

# Continuous Wave Function Collapse in Quantum-Electrodynamics?

Lajos Diósi

*Research Institute for Particle and Nuclear Physics  
H-1525 Budapest 114, P.O.Box 49, Hungary*

**Abstract.** Time-continuous wavefunction collapse mechanisms *not* restricted to markovian approximation have been found only a few years ago, and have left many issues open. The results apply formally to the standard relativistic quantum-electrodynamics. I present a generalized Schrödinger equation driven by a certain complex stochastic field. The equation reproduces the *exact* dynamics of the interacting fermions in QED. The state of the fermions appears to collapse continuously, due to their interaction with the photonic degrees of freedom. Even the formal study is instructive for the foundations of quantum mechanics and of field theory as well.

**Keywords:** wave function collapse, stochastic Schrödinger equation, Lorentz invariance, quantum-electrodynamics

**PACS:** 03.65.Ta,03.65.Pm03.65.Yz,12.20.-m

## INTRODUCTION

The first study of what we can call continuous wave function collapse was Mott's analysis [1] of the particle's track in a cloud chamber. The advanced models [2]-[4] of real continuous collapse have become part of the quantum optics theoretical toolbox. Speculations on fictitious or formal continuous wave function collapse started with [5] by Bohm and Bub and have been discussed in numerous works [6]-[18] of various motivations. As for the mathematical formalisms of both real and fictitious collapses, the triumphing one turned out to be the markovian Stochastic(ally modified) Schrödinger Equation (SSE) [9],[12],[13]. The Lorentz invariant SSE is a hard nut, see e.g. [19] and references therein, as well as [20],[21]. In my opinion, a sensible Lorentz invariant SSE should relax the markovian approximation first. After my early attempts [22]-[24], Strunz published an important result [25] to be followed soon by many others [26]-[33]. I am going to recall the standard markovian and non-markovian SSE, and I shall present and discuss a SSE in the explicit Lorentz invariant context of QED.

## REAL OR FICTITIOUS CONTINUOUS COLLAPSE

Classicality emerges from Quantum via real or fictitious, often time-continuous, measurement (detection, observation, monitoring, e.t.c.) of the wavefunction  $\psi$ . By the real continuous collapse we mean, e.g., the detection of a particle track in a cloud chamber, the photon-counter detection of atomic decay, or the homodyne detection of quantum-optical oscillators. The fictitious continuous collapse means the various theories of spontaneous (universal, intrinsic, primary, e.t.c.) collapse (localization, reduction, e.t.c.). The

items in the parentheses indicate the multitude of close synonyms. To date, the mathematics is the same for both real and fictitious classes! We know almost everything about the mathematical and physical structures if markovian approximation applies. We know much less beyond that approximation. So, let us see what *equation* describes the wave function under time-continuous collapse?

## THE MARKOVIAN STOCHASTIC SCHRÖDINGER EQUATION

The prototype of the SSE has the following structure:

$$\begin{aligned} \frac{d\psi(t,z)}{dt} = & \quad -i\hat{H}\psi(t,z) && \text{hermitian Hamiltonian} \\ & \quad -i\hat{q}z\psi(t,z) && \text{non-hermitian noisy Hamiltonian} \\ & \quad -\frac{1}{2}\gamma\hat{q}^2\psi(t,z) && \text{non-hermitian dissipative Hamiltonian,} \end{aligned} \quad (1)$$

where  $z$  is a complex Gaussian hermitian white-noise:

$$\mathbf{M}[z^*(t)z(s)] = \gamma\delta(t-s). \quad (2)$$

Throughout this work,  $\mathbf{M}[\dots]$  stands for the stochastic average. The eq.(1) is not norm-preserving. We define the physical state by  $\psi/\|\psi\|$  while its statistical weight must be multiplied by  $\|\psi\|^2$ :

$$\psi(t,z) \longrightarrow \frac{\psi(t,z)}{\|\psi(t,z)\|} \equiv |t,z\rangle, \quad (3)$$

$$\mathbf{M}[\dots] \longrightarrow \mathbf{M}[\|\psi(t,z)\|^2 \dots] \equiv \tilde{\mathbf{M}}_t[\dots]. \quad (4)$$

In our case, the state  $|t,z\rangle$  and the noise  $z(t)$  play the roles of the Quantum and the Classical, respectively. Their “mutual influence” is described by the eqs.(1-4)<sup>1</sup>.

The markovian SSE describes perfectly the time-continuous collapse of the wave-function in the given observable(s)  $\hat{q}$ . The state  $|t,z\rangle$  depends on  $\{z(s); s \leq t\}$  causally, i.e., the state at  $t$  depends on the values of the noise at times  $s \leq t$ . The individual solutions  $|t,z\rangle$  can, in principle, be realized by time-continuous monitoring of  $\hat{q}$ . Then  $z(t)$  becomes the classical record explicitly related to the monitored value of  $\hat{q}$ .

Our key-problems will be: causality, realizability, and Lorentz invariance. So far, for the markovian SSE, both causality and realizability hold but Lorentz invariance is missing. To make a progress, we need to relax the markovian approximation.

---

<sup>1</sup> Equivalently, there exists a pair of closed non-linear equations for  $|t,z\rangle$  and for the recorded value of  $\hat{q}$ , cf. [12],[13], or [18].

# THE NON-MARKOVIAN STOCHASTIC SCHRÖDINGER EQUATION

The key to the non-markovian SSE is that the driving noise is a colored noise:

$$\mathbf{M}[z^*(t)z(s)] = \alpha(t-s). \quad (5)$$

The corresponding SSE [26] contains a memory-term:

$$\frac{d\psi(t,z)}{dt} = -i\hat{H}\psi(t,z) - i\hat{q}_z\psi(t,z) + i\hat{q} \int_0^t \alpha(t-s) \frac{\delta\psi(t,z)}{\delta z(s)} ds. \quad (6)$$

This linear SSE is not norm-preserving. We define the physical state by  $\psi/\|\psi\|$  while its statistical weight must be multiplied by  $\|\psi\|^2$  exactly the same way as in eqs.(3,4) for the markovian SSE<sup>2</sup>.

The non-markovian SSE describes the *tendency* of time-continuous collapse of the wavefunction in the given observable(s)  $\hat{q}$ . The state  $|t,z\rangle$  depends on  $\{z(s); s \leq t\}$  causally. The individual solutions  $|t,z\rangle$  can *not* be realized by any known way of monitoring [33]. The non-markovian SSE corresponds mathematically to the influence of a real or fictitious oscillatory reservoir whose instantaneous Husimi function is sampled stochastically, cf. [28]. Disappointedly,  $z(t)$  can *not* be interpreted as a classical record. It only corresponds to fictitious paths in the parameter space of the reservoir's coherent states.

The status of our key problems for the non-markovian SSE is the following. Causality holds, but realizability and Lorentz invariance may not hold at all. Can we enforce Lorentz invariance at least?

## CASE STUDY: QUANTUM-ELECTRODYNAMICS

We choose the well-known quantum theory of electromagnetic interaction as the framework to study the possible form of a Lorentz invariant SSE — without any guarantee that it exists. At least, we shall try to export the Lorentz invariance from QED to SSE.

Let  $x = (x_0, \vec{x})$  denote the four-vector of space-time coordinates. The vector-field  $\hat{A}(x)$  stands for the quantized electromagnetic four-potential, and the Dirac spinor field  $\hat{\chi}(x)$  stands for the quantized electron-positron field. Then the fermionic current is defined by  $\hat{J}(x) = e\hat{\bar{\chi}}(x)\gamma\hat{\chi}(x)$ . Later we shall need the electromagnetic correlation  $D(x) = i\langle \text{e.m.vac} | \hat{A}(x)\hat{A}(0) | \text{e.m.vac} \rangle$  as well. The Schrödinger equation in interaction picture takes this form:

$$\frac{d\Psi(t)}{dt} = -i \int_{x_0=t} d\vec{x} \hat{J}(x)\hat{A}(x) \Psi(t). \quad (7)$$

---

<sup>2</sup> There exists a non-linear non-markovian SSE for  $|t,z\rangle$  [27]. There is no equation for anything like the recorded value of  $\hat{q}$ . To date, there has been no way to define a classical record.

As usual in QED, we suppose the uncorrelated initial state  $\Psi(-\infty) = \psi(-\infty) \otimes |\text{e.m.vac}\rangle$  where  $\psi(-\infty)$  is the initial state of the electrons and positrons before the interaction is switched on. We seek the SSE for the electron-positron wavefunction  $\psi(t)$  continuously localized by the electromagnetic field which plays the role of the “environment” or the “reservoir”. It can be shown that the SSE is driven by the negative-frequency part  $A^-(x)$  of the e.m. “vacuum-field”  $A^+ + A^- = A$ , satisfying:

$$\mathbf{M}[A^-(x)A^+(y)] = \langle \text{e.m.vac} | \widehat{A}(x)\widehat{A}(0) | \text{e.m.vac} \rangle = -iD(x-y). \quad (8)$$

The SSE<sup>3</sup> contains a memory-term:

$$\frac{d\psi(t, A^-)}{dt} = -i \int_{x_0=t} d\vec{x} \widehat{J}(x) A^-(x) \psi(t, A^-) - \int_{x_0=t} d\vec{x} \int_{y_0 \leq t} dy \widehat{J}(x) D(x-y) \frac{\delta \psi(t, A^-)}{\delta A^-(y)}. \quad (9)$$

This linear non-markovian SSE is not norm-preserving. We define the physical state by  $\psi/\|\psi\|$  while its statistical weight must be multiplied by  $\|\psi\|^2$ , cf. eqs.(3,4):

$$\psi(t, A^-) \longrightarrow \frac{\psi(t, A^-)}{\|\psi(t, A^-)\|} \equiv |t, A\rangle, \quad (10)$$

$$\mathbf{M}[\dots] \longrightarrow \mathbf{M}[\|\psi(t, A^-)\|^2 \dots] \equiv \widetilde{\mathbf{M}}_t[\dots]. \quad (11)$$

Similarly to the markovian SSE, the state  $|t, A\rangle$  and the random complex field  $A^-(x)$  play the role of the Quantum and the Classical, respectively. Their “mutual influence” is described by the eqs.(8-11) which are *formally* Lorentz invariant. The solutions of the “relativistic” SSE (9), when averaged over  $A^-$ , describe the exact QED fermionic reduced state:

$$\mathbf{M}[\psi(t, A^-)\psi^\dagger(t, A^+)] = \text{tr}_{\text{e.m.}}[\Psi(t)\Psi^\dagger(t)]. \quad (12)$$

The “relativistic” SSE describes the *tendency* of time-continuous collapse of the fermionic wavefunction in the current  $\widehat{J}$  although the collapse happens in (certain) Fourier components instead of the local values  $\widehat{J}(x)$ . The state  $|t, A\rangle$  depends on the classical field  $\{A(x); x_0 \leq t\}$  causally. The individual solutions  $|t, A\rangle$  can *not* be realized by any known way of monitoring. Therefore the classical field  $A$  can *not* be interpreted as a classical record<sup>4</sup>.

## LORENTZ INVARIANCE?

We can express the solution of the “relativistic” SSE (9), emerging from the initial state  $\psi(-\infty)$ :

$$\psi(t, A^-) = \text{T exp} \left\{ -i \int_{x_0 \leq t} dx \widehat{J}(x) A^-(x) - \int \int_{y_0 \leq x_0 \leq t} dx dy \widehat{J}(x) D(x-y) \widehat{J}(y) \right\} \psi(-\infty), \quad (13)$$

<sup>3</sup> This equation follows from the results of [26]-[28], where a closed non-markovian SSE for the normalized state  $|t, A\rangle$  is also given.

<sup>4</sup> The complex random field  $A^-$  carries information on the quantized e.m. field  $\widehat{A}(x)$ , the details are still to be investigated.

where  $T$  stands for time-ordering of the current operators  $\widehat{J}(x)$ . Consider the expectation value of the local e.m. current at some  $t$ :

$$J(t, \vec{x}, A) = \frac{\psi^\dagger(t, A^+) \widehat{J}(t, \vec{x}) \psi(t, A^-)}{\psi^\dagger(t, A^+) \psi(t, A^-)}. \quad (14)$$

This local current  $J(x, A)$  depends on  $A(y)$  for  $y_0 \leq x_0$  which is causal in the given frame while it may violate causality in other Lorentz frames. To assure Lorentz invariant causality, the local current  $J(x, A)$  must not depend on  $A$  outside the backward light-cone of  $x$ . Since this is not the case, our “relativistic” SSE can not be causal at all.

We can see that, despite our efforts, the status of the key-problems for the “relativistic” SSE is disappointing: causality, realizability, and Lorentz invariance have been all lost. However, the “relativistic” SSE is the prototype of a Lorentz-invariant-looking closed SSE and we must study it if we wish to know why and where exactly Lorentz invariance has gone<sup>5</sup>.

## OUTLOOK

“Classicality emerges from Quantum via real or fictitious, often time-continuous, measurement (detection, observation, monitoring, e.t.c.) of the wavefunction  $\psi$ .” This has been our universal motivation to investigate the corresponding mathematical models. All markovian SSE’s turn out to be mathematically equivalent with standard (though sophisticated) quantum measurements [18]. The non-markovian SSE’s are equivalent with certain quantum reservoir dynamics, i.e., with their formal stochastic decompositions (unravelings) [26]. We have inspected in the previous section that the causality and Lorentz invariance (as well as the realizability) remain problematic even when we start from a true Lorentz invariant dynamics.

Let us ask the following question. Can we construct more general models which would liberate us from the mathematical constraints of the standard quantum theory? The minimalist’s answer would be this. We should replace the concept “Emergence of Classicality from Quantum” by the concept “Coexistence of Classical and Quantum”. The classical entities<sup>6</sup> are certain classical fields  $C(x)$  and the quantum entities are certain quantum fields  $\widehat{Q}(x)$ . We seek a causal and Lorentz invariant coexistence including their “mutual influence” on each other. The loophole is that, unlike in the quantum theory, the “mutual influence” is not necessarily a dynamical or a measurement-like mechanism. In our longstanding struggles with the problem of Classical vs. Quantum, the main issue to overcome has always been the painful lack of a consistent model that “couples” the coexisting classical and quantum entities. Aren’t quantum dynamics and measurement too restrictive? Are there any other consistent mechanisms?

---

<sup>5</sup> Lorentz invariance would already be lost in a single von Neumann-Lüders collapse. Nevertheless, the status of Lorentz invariance in continuous collapse is a theoretical challenge.

<sup>6</sup> According to certain alternative concepts [12],[21],[34],[35] the whole physics might be represented by classical entities so that we may not care if the wave function violates Lorentz invariance and causality.

## ACKNOWLEDGMENTS

I thank the organizers of the conferences in Triest and in Mali Losinj for the kind invitation and the financial support. My research is supported by the Hungarian OTKA under Grant No. 49384.

## REFERENCES

1. N. F. Mott, *Proc. Roy. Soc. A*, **126**, 79–84 (1929).
2. J. Dalibard, Y. Castin, and K. Molmer, *Phys. Rev. Lett.*, 580–583 (1989).
3. H. Carmichael, *An Open Systems Approach to Quantum Optics*, Springer, Berlin, 1989.
4. H. M. Wiseman and G. Milburn, *Phys. Rev. A*, **47** 642–662 (1993).
5. D. Bohm and J. Bub, *Rev. Mod. Phys.* **38**, 453–469 (1966).
6. F. Károlyházi, *Nuovo Cim.*, **52**, 390–402 (1966).
7. P. Pearle, *Phys. Rev. D*, **13**, 857–868 (1976).
8. A. Barchielli, L. Lanz, and G. M. Prosperi, *Nuovo Cim. B*, **72**, 79–121 (1982).
9. N. Gisin, *Phys. Rev. Lett.*, **52**, 1657–1660 (1984).
10. G. C. Ghirardi, A. Rimini, and T. Weber, *Phys. Rev. D*, **34**, 470–491 (1986).
11. L. Diósi, *Phys. Lett. A*, **114**, 451–454 (1986).
12. L. Diósi, *Phys. Lett. A*, **129**, 419–423 (1988).
13. V. P. Belavkin, in *Modelling and Control of Systems*, edited by A. Blanquiere, Lecture Notes in Control and Information Sciences, Springer, Berlin, 1988, **121**, pp. 245–265.
14. P. Pearle, *Phys. Rev. A*, **39**, 2277–2289 (1989).
15. I. C. Percival, *Quantum State Diffusion*, Cambridge University Press, Cambridge, 1998.
16. Ph. Blanchard, and A. Jadczyk, *Annl. Phys.* **4**, 583–599 (1995).
17. S. A. Adler, D. C. Brody, L. P. Hughston, and T. A. Brun, *J. Phys. A*, **34**, 8795–8820 (2001).
18. H. M. Wiseman, and L. Diósi, *Chem. Phys.*, **268**, 91–104 (2001).
19. S. L. Adler, and T. A. Brun, *J. Phys. A*, **34**, 4797–4809 (2001).
20. A. Rimini, *Lecture Notes in Physics*, **622**, 221–231 (2003).
21. R. Tumulka, in this volume.
22. L. Diósi, *Phys. Rev. A*, **42**, 5086–5092 (1990).
23. L. Diósi, in *Stochastic Evolution of Quantum States in Open Systems and in Measurement Processes*, edited by L. Diósi and B. Lukács, World Scientific, Singapore, 1994, pp. 15–24.
24. L. Diósi, *Quant. Semiclass. Opt.*, **8**, 309–314 (1996).
25. W. T. Strunz, *Phys. Lett. A*, **224** 25–30 (1996).
26. L. Diósi, and W. T. Strunz, *Phys. Lett. A*, **235** 569–573 (1997).
27. W. T. Strunz, L. Diósi, and N. Gisin, *Phys. Rev. Lett.*, **82**, 1801–1805 (1999).
28. L. Diósi, N. Gisin, and W. T. Strunz, *Phys. Rev. A*, **61**, 22108 (2000).
29. A. A. Budini, *Phys. Rev. A*, **63**, 012106 (2000).
30. J. T. Stockburger, and H. Grabert, *Phys. Rev. Lett.* **88**, 170407 (2002).
31. A. Bassi, and G. Ghirardi, *Phys. Rev. A*, **65**, 042114 (2002).
32. A. Bassi, *Phys. Rev. A*, **67**, 062101 (2003).
33. J. Gambetta, and H. M. Wiseman, *Phys. Rev. A*, **68**, 062104 (2003).
34. L. Diósi, *Phys. Rev. A*, **43**, 17–21 (1991).
35. F. Dowker, and I. Herbauts, *Found. Phys. Lett.*, **18**, 499–518 (2005).

UK STFC (ST/W006561/1 and ST/S002227/1). D K L Oi acknowledges the support of the UK EPSRC (EP/T517288/1, EP/T001011/1, EP/W027011/1). A Roura is supported by the Q-GRAV Project within the Space Research and Technology Program of the German Aerospace Center (DLR).

## ORCID iDs

Rainer Kaltenbaek  <https://orcid.org/0000-0002-9991-9919>  
Markus Arndt  <https://orcid.org/0000-0002-9487-4985>  
Markus Aspelmeyer  <https://orcid.org/0000-0003-4499-7335>  
Peter F Barker  <https://orcid.org/0000-0001-9669-9853>  
Alessio Belenchia  <https://orcid.org/0000-0002-0347-6763>  
Joel Bergé  <https://orcid.org/0000-0002-7493-7504>  
Garrett D Cole  <https://orcid.org/0000-0001-7149-3218>  
Animesh Datta  <https://orcid.org/0000-0003-4021-4655>  
Uroš Delić  <https://orcid.org/0000-0002-1426-7289>  
Andrew A Geraci  <https://orcid.org/0000-0001-7009-0118>  
Antoine Heidmann  <https://orcid.org/0000-0002-0784-5175>  
James Millen  <https://orcid.org/0000-0002-6950-3461>  
David C Moore  <https://orcid.org/0000-0002-2358-4761>  
Gavin W Morley  <https://orcid.org/0000-0002-8760-6907>  
Lukas Novotny  <https://orcid.org/0000-0002-9970-8345>  
Daniel K L Oi  <https://orcid.org/0000-0003-0965-9509>  
Mauro Paternostro  <https://orcid.org/0000-0001-8870-9134>  
Manuel Rodrigues  <https://orcid.org/0000-0002-8573-8068>  
Loïc Rondin  <https://orcid.org/0000-0002-4833-2886>  
Albert Roura  <https://orcid.org/0000-0002-8049-8982>  
Wolfgang P Schleich  <https://orcid.org/0000-0002-9693-8882>

## References

- [1] Pikovski I, Zych M, Costa F and Brukner č 2015 Universal decoherence due to gravitational time dilation *Nat. Phys.* **11** 668–72
- [2] Zych M 2017 *Quantum Systems Under Gravitational Time Dilation (Springer Theses)* (Cham: Springer)
- [3] Károlyházy F 1966 Gravitation and quantum mechanics of macroscopic objects *Il Nuovo Cimento A* **42** 390–402
- [4] Bassi A, Großardt A and Ulbricht H 2017 Gravitational decoherence *Class. Quantum Gravity* **34** 193002
- [5] Schrödinger E 1935 Die gegenwärtige Situation in der Quantenmechanik *Sci. Nat.* **23** 807–12
- [6] Fein Y Y *et al* 2019 Quantum superposition of molecules beyond 25 kDa *Nat. Phys.* **15** 1242–5
- [7] Riedel C J 2013 Direct detection of classically undetectable dark matter through quantum decoherence *Phys. Rev. D* **88** 116005
- [8] Bateman J, McHardy I, Merle A, Morris T R and Ulbricht H 2015 On the existence of low-mass dark matter and its direct detection *Sci. Rep.* **5** 8058
- [9] Riedel C J and Yavin I 2017 Decoherence as a way to measure extremely soft collisions with dark matter *Phys. Rev. D* **96** 023007
- [10] Kaltenbaek R, Hechenblaikner G, Kiesel N, Romero-Isart O, Schwab K C, Johann U and Aspelmeyer M 2012 Macroscopic quantum resonators (MAQRO) *Exp. Astron.* **34** 123–64
- [11] Kaltenbaek R *et al* 2016 Macroscopic Quantum Resonators (MAQRO): 2015 update *EPJ Quantum Technol.* **3** 5
- [12] Chang D E, Regal C A, Papp S B, Wilson D J, Ye J, Painter O, Kimble H J and Zoller P 2010 Cavity opto-mechanics using an optically levitated nanosphere *Proc. Natl Acad. Sci. USA* **107** 1005–10
- [13] Romero-Isart O, Juan M L, Quidant R and Cirac J I 2010 Toward quantum superposition of living organisms *New J. Phys.* **12** 033015
- [14] Barker P F and Shneider M N 2010 Cavity cooling of an optically trapped nanoparticle *Phys. Rev. A* **81** 023826
- [15] Monteiro F, Afek G, Carney D, Krnjaic G, Wang J and Moore D C 2020 Search for composite dark matter with optically levitated sensors *Phys. Rev. Lett.* **125** 181102
- [16] Afek G, Carney D and Moore D C 2022 Coherent scattering of low mass dark matter from optically trapped sensors *Phys. Rev. Lett.* **128** 101301
- [17] Carney D *et al* 2021 Mechanical quantum sensing in the search for dark matter *Quantum Sci. Technol.* **6** 024002
- [18] Riedel C J 2015 Decoherence from classically undetectable sources: standard quantum limit for diffusion *Phys. Rev. A* **92** 010101
- [19] Branford D, Gagatsos C N, Grover J, Hickey A J and Datta A 2019 Quantum enhanced estimation of diffusion *Phys. Rev. A* **100** 022129
- [20] Carney D, Hook A, Liu Z, Taylor J M and Zhao Y 2021 Ultralight dark matter detection with mechanical quantum sensors *New J. Phys.* **23** 023041
- [21] Joos E and Zeh H D 1985 The emergence of classical properties through interaction with the environment *Z. Phys., B Condens. Matter* **59** 223–43
- [22] Schlosshauer M A 2007 *Decoherence and the Quantum-to-Classical Transition* (Berlin: Springer)
- [23] Voirin T, Bandecchi M, Falkner P and Pickering A 2019 CDF STUDY REPORT (QPPF, European Space Agency) (available at: <http://sci.esa.int/future-missions-department/61074-cdf-study-report-qppf/> CDF-183(C))
- [24] Blencowe M P 2013 Effective field theory approach to gravitationally induced decoherence *Phys. Rev. Lett.* **111** 021302
- [25] Anastopoulos C, Hu B-L 2021 Gravitational decoherence: a thematic overview (arxiv:2111.02462)
- [26] Frenkel A 1990 Spontaneous localizations of the wave function and classical behavior *Found. Phys.* **20** 159

- [27] Diósi L 1987 A universal master equation for the gravitational violation of quantum mechanics *Phys. Lett. A* **120** 377–81
- [28] Penrose R 1996 On gravity's role in quantum state reduction *Gen. Relativ. Gravit.* **28** 581–600
- [29] Diósi L 2007 Notes on certain newton gravity mechanisms of wavefunction localization and decoherence *J. Phys. A: Math. Theor.* **40** 2989
- [30] Penrose R 2014 On the gravitization of quantum mechanics I: quantum state reduction *Found. Phys.* **44** 557–75
- [31] Kaltenbaek R 2022 Feasibility considerations for free-fall tests of gravitational decoherence *AVS Quantum Sci.* **4** 015604
- [32] Gasbarri G, Belenchia A, Carlesso M, Donadi S, Bassi A, Kaltenbaek R, Paternostro M and Ulbricht H 2021 Testing the foundation of quantum physics in space via Interferometric and non-interferometric experiments with mesoscopic nanoparticles *Commun. Phys.* **4** 155
- [33] Martinecz L, Hornberger K, Millen J, Kim M S and Stickler B A 2020 Quantum electromechanics with levitated nanoparticles *npj Quantum Inf.* **6** 1–8
- [34] Pino H, Prat-Camps J, Sinha K, Venkatesh B P and Romero-Isart O 2018 On-chip quantum interference of a superconducting microsphere *Quantum Sci. Technol.* **3** 025001
- [35] Romero-Isart O, Pflanzner A C, Blaser F, Kaltenbaek R, Kiesel N, Aspelmeyer M and Cirac J I 2011 Large quantum superpositions and interference of massive nanometer-sized objects *Phys. Rev. Lett.* **107** 020405
- [36] Nimmrichter S 2014 *Macroscopic Matter Wave Interferometry (Springer Theses)* (Cham: Springer)
- [37] Armano M et al 2016 Sub-femto-g free fall for space-based gravitational wave observatories: LISA pathfinder results *Phys. Rev. Lett.* **116** 231101
- [38] Roura A 2020 Gravitational redshift in quantum-clock interferometry *Phys. Rev. X* **10** 021014
- [39] Zanoni A P, Burkhardt J, Johann U, Aspelmeyer M, Kaltenbaek R and Hechenblaikner G 2016 Thermal performance of a radiatively cooled system for quantum optomechanical experiments in space *Appl. Therm. Eng.* **107** 689–99
- [40] Bateman J, Nimmrichter S, Hornberger K and Ulbricht H 2014 Near-field interferometry of a free-falling nanoparticle from a point-like source *Nat. Commun.* **5** 4788
- [41] Bykov D S, Mestres P, Dania L, Schmöger L and Northup T E 2019 Direct loading of nanoparticles under high vacuum into a Paul trap for levitodynamical experiments *Appl. Phys. Lett.* **115** 034101
- [42] Nikkhou M, Hu Y, Sabin J A and Millen J 2021 Direct and clean loading of nanoparticles into optical traps at millibar pressures *Photonics* **8** 11
- [43] Schmid P et al Trapped nanoparticles for space experiments Technical report Study under contract with ESA, AO/I-6889/11/NL/CBi (2012–2014)
- [44] Oi D K L, Ling A, Grieve J A, Jennewein T, Dinkelaker A N and Krutzik M 2017 Nanosatellites for quantum science and technology *Contemp. Phys.* **58** 25–52
- [45] Kiałka F, Fein Y Y, Pedalino S, Gerlich S and Arndt M 2022 A roadmap for universal high-mass matter-wave interferometry *AVS Quantum Sci.* **4** 020502
- [46] Stickler B A, Papendell B, Kuhn S, Schriniski B, Millen J, Arndt M and Hornberger K 2018 Probing macroscopic quantum superpositions with nanorotors *New J. Phys.* **20** 122001
- [47] Schäfer J, Rudolph H, Hornberger K and Stickler B A 2021 Cooling nanorotors by elliptic coherent scattering *Phys. Rev. Lett.* **126** 163603
- [48] Penny T W, Pontin A, Barker P F 2021 Sympathetic cooling and squeezing of two co-levitated nanoparticles (arXiv:2111.03123)
- [49] SierraLobo 2015 CryoCube (available at: [www.sierralobo.com/cryocube/](http://www.sierralobo.com/cryocube/))
- [50] Wuenschel H 1970 Unique manufacturing processes in space environment. The Space Congress® Proceedings (available at: <https://commons.erau.edu/space-congress-proceedings/proceedings-1970-7th/session-7/4/>)
- [51] Melfi Jr L T, Outlaw R, Hueser J and Brock F 1976 Molecular shield: An orbiting low density materials laboratory *J. Vac. Sci. Technol.* **13** 698–701
- [52] Strozier J, Sterling M, Schultz J and Ignatiev A 2001 Wake vacuum measurement and analysis for the wake shield facility free flying platform *Vacuum* **64** 119–44