X-Ray Emission from Atomic Systems Can Distinguish between Prevailing Dynamical Wave-Function Collapse Models

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In this work the spontaneous electromagnetic radiation from atomic systems, induced by dynamical wave-function collapse, is investigated in the x-ray domain. Strong departures are evidenced with respect to the simple cases considered until now in the literature, in which the emission is either perfectly coherent (protons in the same nuclei) or incoherent (electrons). In this low-energy regime the spontaneous radiation rate strongly depends on the atomic species under investigation and, for the first time, is found to depend on the specific collapse model.

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Introduction.—Quantum mechanics is our most successful physical theory, allowing us to understand and predict a large number of phenomena with extreme precision [1]. At the core of the theory lies the superposition principle, according to which systems are allowed to be in superposition of different states. The theory does not set any boundary on the limit of validity of this principle; yet, we do not observe superpositions of macroscopic objects. This well-known problem is exemplified by the Schrödinger's cat thought experiment [2].

It has been suggested that the linearity of quantum mechanics, from which the superposition principle directly follows, may break down at a certain scale [3–5]. This idea has been extensively developed in the framework of collapse models [6–9], which are phenomenological models that modify the Schrödinger equation by adding nonlinear and stochastic terms that naturally collapse the wave function in space. According to these models, microscopic systems are very weakly affected by the nonlinearities which, however, become dominant when atoms glue together to form larger and larger systems, this way solving the measurement problem. Among the several collapse models proposed, two are of particular relevance: the continuous spontaneous localization (CSL) model [10] and the so-called Diósi-Penrose (DP) model [11,12].

Spontaneous collapses must be random, in order to avoid the possibility of faster than light signaling [13]; this randomness manifests as a diffusive motion of the system [14], which corresponds to a random acceleration of the atoms and, hence, to the emission of radiation from their charged constituents. The experimental search of this spontaneous radiation was performed, for both CSL [15] and DP [16], in the energy domain of the γ rays, by comparing the measured radiation spectrum by high purity germanium crystals with the spontaneous emission rate predicted by the models for the atomic systems which constitute the experimental setup. The obtained strong bounds, combined with constraints provided by other experimental tests and theoretical considerations, are leading to a progressive falsification of the models in their Markovian formulation.

Theoretical efforts have being devoted to the development of non-Markovian collapse models [17–21], in order to counteract the runaway energy increase. These new models require the introduction of a cutoff frequency in the stochastic noise spectrum; for this reason, a systematic scan of the spontaneous radiation phenomenon, as a function of the decreasing energy, is mandatory.

The search for spontaneous radiation emitted by germanium crystals was performed in the x-ray domain in [22] [for $E \in (15–50)$ keV], and more recently in [23] [for $E \in (19–100)$ keV]. In [22] a formula for the expected spontaneous radiation rate from quasifree electrons was applied: the expected radiation depends on the energy as

$1/E$ and is proportional to the number of quasifree electrons [24]. This formula is not suitable to describe the more complex phenomenology of the spontaneous radiation emitted by the whole atomic system; more refined calculations are presented in [15,16], where the CSL and the DP rates are calculated for an atomic system, in the limit in which the spontaneous photon wavelength λ_γ is between the nuclear dimension and the mean radius of the lower laying atomic orbit. Consistently, in [15,16] the data analyses are performed for photons energies in the range (1–3.8) MeV. The rate results to be proportional, for both CSL and DP, to $(N_p^2 + N_e)/E$, where N_p and N_e are, respectively, the number of protons and electrons of the atom under study. In this regime, different collapse models share the same expected shape for the energy distribution of the spontaneous emission rate, the scaling factor being proportional to combinations of constants of nature with the characteristic parameters of the models, which are λ and r_C (strength and correlation length of the collapse noise) for CSL and the correlation length R_0 for DP (the role of the strength being played by the gravitational constant G in this case). The latter theoretical rates were also assumed in the analysis [23], which set the strongest bounds on the parameters of the Markovian models, i.e., $\lambda/r_C^2 < (4.94 \pm 0.15) \times 10^{-1}\ \mathrm{s}^{-1}\,\mathrm{m}^{-2}$, assuming that the white noise is coupled to the particle mass density, and $R_0 > (2.54 \pm 0.03) \times 10^{-10}$ m.

Given the importance of integrating the search of the spontaneous radiation signal in the high-energy domain of the γ rays to the one in the x rays, also in view of future experimental studies, scanning the cutoff energy of the non-Markovian models, we derive in this work the general expression of the radiation emission rates, for both Markovian and non-Markovian formulations of the CSL and DP models. We adopt a semiclassical approach which is valid above 1 keV (see, e.g., [17]), appropriate to the current experimental surveys; for lower energies a fully quantum mechanical analysis is required, which is under development. The general rates are found to exhibit a nontrivial energy dependence, which is strongly influenced by the interplay between the photons wavelengths, the radii of the electronic orbits, and the correlation length of the model under study. Since the correct general spectra strongly differ from the $1/E$ behavior in the x-ray range, a reanalysis of the data in Refs. [22,23] should be considered.

Interestingly, the spontaneous radiation energy spectrum is found, at the atomic λ_γ scale, to depend on the specific model of wave-function collapse under scrutiny. This finding opens new scenarios in the experimental investigation of the spontaneous radiation: a measurement sensitive to this signature of the collapse, would be able to recognize the most probable pattern of dynamical wave-function reduction.

CSL spontaneous emission rate, general expression.— The rate of the spontaneous radiation emitted by an atomic system, in the context of the Markovian CSL model, was derived in [15]:

$$\left.\frac{d\Gamma}{dE}\right|_t^{\mathrm{CSL}} = \frac{\hbar\lambda}{6\pi^2\epsilon_0 c^3 m_0^2 E}\sum_{i,j}\frac{q_i q_j}{m_i m_j} f_{ij}\,\frac{\sin(b_{ij})}{b_{ij}}, \quad (1)$$

where $b_{ij} = 2\pi|\mathbf{r}_i - \mathbf{r}_j|/\lambda_\gamma$, q_j, and m_j represent, respectively, the charge and the mass of the jth particle, at position $\mathbf{r}_j$. m_0 denotes the nucleon mass, ϵ_0 the vacuum permittivity, $\hbar$ and c are, as usual, the reduced Planck constant and the speed of light, E is the energy of the spontaneously emitted photon.

The term f_{ij} encodes the balance between the emitters' distances and the correlation length r_C. A generalized expression for f_{ij} is provided in Sec. A of supplemental material [25]:

$$f_{ij} = \frac{m_i m_j}{2r_C^2} e^{-\frac{(\mathbf{r}_i-\mathbf{r}_j)^2}{4r_C^2}}\left(3 - \frac{(\mathbf{r}_i - \mathbf{r}_j)^2}{2r_C^2}\right). \quad (2)$$

Using Eq. (2), and analyzing the contributions to the spontaneous radiation by the protons in the nucleus, the orbital electrons, and due to the combined electrons-protons emission—see Sec. B of supplemental material [25]— Eq. (1) turns to

$$\left.\frac{d\Gamma}{dE}\right|_t^{\mathrm{CSL}} = \frac{\hbar e^2\lambda}{12\pi^2\epsilon_0 c^3 m_0^2 r_C^2 E}\left\{3N_p^2 + 3N_e + 2\sum_{oo'\,\mathrm{pairs}} N_o N_{o'}\,\frac{\sin\left[\frac{\beta|\rho_o-\rho_{o'}|E}{\hbar c}\right]}{\left[\frac{\beta|\rho_o-\rho_{o'}|E}{\hbar c}\right]}\, e^{-\frac{\beta^2(\rho_o-\rho_{o'})^2}{4r_C^2}}\left(3 - \frac{\beta^2(\rho_o-\rho_{o'})^2}{2r_C^2}\right)\right.$$
$$\left. -2N_p\sum_o N_o\,\frac{\sin\left(\frac{\rho_o E}{\hbar c}\right)}{\left(\frac{\rho_o E}{\hbar c}\right)}\, e^{-\frac{\rho_o^2}{4r_C^2}}\left(3 - \frac{\rho_o^2}{2r_C^2}\right) + \sum_o N_o(N_o-1)e^{-\frac{(\alpha\rho_o)^2}{4r_C^2}}\,\frac{\sin\left(\frac{\alpha\rho_o E}{\hbar c}\right)}{\left(\frac{\alpha\rho_o E}{\hbar c}\right)}\left(3 - \frac{(\alpha\rho_o)^2}{2r_C^2}\right)\right\}, \quad (3)$$

where the electron pair distances are parametrized in terms of the mean radii of the atomic orbits ρ_o ($\rho_{o'}$) by means of the constants α and β which are given below. N_o represents the number of electrons in the oth orbit of the atom. Equation (3) represents a generalization of the spontaneous emission rate which was derived in Ref. [15]:

$$\left.\frac{d\Gamma}{dE}\right|_t^{\mathrm{CSL}} = \frac{\hbar e^2 \lambda}{4\pi^2 \epsilon_0 c^3 r_C^2 m_0^2 E}(N_p^2 + N_e), \tag{4}$$

under the condition that λ_γ is intermediate between the nuclear and atomic dimensions, i.e., the energy range under scrutiny belongs to the γ-ray domain. In that case, protons emit coherently (proportionally to the square of their number) and electrons emit independently (linear dependence). More complex is the situation described by Eq. (3).

If r_C exceeds the distance between the emitters (as confirmed by radiation experiments for a Markovian CSL [15,22]) then the stochastic field "shakes" them coherently. If λ_γ becomes also of the order of the mean orbit radii of the atom, then the electrons of the corresponding orbits start to emit coherently, i.e., quadratically. Nonetheless, the corresponding increase in the expected spontaneous emission rate is counteracted by the *cancellation*, among oppositely charged particles whose distance is smaller than λ_γ. In the limit in which λ_γ is also much bigger than the atomic size, Eq. (3) reduces to

$$\left.\frac{d\Gamma}{dE}\right|_t^{\mathrm{CSL}} = \frac{\hbar e^2 \lambda}{4\pi^2 \epsilon_0 c^3 m_0^2 r_C^2 E}[N_p^2 - 2N_p N_e + N_e^2], \tag{5}$$

which vanishes for neutral atoms.

Intermediate regimes for λ_γ and r_C, in comparison with $|\mathbf{r}_i - \mathbf{r}_j|$, give rise to a new, interesting pattern, in which the shape of the expected spontaneous radiation spectrum exhibits a nontrivial energy dependence, which is influenced by the atomic structure. This is exemplified in Fig. 1. The top panel of Fig. 1 shows (dashed line) the general spontaneous emission rate predicted by Eq. (3) compared to the simple (solid line) case described by Eq. (4), for germanium (Ge targets were indeed used in various experiments, e.g., [15,23]). The grey-shaded area corresponds to the values $\beta = 1.04$ and α spanning in the range (1–1.5), according to the literature [26,27], see Sec. B of supplemental material [25]. The same rates for a xenon target are shown in the bottom panel of Fig. 1. In Eq. (3) the value of r_C is set to 1.15×10^{-8} m, consistently with the results of Ref. [15], which are obtained by applying Eq. (4) in the γ-ray regime [(1–3.8) MeV], where Eq. (4) is an excellent approximation. $r_C = 1.15 \times 10^{-8}$ m corresponds to the intersection among the experimental bound and the theoretical constraint (corresponding, respectively, to the orange and gray lines in Fig. 4 of Ref. [15]). The mean radii of the orbits are obtained based on a density functional theory (DFT) [28] all-electron calculation, for an isolated atom; the DFT code GPAW [29] is adopted. The distributions are normalized to the common constant prefactors to evidence differences in shape. As expected, the simple (solid line) and the general (dashed line) rates converge for high energies (above 200 keV), where λ_γ becomes sizably smaller than the lower atomic orbit radii. Since r_C is much greater than the size of the germanium atom, the x-ray regime is, instead, characterized by a balance among electrons and protons coherent emission and the cancellation of their contributions. The analyses performed in Refs. [22,23] should be reconsidered based on this low-energy complex pattern. On one side the smaller expected rate could result in less stringent bounds on the parameters of the CSL model, which could require collecting more statistics. On the other side, the sensitivity may be enhanced by the new structure of the energy spectrum of the spontaneous radiation, which could help to better disentangle the expected signal from the background components.

The general expression of the spontaneous emission rate in Eq. (3) encodes the phenomenology for future investigations of the spontaneous radiation at low energies (x rays). Comparison of the theoretical expectation with the measured spectra requires a recursive analysis: in the first step, a suitable prior has to be assumed for r_C, an updated value for r_C will be obtained, which will serve as input for

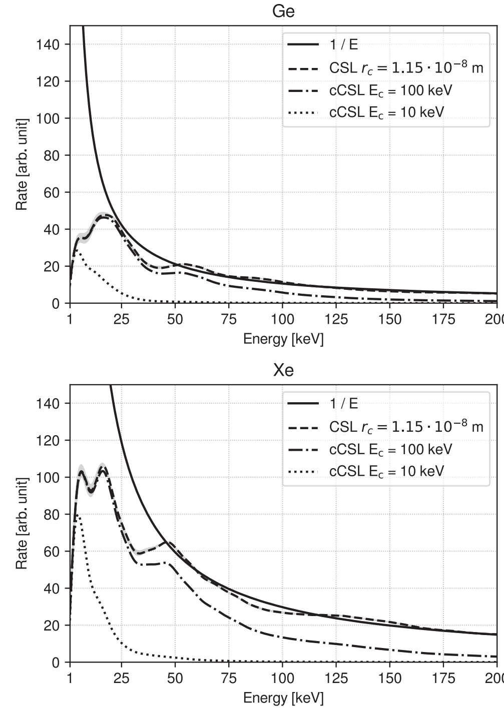

FIG. 1. The top panel of the figure shows (solid line) the $1/E$ dependence Eq. (4), for the spontaneous radiation rate of a Markovian CSL model, which is valid only in the high-energy domain. This is compared to the general rate in Eq. (3) (dashed line) for a prior value of the correlation length $r_C = 1.15 \times 10^{-8}$ m. The distributions are calculated for a germanium atom and normalized to the common constant prefactors. The bottom panel of the figure shows the shapes of the same rates, calculated for a xenon atom. The dotted and dash-dotted curves in the top and bottom panels, represent the corresponding spontaneous emission rates in Ge and Xe for a non-Markovian CSL model Eq. (6), when $E_c = 10$ keV (dotted) and $E_c = 100$ keV (dash-dotted).

the new prior. The analysis should then be iterated till convergence of the r_C values, within the experimental sensitivity, is reached.

The dependence of the expected rate on the atomic structure becomes evident comparing the top panel of Fig. 1 with the bottom one, which shows as a dashed line the general spontaneous emission rate given by Eq. (3) for xenon (high sensitivity bounds on the spontaneous collapse could be set by the XENON experiment [30] by exploiting a xenon target); the solid line describes again the simple rate of Eq. (4). Equation (3) predicts a strong dependence of the spontaneous radiation yield on the atomic number Z; as such a survey of the spontaneous collapse induced emission over Z would greatly improve the experimental sensitivity on this, new physics, phenomenon.

The generalization of Eq. (3) to the non-Markovian case requires to multiply the right-hand side by the Fourier transform of the noise correlation function (see, e.g., [17,19–21,31] and the derivation of the colored DP model emission rate below). Assuming, e.g., an exponentially decaying noise correlation function $[f(t-s) = (\Omega/2)e^{-\Omega|t-s|}]$, characterized by a correlation time Ω^{-1}, the rate becomes

$$\left.\frac{d\Gamma}{dE}\right|_t^{\mathrm{cCSL}} = \left.\frac{d\Gamma}{dE}\right|_t^{\mathrm{CSL}} \times \frac{E_c^2}{E_c^2+E^2}, \tag{6}$$

where $E_c = \hbar\Omega$ and cCSL denote results for a colored (non-Markovian) CSL model. The dotted and dash-dotted curves in Fig. 1 represent the general spontaneous emission rates for a non-Markovian CSL model Eq. (6), for $E_c = 10$ keV (dotted) and $E_c = 100$ keV (dash-dotted).

DP spontaneous emission rate, general expression.— The rate of the spontaneous radiation emitted by an atomic system, according to a Markovian DP model, was derived in Ref. [16]:

$$\left.\frac{d\Gamma}{dE}\right|_t^{\mathrm{DP}} = \frac{\mathrm{G}e^2}{12\pi^{5/2}\epsilon_0 c^3 R_0^3 E}(N_p^2 + N_e), \tag{7}$$

assuming a spontaneous photon wavelength which is much bigger than the nuclear size and much smaller than the lower lying atomic orbit mean radius. Note that Eq. (7) differs from the result presented in [16] by a factor 8π. This is because in [16] we adopted the convention introduced in [12] while we refer here to the original model introduced by Diósi [32], in which the factor 8π was not present.

The general structure of the rate is derived in Sec. C of supplemental material [25], where the non-Markovianity of the noise time correlation is also considered. For an exponential time correlation we have

$$\begin{aligned}\left.\frac{d\Gamma}{dE}\right|_t^{cDP} &= \frac{G}{6\pi^2\epsilon_0 c^3 E}\sum_{i,j} q_i q_j f_{ij}\frac{\sin(b_{ij})}{b_{ij}}\frac{E_c^2}{E_c^2+E^2} \\ &= \left.\frac{d\Gamma}{dE}\right|_t^{\mathrm{DP}}\frac{E_c^2}{E_c^2+E^2},\end{aligned} \tag{8}$$

with G the Newton constant. The rate for the Markovian model is recovered in the limit $E_c \to \infty$. In analogy with the expected rate for the CSL model [Eq. (1)], the interplay between the particle mean distances and the wavelength of the spontaneously emitted photon is contained in the terms $\sin(b_{ij})/b_{ij}$. The dependence on the particle distances in relation to the correlation length of the model R_0 is instead specified by the terms f_{ij}. As it is shown in Sec. D of supplemental material [25], f_{ij} is a measure of the overlap between the mass densities of the particles ith and jth ($g_{i,j}$), whose spatial resolution is measured by R_0. In formula

$$f_{ij} = 4\pi\int d\boldsymbol{r} g_i(\boldsymbol{r}-\boldsymbol{r}_i, R_0) g_j(\boldsymbol{r}-\boldsymbol{r}_j, R_0). \tag{9}$$

Assuming Gaussian mass density profiles (following, e.g., [10,16] $g_i(\boldsymbol{r}-\boldsymbol{r}_i, R_0) = (2\pi R_0^2)^{-3/2} e^{-[(\boldsymbol{r}-\boldsymbol{r}_i)^2/2R_0^2]}$) and specifying the rate for the mean radii of the atomic orbits we obtain (see Sec. D of supplemental material [25]):

$$\begin{aligned}\left.\frac{d\Gamma}{dE}\right|_t^{\mathrm{DP}} = \frac{\mathrm{G}e^2}{12\pi^{5/2}\epsilon_0 c^3 R_0^3 E}\Bigg\{ & N_p^2 + N_e + 2\sum_{oo'\,\mathrm{pairs}} N_o N_{o'} \frac{\sin\left[\frac{\beta|\rho_o-\rho_{o'}|E}{\hbar c}\right]}{\left[\frac{\beta|\rho_o-\rho_{o'}|E}{\hbar c}\right]} e^{-\frac{\beta^2(\rho_o-\rho_{o'})^2}{4R_0^2}} \\ & + \sum_o N_o(N_o-1) e^{-\frac{(\alpha\rho_o)^2}{4R_0^2}} \frac{\sin\left(\frac{\alpha\rho_o E}{\hbar c}\right)}{\left(\frac{\alpha\rho_o E}{\hbar c}\right)} - 2N_p\sum_o N_o \frac{\sin\left(\frac{\rho_o E}{\hbar c}\right)}{\left(\frac{\rho_o E}{\hbar c}\right)} e^{-\frac{\rho_o^2}{4R_0^2}}\Bigg\}.\end{aligned} \tag{10}$$

Again, the simple rate described by Eq. (7) is a good approximation of Eq. (10) for energies belonging to the γ-ray domain. Figure 2 compares the general expression of the Markovian DP spontaneous emission rate [Eq. (10)] given by the dashed curve, with the simple expression in Eq. (7) given by the solid curve. The prior value $R_0 = 0.54$ Å is chosen, consistently with the result [16], which is obtained by applying Eq. (7) in the γ-ray regime

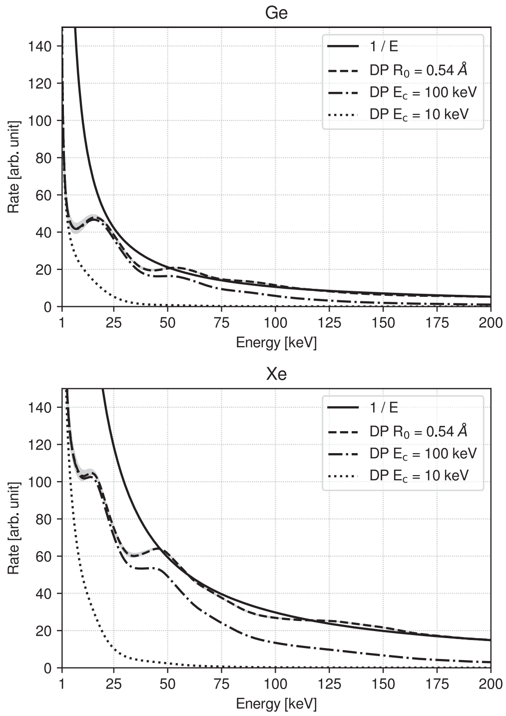

FIG. 2. Top panel of the figure shows (solid line) the $1/E$ dependence Eq. (7), for the spontaneous radiation rate of the Markovian DP model, which is valid only in the high-energy domain. This is compared to the general rate Eq. (10) (dashed line) for a prior value of the correlation length $R_0 = 0.54$ Å. The distributions are calculated for a germanium atom and normalized to the common constant prefactors. The bottom panel of the figure shows the shapes of the same rates, calculated for a xenon atom. The dotted and dash-dotted curves in the top and bottom panels, represent the corresponding spontaneous emission rates in Ge and Xe for a non-Markovian DP model, Eq. (8), when $E_c = 10$ keV (dotted) and $E_c = 100$ keV (dash-dotted).

[(1–3.8) MeV], where Eq. (4) is an excellent approximation. A strong departure from the approximate expression is evident, at low energy, for both germanium (top panel) and xenon (bottom panel) atoms. A significant dependence of the generalized rate on the atomic number is evident for the DP model as well. The dotted and dash-dotted curves in Fig. 2 represent the general spontaneous emission rates for a non-Markovian DP model Eq. (8), for $E_c = 10$ keV (dotted) and $E_c = 100$ keV (dash-dotted).

Discussion.—Figures 1 and 2 summarize our findings and unveil the most interesting consequence of the cancellation phenomenon, for which the contribution to the spontaneous radiation emission from oppositely charged particles, whose distance is exceeded by the model's correlation length and by the observed photon wavelength, cancels. In the low-energy regime, for correlation lengths of the models of the order, or bigger, than the atomic orbits radii, the shapes predicted for the spontaneous emission rates distributions of the CSL and DP models strongly differ. This is due to both the different mathematical structure of the terms f_{ij} and the different values of the correlation lengths, r_C and R_0, of the two models. In the simple scenario in which λ_γ is much smaller than the atomic size, any difference is washed out and the shapes of the spontaneous radiation rates of the two models just differ by a scaling factor.

The energy ranges analyzed in Refs. [22,23] do not allow to unveil differences among the spontaneous emission spectra of the CSL and the DP. Recent experimental works [33,34] present data in the ranges $E \in (5\text{–}30)$ keV (with lower statistics with respect to Ref. [23]) and $E \in (1\text{–}140)$, respectively. The analysis of these data, based on the predicted rates Eqs. (3) and (10), would provide a first test of this prediction for a Ge target [33] and a Xe target [34]. A measurement complementary to [33], with improved statistics, and including the energy interval $E \in (1\text{–}5)$ keV, would be important. Moreover, the prediction of a strong dependence of the spontaneous emission rate on the atomic structure should be exploited by performing a dedicated experiment, performing a search for spontaneous radiation signal by scanning over several targets (under the same experimental conditions) in the energy ranges in which the CSL and DP rates differ for the considered targets. Such a measurement would further improve the sensitivity and would allow us to distinguish an eventual signal detection from systematic effects.

In this Letter, we derived the fundamental theoretical formulas for these future investigations, both for the white noise models and their non-Markovian generalizations.

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