y}|^2\rho(\mathbf{x},\mathbf{y}), \tag{12}$$

where $\rho(\mathbf{x},\mathbf{y})$ is the density matrix of the centre of mass. This equation implies the decoherence time (cf equation (4))

$$\frac{2\hbar}{M\omega_G^2}\frac{1}{|\mathbf{x}-\mathbf{y}|^2}=\frac{2\hbar R^3}{GM^2}\frac{1}{|\mathbf{x}-\mathbf{y}|^2} \tag{13}$$

for the decay of the superposition $|\mathbf{x}\rangle+|\mathbf{y}\rangle$ [5, 6, 9].

Many of the studies [1–16] agree that the heuristic mass density, e.g., postulating a bulk homogeneous ball, yields plausible localization (11) and decoherence (13) scales. In general, the Newtonian localization and decoherence can be ignored for atomic systems while the quantum dynamics of massive bodies becomes dominated by them. It turns out, however, that the predicted scales depend on the precise definition of the mass density $f(\mathbf{r}|X)$.

4. Point-like objects—divergence, early cutoff

The Newtonian self-energy $U(X,X')$ diverges for point-like particles when, e.g.,

$$f(\mathbf{r}|\mathbf{x})=M\delta(\mathbf{r}-\mathbf{x}). \tag{14}$$

This divergence could paralyze both our localization and decoherence models above. Interestingly, the Schrödinger–Newton equation (2) remains regular for point-like particles as well. But the von-Neumann–Newton equation (3) becomes divergent. Let us follow the analysis by Gian-Carlo Ghirardi, Renata Grassi and Alberto Rimini [7]. The von-Neumann–Newton equation does not conserve the energy. The rate of increase of the translational energy for a rigid ball can be exactly calculated:

$$\frac{\mathrm{d}E}{\mathrm{d}t}=\frac{G\hbar}{2M}\int f^2(\mathbf{r})\,\mathrm{d}\mathbf{r}=\frac{3}{8\pi}\hbar\omega_G^2. \tag{15}$$

This rate diverges for a point-like object. Comparing the above 'heating rate' with certain experimental evidence, Ghirardi *et al* come to the conclusion that the cutoff on spatial mass density resolution must be as early as $a=10^{-5}$ cm. (The present author used 10^{-12} cm, ignorantly, cf [6] and also [17].) The cutoff can technically be realized by the corresponding regularization of the Newtonian kernel $1/r$ or, alternatively, of the mass density $f(\mathbf{r}|X)$.

The Ghirardi *et al* choice is the smoothed $f(\mathbf{r}|X)$:

$$f(\mathbf{r}|X)=(2\pi a^2)^{-3/2}\int\exp\left(-\frac{1}{2a^2}|\mathbf{r}-\mathbf{r}'|^2\right)f_0(\mathbf{r}'|X)\,\mathrm{d}\mathbf{r}', \tag{16}$$

where $f_0(\mathbf{r}|X)$ is the microscopic mass distribution of the point-like or extended constituents. Eventually, Ghirardi and co-workers adapted their continuous spontaneous localization (CSL) theory to the smoothed mass density $f(\mathbf{r}|X)$. The 'mass-proportional CSL', cf e.g. [18], uses the simple contact potential

$$U_{\mathrm{CSL}}(X,X')=-\gamma\int f(\mathbf{r}|X)f(\mathbf{r}|X')\,\mathrm{d}\mathbf{r} \tag{17}$$

rather than the original Newtonian version (1). In CSL, the strength-parameter γ is no longer related to Newton's G, although γ is considered a universal parameter. Its ultimate range is under careful investigation by Adler [19].

5. Interatomic resolution

How does the interaction potential (1) change if, not imposing the early cutoff 10^{-5} cm of section 4, we increase the resolution of the mass density towards the interatomic scales?

For ball geometry, equation (8) shows that the gravitational frequency ω_G grows with the spatial fluctuations of the mass density. To model the fine structure beyond the constant average mass density $\bar{f}$, suppose the ball consists of identical atoms of mass m each. Assume, furthermore, that the atomic mass is blurred on a certain distance σ. One could take a spatial Gaussian distribution of linear spread σ. For our purposes, little homogeneous balls of radius σ will suitably represent the individual atoms. Suppose the scale σ is much smaller than the interatomic distance, yet much greater than the coherent displacements $|\mathbf{x}' - \mathbf{x}|$ (quantum mechanical spread) of the centre of mass. Then, the total atomic contribution to the rhs of (8) yields

$$\omega_G^2 = \frac{4\pi}{3} G \bar{f}_\sigma = \frac{GM}{\sigma^3}, \tag{18}$$

where $\bar{f}_\sigma = 3m/4\pi\sigma^3$ is the average density of the blurred atoms. Comparing this result to (9), we can resume that the microscopic resolution of the mass density *enhances* the proposed Newtonian gravitational mechanisms. The enhancement can simply be characterized via replacing the Newton constant G by the following effective constant $\widetilde{G}$:

$$\widetilde{G} = \frac{\bar{f}_\sigma}{\bar{f}} G. \tag{19}$$

The higher the atomic density $\bar{f}_\sigma$ the stronger will be the proposed gravitational localization (11) and decoherence (13) effects.

Such enhancement depends on the geometry of the massive object. If the object is a rectangular slab rather than a ball, the gravitational frequency ω_G (9) as well as the effective Newton constant $\widetilde{G}$ will be re-calculated easily.

6. Closing remarks

One witnesses a growing number and variety of proposals that point towards possible experiments in the near future that will test the predicted decoherences at least (see, e.g., [20]). Testing the proposed spontaneous mechanisms of macroscopic localization is, however, completely out of question (unless the test of decoherence is considered an indirect test of localization as well). In some cases, the decoherence effects predicted by the Newtonian mechanism as in equations (3) and/or (4) would be too weak to be observed [21]. This tendency may, however, change if the models resolve the mass density over interatomic scales, see e.g. section 5, provided the reasons of the earlier length-cutoff are somehow neutralized. We can thus see the issue of mass density resolution is definitive from the experimental viewpoint.

From the theoretical viewpoint, the divergence of the von-Neumann–Newton equation and the corresponding decoherence time for point-like particles represent a serious difficulty. Any cutoff turns the parameter-independent model into a less attractive one-parameter model. Yet, we do not know whether the Newtonian mechanism, i.e. the structure $U(X, X')$, plays a role in spontaneous decoherence or, alternatively, the simple contact structure $U_{\mathrm{CSL}}(X, X')$ of Ghirardi *et al* is the real one, while CSL has been a two-parameter model from the beginning.

7. Outlook

If we assume that the Newtonian mechanisms are responsible for the emergent classical behaviour of the massive non-relativistic quantized matter then, in the spirit of figure 1, the intellectual perspective includes not only the proposed modifications of the standard quantum mechanics but the refinement of our concept of gravity and spacetime. If we modify or refine our concept of gravity too, this perspective may overcome the encountered difficulties of the modified quantum equations. All this should lead to an autonomous theory of some, still unknown, new quality of physical phenomena. I wrote this in 1992 and put a question mark below the edge connecting $\hbar$ and G. It marks the radically new phenomenon that will follow from the autonomous theory—provided such theory exists and we discover it. As it happened already to the Einstein (c plus G) and the Dirac (c plus $\hbar$) theories. These two theories led us to the radically new things such as the positron and the black hole, respectively. But the new physics had simple pre-phenomena in the form of slight anomalies such as the Lamb shift (c plus $\hbar$) and the gravitational light deflection or the Merkur perihelium anomaly (c plus G). In our case ($\hbar$ plus G), it is not clear which scenario wins: shall we anticipate the new physics through the little excess of decoherence [20] or through the radically new phenomena that we should become able to predict from a more complete theory.

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