On coexistence of Classical Continuum and Quantum Theory

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Vocabulary
The physics issue
Models - candidates
On mathematics of coexistence
Continuous "measurement"
Conjecture & Implications

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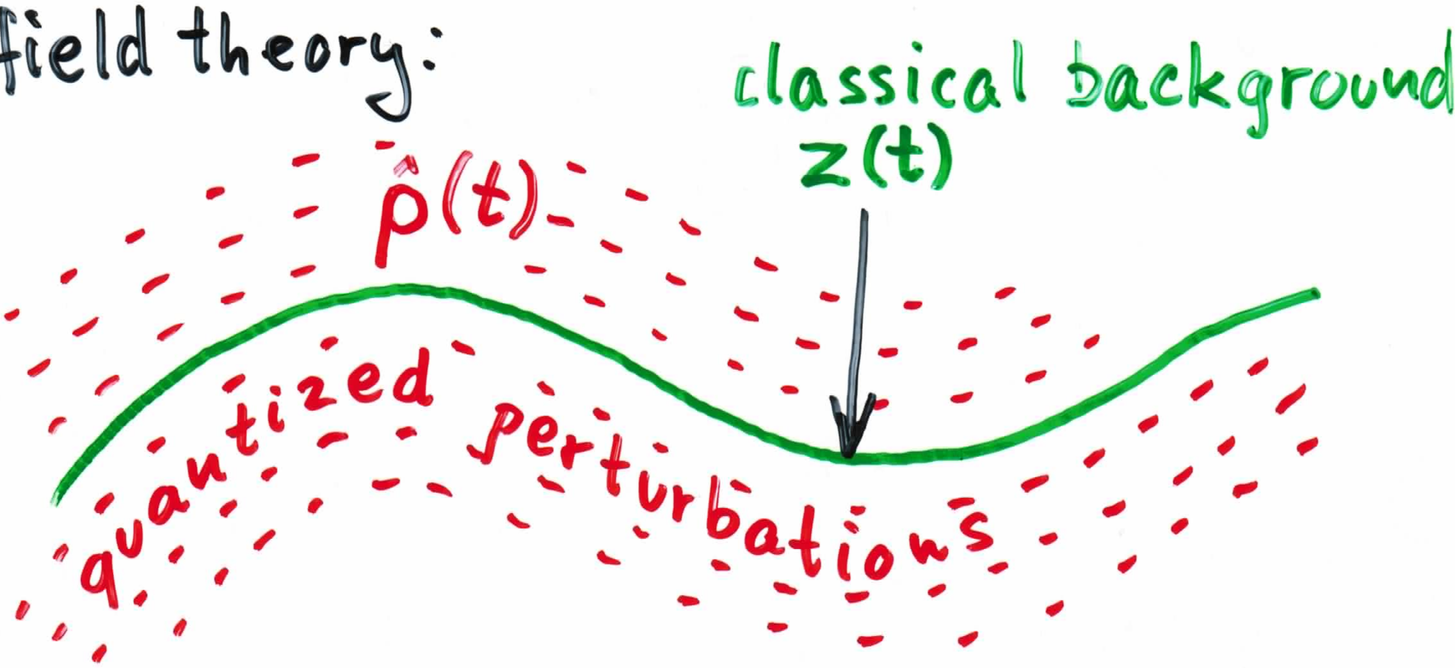
Vocabulary

- field theory:
  queen of conservative physics,
  a desired framework.
- classical continuum:
  a smooth function $z(t)$ of time.
- causality:
  perturbation at $t_2 > t_1$ has no effect at t_1.
- coexistence:
  quantum state $\rho(t)$ and
  classical continuum $z(t)$
  do depend on each other.
- measurement:
  name of mathematical procedure on ρ

The physics issue

Q-field theory:



no backreaction $\hat{p}(t) \rightarrow z(t)$

for strong perturbations
no fundamental theory

What is the main obstacle ?

- nonperturbative effects ?
- infinite degrees of freedom ?
- dynamic incompatibility ?
- Lorentz-invariance ?
-

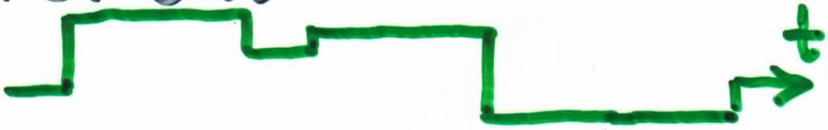
Study the coexistence of $\hat{p}(t)$ & $z(t)$!

Models-candidates for Coexsistence $\rho(t)$ & $z(t)$

- Measurement
- Mean-field
- Bohm
- Nondemolition measurement
- Decoherence
- Continuous measurement
- Decoherent histories
- Hybrid dynamics

Models-candidates for Coexistence $\rho(t)$ & $z(t)$

- **Measurement**

  $z(t) =$  — not contin.

- **Mean-field**

  $z(t) = \mathrm{tr}(q\rho(t))$ — not 'true'

- **Bohm**

  $w_t(z(t)) = \mathrm{tr}(\delta(z(t)-q)\rho(t))$ — not 'true'

- **Nondemolition measurement**

  $\{q(t);\ \text{commutative}\} \longrightarrow \{z(t)\}$ — not smooth

- **Decoherence**

  $z(t) = ?$ — elusive

- **Continuous measurement**

  $z(t) =$  — not smooth

- **Decoherent histories**

  same as • Measurement, • Cont. Meas.

- **Hybrid dynamics**

  $\dot{\rho}(z,t) = \mathcal{L}\rho(z,t)$ — not 'true'

On mathematics of coexistence

$\exists \quad \rho_t \;\&\; [z;t] \;\&\; w_t[z;t]$

Notation: $\rho_t[z;t]$ = contitional Q-state;

$$\int \rho_t[z;t] D[z;t] = \text{un-} \;\text{''}\; - \;\text{''} -$$

Time evolution:

- $\rho_t = \dfrac{1}{w_t[z;t]} M_t[z;t]\rho_0$

M is CPM $\quad \int M_t^\dagger[z;t] D[z;t] 1 = 1$

- $w_t[z;t] = \mathrm{tr}\left(M_t[z;t]\rho_0\right)$

Causality conditions:

$$w_t[z;t] = \int w_{t'}[z;t'] D[z;t',t], \quad t' > t$$

Interactivity conditions:

$$\frac{\delta \rho_t}{\delta z_{t'}}, \frac{\delta \rho_t}{\delta H_{t'}}, \frac{\delta w_t}{\delta H_{t'}} \neq 0, \quad t' < t$$

$\uparrow$ action $\qquad \uparrow$ back-action

Continuous measurement of q

- $\rho_t = \frac{1}{w_t} \mathcal{M}_t[z;t]\rho_0 = \frac{1}{w_t} V_t[z;t]\rho_0 V_t^\dagger[z;t]$

$$\partial/\partial t\, V_t = -iH_t V_t - \frac{\gamma}{2}(q - z_t)^2 V_t; \quad V_0 = 1$$

- $w_t[z;t] = \mathrm{tr}\left(V_t^\dagger[z;t] V_t[z;t]\rho_0\right)$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

ρ_t & z_t depend on each other causally

But: $z_t = \mathrm{tr}(q\rho_t) +$ white noise

i.e.: classical continuum is <u>not</u> smooth

Wash out white noise!

$$V_t[z;t] = \mathrm{T}\exp\left\{-\frac{\gamma}{2}\int_0^t (q_\tau - z_\tau)^2 d\tau\right\}$$

$$\int_0^t (q_\tau - z_\tau)^2 d\tau \longrightarrow \int_0^t\int_0^t (q_\tau - z_\tau)\tilde{\delta}(\tau - s)(q_s - z_s)\, d\tau ds$$

Causality fails:

$$\frac{\delta w_t[z;t]}{\delta H_{t'}} \neq 0 \text{ if } t' > t.$$

Conjecture

The interactive coexistence of a smooth classical continuum $z(t)$ with a quantum system $\rho(t)$ is impossible. The main obstacle is causality.

Implications

There is no local "field theory" for the interactive coexistence of $\varphi(t,\underline{x})$ and $z(t,\underline{x})$. Everyday $\underline{H}(t,\underline{x})$, $\underline{E}(t,\underline{x})$, $g_{ik}(t,\underline{x})$ are emerging via short scale acausal mechanisms, understood at FAPP level.

Who is capable of doing an acausal yet consistent theory for $z(t)$?