

Emergence of Classicality from Quantum  
Physics:  
Continuous Spontaneous Measurement  
xxx  
(extension of the accepted Thesis)

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## Preface

For occasional readers, not familiar with the academic system in Hungary, this booklet needs preliminary explanation. The Hungarian Academy of Sciences offers the particular degree "Doctor of Academy" for successful researchers. The degree is informally considered an equivalent of professorship for those who work at research institutes instead of universities. The rules to apply for the above degree differ definitely from the common procedure of university habilitation. They are much like a Ph.D. procedure. A wise option permits one a brief summary to write instead of a full "Thesis"...

Hungarian version of this booklet brought about frustrating criticism by experts devoted to purest academic values and national scientific standards. 7 years later, with the same devotion in my heart, I revise and extend the original material, without the intention to include all new aspects that have appeared in the meantime.

# 1 Introduction

According to widespread views, quantum mechanics is in perfect agreement with all definite physics experiments. None of them forces us either to correct or to complete the theory. On the other hand, it has been recognized since the earliest time of understanding quantum mechanics that it contradicts our general macroscopic world view. Attempts to reconcile quantum theory with our everyday macroscopic experience have always been the subject of investigations. I realized in the early nineties (1992) that quantum cosmology justifies one's longstanding discontent with the notorious concept of "wave function reduction". The issue grew physical.

Seldom did I enter the interpretational debates on the issue. Rereading my original papers and recollecting my old talks I dare say that my equations constitute a consistent story while my motivations and interpretations are more eclectic. Distinguished reviewers from various fields, e.g. Bell (1990), Khalfin (1992), Penrose (1994), Hawking and Penrose (1996), Giulini *et al.* (1996), Percival (1997,1998), Carmichael (1997), Plenio and Knight (1998) quote my results when presenting their own philosophies. Readers of expertise different from quantum fun-

damentals or quantum optics may, from the above works, get an insight into a piece of contemporary physics: the context of my Thesis.

In Part 2, I formulate the issue of reconciliation between the quantum theory and the macroscopic world view in such terms that an efficient research strategy, in Part 3, derives from it. The main items of my research, along with some historically related works, are then presented in Parts 4-7.

## 2 The issue

Quantum mechanics has turned out to be a *dichotomic* theory. In standard theory, the wave function  $\psi$ , describing the state of the closed quantized system, evolves continuously and deterministically according to Schrödinger's equation:

$$\dot{\psi} = -\frac{i}{\hbar}H\psi, \quad (1)$$

here  $H$  is the Hamilton-operator. Obviously enough, the wave function  $\psi$  needs an interpretation in classical terms. In other words, one should specify how classical concepts ("classicality") emerge from the quantum. According to the present state of art in quantum theory, the emergence of classicality shall be explained by the

concept of *quantum measurement*.

Whenever the quantized system is brought into contact with a measuring apparatus capable to measure a Hermitian observable  $A$  the wave function  $\psi$  will change at random. The measured "classical" value is equal to one of the eigenvalues  $\lambda_z$  of  $A$  and an instantaneous *reduction* of the wave function will take place into the corresponding eigenstate  $\phi_z$ :

$$\psi \rightarrow \phi_z \quad , \quad (2)$$

where the different eigenstates  $\{\phi_z; z = 1, 2, \dots\}$  are orthogonal to each other. Hence, the outcome of quantum measurement is stochastic. The probability of the  $z$ 'th outcome is

$$w_z = |\langle \phi_z | \psi \rangle|^2. \quad (3)$$

For continuous observable  $A$  the perfect reduction (2) never takes place. A *localization* effect will substitute it, i.e., the measurement makes the quantum uncertainty  $\Delta A$  decrease to a finite value instead of zero. The ultimate form of the above theory of quantum measurement belongs to von Neumann.

If quantum theory is believed to be universal, i.e. valid for microscopic as well as macroscopic systems, then it should not be dichotomic. The measurement theory should not be an independent discipline. Rather it should, for

instance, be incorporated into a suitably modified version of the Schrödinger-equation (1).

We shall concentrate on *two* of the many other *issues* related to quantum measurements. First is that the standard measurement theory specifies the result of the measurement but does not specify the *process* leading to it. Second, the emergence of the universal classicality of the common macroscopic world could be explained from a spontaneous and *universal* measurement which exercised by the Nature on itself.

### 3 My Research Strategy

In the second half of the eighties, I concentrated my researches on the concrete tasks which followed from the chosen particular formulation of the fundamental issues. Such a formulation was relevant and fertilizing, first of all because the corresponding technical tasks turned out to be solvable.

- I constructed a flexible mathematical model for continuous measurement (see Part 4).
- Based on plausible physical motivations, I modified the quantum theory in order to model the universal reduction of the wave function (see Part 5).

The ultimate role of these achievements (as well as

those by other people) in removing the dichotomy of the standard quantum theory is, even today, an open question. In the early nineties, nonetheless, it was worth to explore the perspectives of the concept beyond the non-relativistic regime.

- I investigated the possibility of relativistic invariant models for (continuous) quantum measurement (see Part 6).

In the meantime, Gell-Mann and Hartle developed a new concept to explain the emergence of classicality from the quantum, alternative to the von Neumann measurement theory. The ultimate goal of their theory of *decoherent histories* is an explanation of the emergence of classical cosmology from a fully quantized Universe. This concept became so much appreciated in the literature that I felt unavoidable to discuss its relation to my approach.

- I investigated the relation between the concepts of decoherent history and continuous quantum measurement (see Part 7).

## 4 The model of non-relativistic continuous measurement

The fundamental dichotomy of quantum theory can,

in one way, be removed if we treat the Schrödinger evolution (1) and the reduction process (2,3) parallelly and we unify them into a single process. This is called *continuous measurement*. It is modeled by two coupled stochastic equations. One is a Schrödinger equation for  $\psi(t)$ , modified by certain stochastic terms:

$$\frac{d\psi}{dt} = -\frac{i}{\hbar}H\psi + \textit{stochastic nonlinear term}. \quad (4)$$

The other one is a stochastic equation for the continuously measured value  $z(t)$  of the observable  $A$ . It turns out to have the form:

$$z = \langle \psi | A | \psi \rangle + \textit{stochastic term}. \quad (5)$$

The above model originates, on one hand, from the formal unification of the Schrödinger evolution and von Neumann reduction. It becomes, on the other hand, the true theory of quantum measurement wherever the measuring apparatus really detects permanently, realizing an (unsharp) continuous measurement. Indeed, the joint stochastic evolutions of the measured wave function  $\psi(t)$  and of the measurement record  $z(t)$  are described by equations like (4) and (5).

All relevant equations (4,5) are constrained by *master equations*. If we introduce the system's density operator



$\rho$  as the stochastic mean of the pure quantum states, i.e.,:

$$\rho(t) = M[|\psi(t)\rangle\langle\psi(t)|] \quad (6)$$

then the stochastic evolution of  $\psi(t)$  must be such that  $\rho(t)$  satisfy a linear Liouville equation:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \text{linear term}. \quad (7)$$

In many cases of research, one finds first the master equation above. Then one proceeds to the construction of the pure state stochastic equation (4) as well as of the equation for the measured record (5). This latter procedure is called the *stochastic unraveling* of the master equation (7).

Formally, there are legions of stochastic unravelings for the same master equation. One therefore requires that the rule of constructing equations (4,5) from (7) be independent of the mathematical representation of (7). We speak about *invariant unraveling* in this case. Surprisingly, there is just a single Gauss and, alternatively, a single Poisson stochastic unraveling for a given master equation. They are called *quantum state diffusion* or *orthojumps*, respectively.

History, related works

As for the equation (4), Bohm and Bub (1966) propose deterministic evolution equation with random initial conditions. Non-linear Wiener process is introduced by Pearle (1982), he finds

an amazing gambler's game analogy, too. Gisin (1984,1989) derives the linearity of the master equation (7) from the principle of locality; constructs the first nonlinear Wiener process satisfying (7). I (1988a) prove how a unique nonlinear (Wiener or, alternatively, Poisson) process follows from (7). Gisin and Percival (1992,1993) presents the Ito equations of the unique Wiener process. This has been known as quantum state diffusion theory.

As for the mathematical model of unsharp continuous von Neumann reductions, the idea of restricting Feynman-path integrals for a tube along  $z(t)$  is due to Mensky (1979). Barchielli *et al.* (1982,1983) introduce Gaussian "tubes" and derive generating functionals for the distribution functions of the processes  $\psi(t)$ ,  $z(t)$ . Caves and Milburn (1987) invent the feed-back mechanism in the path-integral formalism. I (1988ab) derive 2 separate Ito-equations: one for  $\psi(t)$  and one for  $z(t)$ . The first one turns out to be Gisin's fenomenologic equation (1984). Belavkin and Staszewski (1992) derives this couple of equations from Belavkin's quantum filtering theory. I *et al.* (1995) prove that the standard quantum state diffusion equation for  $\psi(t)$  corresponds to continuous measurement of the Lindblad generators, accompanied by a certain feed-back *à la* Caves and Milburn (1987).

These stochastic Schrödinger equations, invented for fundamental purposes, are in the nineties reintroduced in quantum optics. The concept, called *quantum trajectories*, is pioneered by Dalibard *et al.* (1992), and by Carmichael (1993). An increasing number of applications and reviews have since been appearing, see Bocko (1996), Carmichael (1997), Plenio (1998).

My contribution

T1. (Diósi 1988a). I proved that, in modified quantum theories, the linearity of the master equation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \textit{linear term}$$

of the density operator could only be violated at the price of altering the usual set of observables. I proved the existence of a unique Gauss and a unique Poisson  $\psi$ -valued process, *unraveling* the same master equation. I constructed these processes. Via simple examples I proved that, for long times, both  $\psi$ -valued processes approach the standard von Neumann wave function reduction.

T2). (Diósi 1988b). In a heuristic model of continuous position measurement of a free particle, where the master equation takes the form:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] - \frac{\gamma}{2}[q, [q, \rho]],$$

I pointed out that the wave function  $\psi(t)$  and the measured record  $z(t)$  are governed by the following pair of Ito-stochastic differential equations:

$$\begin{aligned} \frac{d\psi}{dt} &= -\frac{i}{\hbar}H\psi - \frac{\gamma}{2}(q - \langle q \rangle)^2 \psi + w\sqrt{\gamma}(q - \langle q \rangle) \psi, \\ z &= \langle q \rangle + \frac{1}{2\sqrt{\gamma}}w, \end{aligned}$$

with  $\langle q \rangle = \langle \psi | q | \psi \rangle$ . Here  $H$  is the free particle's Hamiltonian,  $q$  is the position operator, and  $w(t)$  is the standard Wiener process. The constant  $\gamma$  controls the accuracy of the continuous measurement. Thus I opened the way to the general application of Ito equations in the theory of continuous quantum measurement.

T3). (Diósi 1988c). I pointed out that the stationary solutions of the Ito-Schrödinger equations of continuous position measurement for an otherwise free particle are localized wave packets of widths  $\sigma = (\hbar/4\gamma m)^{1/4}$ , imitating classical trajectories of the particle ( $m$  is the particle's mass). I showed that the wave packets perform tiny random walks along the classical trajectories:

$$\frac{d\langle p \rangle}{dt} = \hbar\sqrt{\gamma}w$$

$$\frac{d\langle q \rangle}{dt} = \frac{\langle p \rangle}{m} + 2\sqrt{\gamma}\sigma^2w$$

where  $\langle p \rangle, \langle q \rangle$  are the canonical coordinates of the wave packet's center.

## 5 Universal continuous measurement

In everyday experiences, Nature shows spontaneous classicality, at least for macroscopic systems. If the quantum theory is taken valid universally then, according to

its standard rules, no classicality could emerge from the wave function but via the von Neumann reductions (2,3) which postulate the presence of measuring apparatuses. This is not likely to be a reasonable explanation for everyday classicality of macroscopic systems. On the other hand, macroobjects are never isolated and their environment acts formally as apparatus. This sort of spontaneous (and usually continuous) measurement is called *environmental measurement*. But we can hardly accept that classicality is only due to the particular, e.g., thermal environment. A certain "environment", acting everywhere, is to be assumed that guarantees the observed universality of spontaneous continuous reduction (i.e., of spontaneous and universal emergence of classicality).

*Universal* continuous measurement will, technically, be described by stochastic equations like (4) and (5). For microscopic systems, the (suitably generalized) equation (4) should asymptotically reduce to the Schrödinger equation (1). For macroscopic systems it should always tend to the corresponding dynamic equations of classical mechanics. Furthermore, these universal equations should automatically imply the quantum mechanical measurement (2,3) whenever they applied to a coupled pair of a measured microsystem and of the macroscopic measuring

apparatus. By means of this desired theory of continuous and universal quantum measurement one obtains unified description of both the microscopic and macroscopic worlds. The distinguished interpretation of measurement is not needed anymore. The *dichotomy* disappears from quantum theory.

The basic assumption in universal continuous measurement models is that about the spontaneously measured Hermitian observables. An *ad hoc* choice could be the position operators of all elementary constituents. A better choice, motivated by gravitational considerations, is the operators of non-relativistic mass distribution  $f(r)$  at each points  $r$ . The strength of the measurement can thus be set by the Newton constant  $G$  of gravity. First, one constructs the master equation (7). This can most easily be done by starting with the hypothesis that the classical Newtonian gravitational field has a tiny white noise fluctuation with correlation

$$\frac{\hbar G}{|r - r'|} \delta(t - t'). \quad (8)$$

Then the master equation (7) takes the form:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar} [H, \rho] - \frac{G}{2\hbar} \iint [f(r), [f(r'), \rho]] \frac{d^3r d^3r'}{|r - r'|}. \quad (9)$$

The interpretation of the mass density  $f(r)$  needs special

care. The master equation (9) diverge for point-like particles. A certain cutoff is needed. Hence, we replace the sharp distribution  $f(r)$  by the coarse grained one:

$$f(r) \rightarrow (2\pi\Delta)^{-3/2} \int f(r+x) \exp\left(-\frac{x^2}{2\Delta^2}\right) d^3x \quad (10)$$

where  $\Delta$  is thought to be about  $10^{-5}$  centimeter. For condensed matter, in particular, the microscopic details become irrelevant and  $f(r)$  is simply the operator of the macroscopic density.

The master equation (9) has all the necessary properties. The term, proportional to the Newton constant, can completely be neglected for atomic systems. For macroscopic systems, however, it destroys coherence between macroscopically different mass distributions. In particular, a superposition

$$\psi = \frac{\phi_1 + \phi_2}{\sqrt{2}} \quad (11)$$

where  $\phi_1, \phi_2$  represent mass distributions  $f_1(r)$  and  $f_2(r)$ , respectively, will decay at the characteristic time scale

$$\tau = \frac{\hbar}{E_{11} + E_{22} - 2E_{12}}. \quad (10)$$

$E_1$  and  $E_2$  are the Newtonian self-energies of the respective mass distributions  $f_1(r), f_2(r)$  while  $E_{12}$  is the Newtonian interaction energy of them. If, for instance,  $\phi_1$

and  $\phi_2$  belong to the center of mass wave functions of a solid of size  $R$ , situated far away from each other then

$$\tau = \text{const} \times \frac{\hbar R}{Gm^2} \quad (11)$$

which would yield some  $10^{-19}$  seconds for ordinary objects of size about 1 centimeter. The very short decay time reassures that such a macroscopic superposition can never be created within our model. Hence a most crucial condition for the emergence of everyday classicality fulfills.

#### History, related works

Feynman (1962) puts forward an idea that gravity might be responsible for the emergence of macrophysics from the quantum. Károlyházy (1966) outlines, in sketchy formalism, a stochastic process where unnatural quantum superpositions decay by gravitational fluctuations of a special spectrum. Zeh (1970) argues in favor of environmental spontaneous measurement. I and Lukács (1985) derive a plausible white-noise spectrum (8) for gravitational fluctuations. Incorporating it, I construct (1986) stochastic equations of generalized Poisson type for the wave function. For macroscopic superpositions, I (1987) derive a decay time (10), also favored by Penrose (1994,1996), proportional to Newton's constant. Ghirardi *et al.* (1986) propose a simple Poisson process (the GRW-model), corresponding to universal unsharp position measurements on microscopic constituents. I propose (1989) gravitational Ito-Schrödinger equations, for the universal measurement of mass distribution, in the framework



of quantum state diffusion theory. Gisin (1989), Ghirardi *et al.* (1990a) construct independently their Ito-stochastic equations, unrelated to gravity. Sánchez-Gómez (1994) and Percival (1994) suggest further models of universal wave function reduction, related to space-time uncertainties.

Károlyházy *et al.* (1982) suggest experiment aboard satellite to test his theory (1966). Ghirardi *et al.* (1990b) point out that a simple cutoff, equal to the GRW length parameter (1986) is needed for the consistency of my theory (1989). Bell (1990), in his last paper, as well as Penrose (1994,1996), give chances to a theory based on my realization of the original GRW (1986) concept. Pearle and Squires (1994) prove that nucleon-decay experiments disclose GRW and favor (a version of) my theory (1989). Bose *et al.* (1997) finds that testing my theory (1989) in quantum optics is still beyond current technology. Percival and Strunz (1997) proposes atomic interference to test Percival's universal theory. After 1993, an incurable cutoff problem in the Károlyházy model (1966) is repeatedly noticed by myself and Lukács (1993ab), Rosales and Sánchez-Gómez (1995), Percival (1995), and Fu (1997).

#### My contribution

T4). (Diósi 1988d). I pointed out that a natural limit of the circuitous jump process of the original GRW theory corresponds to the concept of universal continuous measurement of the particles' positions. I applied this limit theory to macroscopic rigid bodies and calculated the coherent localization of the rotating angles.

T5). (Diósi 1987). I constructed a master equation, modifying the standard quantum mechanics, taking the hypothetical white-noise fluctuations of gravity into the account:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] - \frac{G}{2\hbar} \iint [f(r), [f(r'), \rho]] \frac{d^3r d^3r'}{|r - r'|}$$

where  $f(r)$  is the operator of mass density at point  $r$  and  $G$  is the Newton constant. This equation has since been the best candidate equation capable to account for the spontaneous and universal emergence of classicality.

T6). (Diósi and Lukács 1989). I revisited the derivation of the minimum gravitational uncertainties, pointing out the reasons why Károlyházy had got the result

$$\Delta s \sim s^{2/3},$$

different from mine:

$$\Delta s \sim \sqrt{s},$$

where  $s$  is the geodetic length and  $\Delta s$  is its fundamental uncertainty. I forecasted that Károlyházy's option might not estimate consistently the strength of bulk gravitational fluctuations.

T7). (Diósi 1989). I constructed the theory of universal continuous measurement of the non-relativistic mass

distribution  $f(r)$ . I verified (on examples common from related works) that the theory reduces for ordinary quantum mechanics for single microsystems and, on the other hand, it accounts for the emergence of classical physics for macroobjects. I proved, in particular, that a superposition

$$\psi = \frac{\phi_1 + \phi_2}{\sqrt{2}}$$

where  $\phi_1, \phi_2$  represent mass distributions  $f_1(r)$  and  $f_2(r)$ , respectively, will randomly reduce into  $\phi_1$  or  $\phi_2$  at the characteristic time scale

$$\tau = \frac{\hbar}{E_{11} + E_{22} - 2E_{12}}.$$

$E_1$  and  $E_2$  are the Newtonian self-energies of the respective mass distributions  $f_1(r), f_2(r)$  while  $E_{12}$  is the Newtonian interaction energy of them.

## Beyond Markovian Approximation

The theory of continuous measurement of the eighties relies upon the validity of Markovian approximation of the system's dynamics. This is, in the presence of continuous measurement, not exactly the case. Just oppositely, non-Markovian counterparts of the equations (4-7) must be considered. The study of non-Markovian generaliza-

tion is the key to any relativistic model of continuous measurement.

Non-Markovian master equations can most easily be obtained as the reduced dynamics of a system coupled to an environment consisting of harmonic oscillators. A typical interaction Hamiltonian is

$$H_{int} = q \sum_{\omega} x_{\omega} \equiv qX \quad (12)$$

where  $q$  is the system's coordinate operator and the  $x_{\omega}$ 's are the coordinate operators for the environment oscillators with the corresponding frequencies  $\omega$ . The system's density operator satisfies a linear non-Markovian master equation. Assume that the initial state is the product of the system's initial state  $\rho_0$  and the oscillators' thermal equilibrium state  $\rho_{thermal}$ . The exact master equation reads, in Feynman integral form:

$$\begin{aligned} \rho(t) = & \iint D_F[q] D_F[q'] |q_t\rangle \langle q'_t| \times \\ & \exp\left\{-\int_0^t ds \int_0^s dr (q_s - q'_s) [\alpha(s, r) q_r - \alpha(r, s) q'_r]\right\} \langle q_0 | \rho_0 | q'_0 \rangle, \end{aligned} \quad (13)$$

where  $\alpha(s, r)$  stands for the correlation function in thermal equilibrium:

$$\alpha(r, s) = Tr[X(r)X(s)\rho_{thermal}]. \quad (14)$$

In the high temperature limit this equation reduces to the Markovian form (7).

Generalizing the Markovian case, the exact master equation (13) also possesses a stochastic unraveling in terms of non-Markovian stochastic equations.

$$\frac{d\psi(t)}{dt} = -\frac{i}{\hbar}H\psi(t) + \xi(t)q\psi(t) - q \int_0^t \alpha(t, s) \frac{\delta\psi(t)}{\delta\xi(s)} ds \quad (14)$$

where  $\xi(t)$  is a colored complex Gaussian noise of correlation

$$M[\xi^*(t)\xi(s)] = \alpha(t, s). \quad (15)$$

These non-Markovian results can formally be extended for quantum field theories. In particular, we can identify the system and its environment with the relativistic quantized electron-positron field and the quantized photon field, respectively. Then an exact master equation can, e.g. in superoperator formalism, be written down for the system's density operator:

$$\rho(t) = Texp\left\{\frac{i}{2} \int_{x_0 \langle t} dx \int_{y_0 \langle t} dy [J_+(x)D_F(x-y)J_+(y) + J_-D_{\bar{F}}J_- - J_+D_+J_- - J_-D_-J_+]\right\}\rho(-\infty), \quad (16)$$

where the  $D$ -terms denote standard photon propagators and  $J_{\pm}$  stand for the superoperator of the electromagnetic

current.

#### History, related works

Master equations, governing the evolution of the density operator, contain superoperators acting on the space of (ordinary) operators. For a system embedded into a larger one (environment), Zwanzig (1960) introduces projection superoperators and derives the generic form of exact non-Markovian master equations. In Feynman's path integral formalism, Caldeira and Leggett (1983) derive an exact non-Markovian master equation (13) for a system embedded into thermal harmonic oscillator environment. In irreversible field and many-body theories, independent investigations are performed. Various theories and formalisms of quantum irreversibility appear under different titles, like Schwinger (1961) and Keldysh (1965) closed time-path formalisms, Umezawa's (1982) thermofield formalism. I (1990a) apply a superoperator method to quantum-electrodynamics, obtain the exact relativistic master equation (16) of the electron-positron field. I (1993ab) derive the Lindblad-form of the Caldeira-Leggett equation's Markovian limit, using superoperator formalism.

Exact stochastic unraveling of non-Markovian master equations is considered hopeless a goal until I present a formal construction (1990b,1994a). Strunz (1996) finds a path-integral expression unraveling the Caldeira-Leggett master equation. I and Strunz (1997) construct the corresponding non-Markovian stochastic Schrödinger equation. Strunz, I, and Gisin make this equation suitable for Monte-Carlo simulations and apply it to a number of concrete examples (1998).

My contribution

T8). (Diósi and Strunz 1997). I constructed the non-Markovian stochastic Schrödinger equation

$$\frac{d\psi(t)}{dt} = -\frac{i}{\hbar}H\psi(t) + \xi(t)q\psi(t) - \int_0^t \alpha(t, s) \frac{\delta\psi(t)}{\delta\xi(s)} ds$$

unraveling the exact Caldeira-Leggett master equation. Here  $H$  is the system Hamiltonian,  $q$  is the system coordinate coupled to the heat bath, and  $\alpha(t, s)$  is the equilibrium correlation function of the bath's coordinate coupled to  $q$  linearly. I pointed out that this equation reduces to the usual Markovian quantum state diffusion theory in the limit when  $\alpha(t, s)$  approaches  $\delta(t - s)$ .

T9). (Diósi 1990a). I constructed an exact field-theoretical master equation for the density operator  $\rho$  of the relativistic electron-positron system, integrating over the photonic degrees of freedom:

$$\rho(t) = T \exp \left\{ \frac{i}{2} \int_{x_0 \langle t} dx \int_{y_0 \langle t} dy [J_+(x) D_F(x-y) J_+(y) + J_- D_{\bar{F}} J_- - J_+ D_+ J_- - J_- D_- J_+] \right\} \rho(-\infty),$$

where  $T$  stands for time-ordering,  $J$  is the superoperator of the electromagnetic current, and the  $D$ -terms denote standard photonic propagators.

## Toward quantum cosmology

The problem of fundamental dichotomy of quantum theory and the related measurement problem arose originally at non-relativistic level. Later, however, it became clear that the theory of relativity and especially the principle of locality-causality imply serious constraints on any alternative theories of measurement. Finally, when we introduce a single wave function for the whole Universe, then the dichotomic quantum theory fails completely. Why, there is nothing but the Universe and its wave function! Nothing like a separate measuring apparatus, or a separate environment can be invoked to explain the emergence of classicality from the Universe's wave function.

No doubt the problem of dichotomy, which used to be purely a matter of philosophical discontent for many experts, grows concrete and physical. Yet, there is no generally appreciated concept to treat the problem. One can, for instance, explore a possible relativistic generalization of the concept of continuous measurement, so successful in the non-relativistic regime. A seemingly independent concept, that of *consistent histories* promises also a solution. It was devised to interpret the fully quantized Universe. It is, nonetheless, a theory for ordinary closed



quantum systems as well. Its objective is substituting von Neumann measurement theory with another scheme which does not refer to anything like measurements.

The concept of continuous measurement and decoherent histories turns out to be closely related to each other. Central to the latter is the decoherence functional

$$D[a, a'] = \text{Tr}[h_a h_{a'} \rho] \quad (17)$$

defined between two histories  $h_a$  and  $h_{a'}$ , respectively. A given quantum state  $\rho$  can consistently be interpreted in terms of decoherent histories  $\{h_a\}$  if and only if the decoherence functional (17) is diagonal. This is, in particular, expected in certain subsystems of a larger one. The subsystem's density operator, at certain conditions, obeys to Markovian master equation like equation (7) in the theory of continuous measurement. If we unravel this evolution according to the quantum state diffusion equations, i.e. we construct the solutions of the corresponding stochastic Schrödinger equation (4) then these solutions will allow for a diagonal decoherence functional (17). Namely, the physically unique set of variables that localize in the quantum state diffusion picture also define an approximately decoherent set of histories in the decoherent history approach. The degree of localization is related to the degree of decoherence, and the probabilities

for histories prescribed by each approach are essentially the same. What the theory of continuous measurement singles out of the infinity of quantum histories are just those consistent in the sense of the entirely independent concept of Gell-Mann and Hartle.

#### History, related works

Motivated by the discontent with ordinary quantum measurement theory, Griffith (1984) suggests a history interpretation of quantum states. Closely related to his theory, Gell-Mann and Hartle (1990) brings up the theory of decoherent histories, this time the motivation is definitely the quantum cosmology issue. I *et al.* (1995) point out that the continuous measurement formalism (quantum state diffusion, in particular) is physically equivalent with the formalism of decoherent histories. I (1994b) prove that the orthojump process generates exact decoherent histories. Brun (1997ab) demonstrates this connection on a realistic quantum optical model.

#### My contribution

T10). (Diósi, Gisin, Halliwell, and Percival 1995). I demonstrated a close connection between the decoherent history approach to quantum mechanics and the quantum state diffusion picture. I proved that the physically unique set of variables that localize in the quantum state diffusion picture also define an approximately decoherent set of histories in the decoherent history approach. The degree of localization is related to the degree of de-

coherence, and the probabilities for histories prescribed by both approaches are essentially the same.

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