Relativistic formulation of multiple localized quantum measurements

Lajos Diósi

Wigner Center, Budapest

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Von Neumann detector

S+D initial state $|in\rangle|D\rangle$
measured S observable $\hat{A}$, measured value A;
von Neumann Detector $[\hat{q},\hat{p}]=i$, $\hat{H}_D=0$; pointer $\hat{p}$
von Neumann coupling: $\hat{H}(t)=-\delta(t)\hat{q}\hat{A} \Longrightarrow \hat{S}=\exp[i\hat{q}\hat{A}]$

$$\hat{p}^f=\hat{S}^\dagger\hat{p}^i\hat{S}=\hat{p}^i+\hat{A}$$

If $\langle\Psi|\hat{p}^i|\Psi\rangle=0$, $\langle\Psi|(\hat{p}^i)^2|\Psi\rangle=\sigma^2$ then $A=p^f\pm\sigma$.

$$w(A)=\langle\Psi|\delta(A-\hat{p}^f)|\Psi\rangle$$

D's initial wave function: $\langle p|D\rangle=C\times\exp[-p^2/4\sigma^2]$
Key mechanism: $\langle p|\hat{S}|D\rangle=C\times\exp[-(p-\hat{A})^2/4\sigma^2]$
Outcome distribution without D variables:

$$w(A)=C^2\langle in|\exp[-(p-\hat{A})^2/2\sigma^2]|in\rangle$$

Many von Neumann detectors

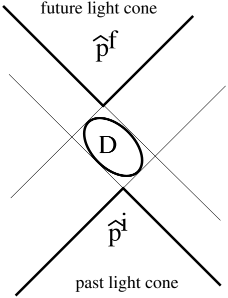
S+D initial state $|in\rangle|D\rangle$
measured S observables $\hat{A}_1, \hat{A}_2, \dots \hat{A}_N$, at different times
measured values $A_1, A_2, \dots, A_N$
N separate von Neumann Detectors
Outcome distribution without D variables:

$$w(A) = \langle out; A|out; A\rangle$$

$$|out; A\rangle = \mathcal{T}\prod_{n=1}^{N} C_N \exp[-(A_n - \hat{A}_n)^2/4\sigma_n^2]|in\rangle$$

$\mathcal{T}$ =time-ordering of $\hat{A}_n$'s.
Advantage of this form: valid relativistically too.
Same math as GRW, where $\sigma_n \equiv 10^{-5} cm$*,* $\hat{A}_n =$ *any constituent position taken (hit) at rate* $\lambda = 10^{-17} Hz$*.*

Von Neumann detector in space-time



$$\hat{p}^f = \hat{p}^i + \hat{A}$$

space–time points $x=(t,\mathbf{r})$

relativistic quantum field $\hat{\Phi}(x)$

measured observable $\hat{A}$

$$\hat{A} = \int g(x)\, \hat{\Phi}(x) d^4x$$

local coupling $g(x)$

von Neumann Detector $[\hat{q},\hat{p}]=i,\ \hat{H}_D=0$

initial/final value of pointer $\hat{p}^{i/f}$

von Neumann coupling:

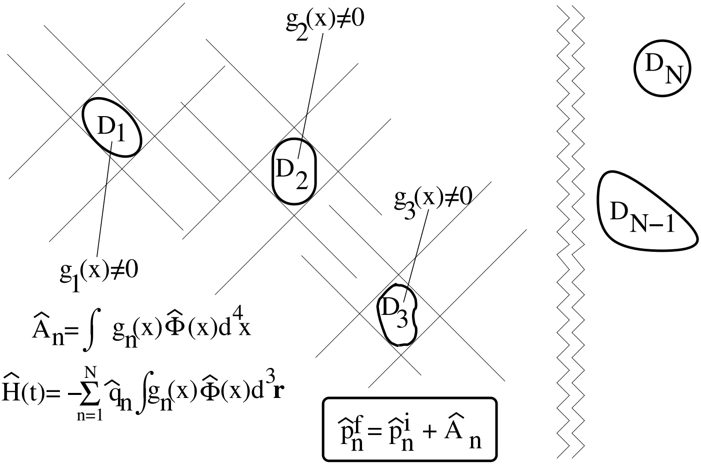
$$\hat{H}(t) = -\hat{q} \int g(x)\, \hat{\Phi}(x) d^3\mathbf{r}$$

$$d\hat{p}/dt = i[\hat{H},\hat{p}] = \int g(x)\, \hat{\Phi}(x) d^3\mathbf{r}$$

If $\langle\Psi|\, \hat{p}^i\, |\Psi\rangle = 0, \quad \langle\Psi|\, (\hat{p}^i)^2\, |\Psi\rangle = \sigma^2$ then $A = p^f \pm \sigma$

$$w(A) = \langle\Psi|\, \delta(A - \hat{p}^f)\, |\Psi\rangle$$

Many von Neumann detectors in space-time



$$\hat{A}_n = \int g_n(x)\hat{\Phi}(x)d^4x$$

$$\hat{H}(t) = -\sum_{n=1}^{N} \hat{q}_n \int g_n(x)\hat{\Phi}(x)d^3\mathbf{r}$$

$$\hat{p}_n^f = \hat{p}_n^i + \hat{A}_n$$

$$w(A_1,A_2,\dots,A_N) = \langle\Psi|\,\delta(A_1-\hat{p}_1^f)\,\delta(A_2-\hat{p}_2^f)\dots\delta(A_N-\hat{p}_N^f)\,|\Psi\rangle$$

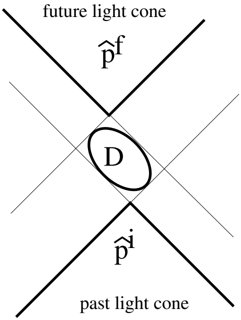
Elimination of the detector variables

Composite initial state ($N = 1$): $|\Psi\rangle = |in\rangle|D\rangle$
Interaction, observable:

$$\hat{H}(t) = \hat{q}\int g(x)\hat{\phi}(x)d^3\mathbf{r}, \quad \hat{A} = \int g(x)\hat{\phi}(x)d^4x$$

D's initial wave functions: $\langle p|D\rangle = C \times \exp(-p^2/4\sigma^2)$
$\hat{S}$-matrix:
$\hat{S} \equiv \mathcal{T}\exp\left(-i\int\hat{H}(t)dt\right) = \mathcal{T}\exp\left(-i\hat{q}\int g(x)\hat{\phi}(x)d^4x\right)$
Outcome distribution:
$w(A) = \langle\Psi|\delta(A-\hat{p}^f)|\Psi\rangle = \langle in|\langle D|\hat{S}^\dagger\delta(A-\hat{p})\hat{S}|D\rangle|in\rangle$
Key mechanism: $\langle p|\hat{S}|D\rangle = C\mathcal{T}\exp[-(p-\hat{A})^2/4\sigma^2]$
Otcome distribution without the detector variables:
$w(A) = C^2\langle in|\tilde{\mathcal{T}}\exp[-(A-\hat{A})^2/4\sigma^2]\mathcal{T}\exp[-(A-\hat{A})^2/4\sigma^2]|in\rangle$

Covariant form of D's outcome (single D)



$$\hat{p}^f = \hat{p}^i + \hat{A}$$

space–time points $x=(t,\mathbf{r})$

relativistic quantum field $\hat{\Phi}(x)$

measured observable $\hat{A}$

$$\hat{A} = \int g(x)\,\hat{\Phi}(x)d^4x$$

If $\langle\Psi|\hat{p}^i|\Psi\rangle=0$, $\langle\Psi|(\hat{p}^i)^2|\Psi\rangle=\sigma^2$

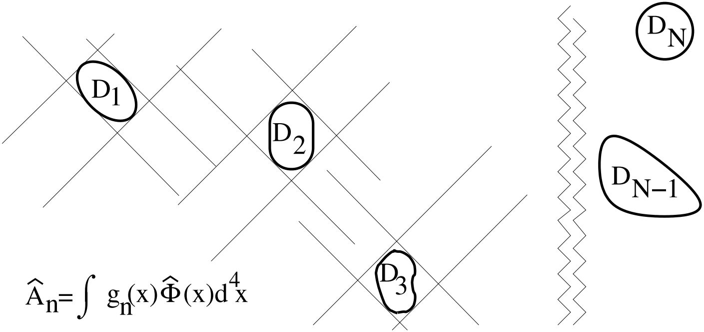
$$w(A)= \langle\Psi|\,\delta(A-\hat{p}^f)\,|\Psi\rangle$$

Covariant result:

$$w(A)=\langle out;A|out;A\rangle$$

$$|out;A\rangle= \mathcal{T}C\exp[-(A-\hat{A})^2/4\sigma^2]|in\rangle$$

Covariant form of D's outcome, many D's



$$\hat{A}_n = \int g_n(x)\hat{\Phi}(x)d^4x$$

$$w(A_1, A_2, \ldots, A_N) = \langle\Psi|\,\delta(A_1 - \hat{p}_1^f)\,\delta(A_2 - \hat{p}_2^f)\ldots\delta(A_N - \hat{p}_N^f)\,|\Psi\rangle$$

Covariant form of D's outcome:

$$w(A) = \langle out;A|out;A\rangle$$

$$|out;A\rangle = \mathcal{T}\prod_{n=1}^{N} C_n \exp[-(A_n - \hat{A}_n)^2/4\sigma_n^2]\,|in\rangle$$

Lesson for RGRW

Localized measurements of $\hat{A}_1 = \int g_1(x)\hat{\phi}(x)d^4x,\ \hat{A}_2 = \dots$
General covariant form of outcome statistics:
$w(A_1, A, 2, \dots) = \langle out; A_1, A_2, \dots | out; A_1, A_2, \dots \rangle$
$|out; A\rangle = \mathcal{T} \prod_{n=1}^N C_N \exp[-(A_n - \hat{A}_n)^2/4\sigma^2]|in\rangle$
Non-relativistically: $g_n(x) \sim \delta(t)$, $\mathcal{T} \prod \hat{A}_n = \dots \hat{A}_2 \hat{A}_1$. Then "sudden collapses" also make sense:
$|1; A_1\rangle = C_1 \exp[-(A_1 - \hat{A}_1)^2/4\sigma^2]|in\rangle$
$|2; A_1, A_2\rangle = C_2 \exp[-(A_2 - \hat{A}_2)^2/4\sigma^2]|1; A_1\rangle$
...
$|out; A_1, A_2, \dots, A_N\rangle = |N; A_1, A_2, \dots, A_N\rangle$
Exactly like GRW!
In relativistic case "sudden collapse" is nonsense.
RGRW might respect/exploit field theory.