FRICTIONAL SCHRÖDINGER-NEWTON EQ IN MODELS OF WAVEFUNCTION COLLAPSE

Lajos Diósi, Budapest

CONTENT:

- **Is Q or G the bottle-neck of Quantum Gravity?**
- **The 'rigid ball' Schrödinger cat and its Newtonian $U(\mathrm{x}-\mathrm{x}')$**
- **Decoherence**
- **Pointer states: SN-equation?**
- **Pointer states: frSN-equation!**
- **Matching decoherence with pointer states**

PEOPLE:

- **SN-equation: D.,Jones,Moroz,Penrose,Tod,Geszti,Elze,Carlip,...**
- **FrSN-equation: D.; FrS-equation: Hasse,D.,Gisin,Pearle,GRW,... ...,Halliwell,Adler,Bassi,Kiefer**

BOTTLE-NECK OF QUANTUM GRAVITY: Q OR G?

- **Mainstream opinion: concept of space time has to be changed**
- **Sidestream opinion: concept of q-measurement has to be changed**
- **Scheme of physics building**

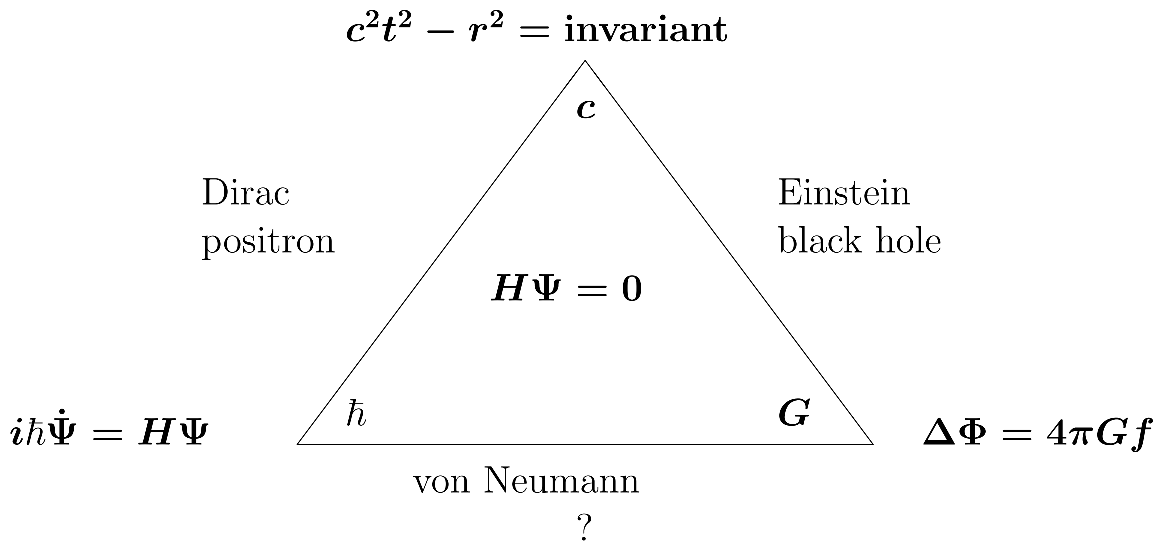


FIG. 1: c =velocity of light, G =Newton's gravitational constant, $\hbar$ =Planck constant. The corners of the triangle represent the three fundamental theories, the sides correspond to partially unified theories while the middle symbolises the fully unified theory.

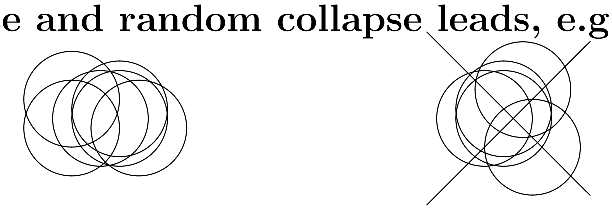
The path upto a relativistic theory of a quantised Universe *may* **go through the non-relativistic theory of Newtonian Quantum Gravity explaining the quantised motion of common macroscopic objects. One seeks for a gravity-related (but non-Hamiltonian)** *theory of spontaneous* **(i.e. non-environmental)** *collapse* **of macro-objects' wave function.**

TRADITIONALLY, TAKE THE EXAMPLE OF THE RIGID MASSIVE BALL!

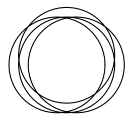
'RIGID BALL' SCHRÖDINGER CAT AND THE NEWTONIAN $U(\mathrm{x}-\mathrm{x}')$

- **Distant initial superposition:**

- **Quick decoherence and random collapse leads, e.g., to:**



- **After longer time, a pointer state is formed:**



- **We need equations! Key expression: formal Newtonian interaction potential of two hypothetical interpenetrating copies of our rigid ball centered at x and x′:**

$$U(\mathrm{x}-\mathrm{x}') =: -G\int \frac{f(\mathrm{r}|\mathrm{x})f(\mathrm{r}'|\mathrm{x}')}{|\mathrm{r}'-\mathrm{r}|}d\mathrm{r}d\mathrm{r}'$$

where $f(\mathrm{r}|\mathrm{x}) = (3M/4\pi R^3)\theta(|\mathrm{r}-\mathrm{x}| \leq R)$ is the mass density at r; M, R are ball mass and radius, resp.

$$U(\mathrm{x}-\mathrm{x}') \sim \begin{cases} -GM^2/|\mathrm{x}-\mathrm{x}'| & \text{for } |\mathrm{x}-\mathrm{x}'| \gg R \\ U(0)+\frac{1}{2}M\omega_G^2|\mathrm{x}-\mathrm{x}'|^2 & \text{for } |\mathrm{x}-\mathrm{x}'| \ll R \end{cases}$$

where $\omega_G^2 = GM/R^3$.

What is the equation of the c.o.m. decoherence?
What is the equation of the pointer state?
What is the equation of both?

THE EQUATION OF C.O.M. DECOHERENCE TIME

• **Postulated 'gravitational' decoherence time:**

$$t_G =: \frac{\hbar}{U(\mathrm{x}-\mathrm{x}')-U(0)}$$

For distant superposition we get:

$$t_G \sim -\hbar/U(0) \sim \hbar R/GM^2$$

For atomic masses, t_G is extremely long and the postulated effect is irrelevant. For nano-objects, t_G is shorter and the postulated effect may compete with the inevitable environmental decoherence. For macro-objects t_G is unrealisticly short.

• **Divergence Problem: for pointlike massive ball ($R = 0$) as well as for any object containing pointlike massive constituents $U(0)$ is ∞ therefore t_G would be zero!**

POINTER STATES: SN-EQUATION?

- We postulate the SN-eq.:

$$\frac{d\psi(\mathrm{x})}{dt} = \text{standard q.m. terms} - \frac{i}{\hbar}\int U(\mathrm{x}-\mathrm{x}')|\psi(\mathrm{x}')|^2 d\mathrm{x}'\ \psi(\mathrm{x})$$

Its ground state solution is a standing soliton of width $\Delta\mathrm{x}_G$. Galilean translations and boosts yield the overcomplet set of pointer states.

For atomic particles, $\Delta\mathrm{x}_G$ is extremely large and the localization effect is irrelevant. For nano-objects, the localization effect becomes relevant. For rigid ball of common density the approximation $\Delta\mathrm{x}_G \ll R$ is valid if $R \gg 10^{-5}cm$, $M \gg 10^{-15}g$. Then, $U(\mathrm{x}-\mathrm{x}') \approx U(0) + \frac{1}{2}M\omega_G^2|\mathrm{x}-\mathrm{x}'|^2$ and the SN-equation reduces to:

$$\frac{d\psi(\mathrm{x})}{dt} = \text{standard q.m. terms} - \frac{i}{2\hbar}M\omega_G^2|\mathrm{x}-\langle\mathrm{x}\rangle|^2\psi(\mathrm{x})$$

Exact ground state solution is easy (if there is no external potential):

$$\psi(\mathrm{x}) = \mathcal{N}exp\left(-\frac{\mathrm{x}^2}{4\Delta x_G^2}\right), \qquad \Delta\mathrm{x}_G^2 = \sqrt{\frac{\hbar}{M\omega_G}} = \left(\frac{\hbar^2}{GM^3}\right)^{1/4} R^{3/4}$$

- Reversible non-linear eq, no divergence problem for $R = 0$!
- But: no interpretation for the rest of the solutions which are not simple solitons.

POINTER STATES: FRICTIONAL SN-EQUATION!

- **Alternatively to the SN, we postulate the frSN-eq.:**

$$\frac{d\psi(\mathrm{x})}{dt} = \text{standard q.m. terms} - \frac{1}{\hbar}\int U(\mathrm{x}-\mathrm{x}')|\psi(\mathrm{x}')|^2 d\mathrm{x}'\ \psi(\mathrm{x}) + \frac{1}{\hbar}U_G\psi(\mathrm{x})$$

where $U_G = \iint U(\mathrm{x}''-\mathrm{x}')|\psi(\mathrm{x}')\psi(\mathrm{x}'')|^2 d\mathrm{x}' d\mathrm{x}''$.

Its ground state solution is a standing soliton of width $\Delta\mathrm{x}_G$ **of the same order like for the SN-equation. Similarily to SN, Galilean translations and boosts yield the overcomplet set of pointer states.**

For atomic particles $\Delta\mathrm{x}_G$ **is extremely large and the localization effect is irrelevant. For nano-objects the localization effect becomes relevant. For rigid ball of common density the approximation** $\Delta\mathrm{x}_G \ll R$ **is valid if** $R \gg 10^{-5}cm$, $M \gg 10^{-15}g$. **Then,** $U(\mathrm{x}-\mathrm{x}') \approx U(0) + \frac{1}{2}M\omega_G^2|\mathrm{x}-\mathrm{x}'|^2$ **and the frSN-equation reduces to:**

$$\frac{d\psi(\mathrm{x})}{dt} = \text{standard q.m. terms} - \frac{1}{2\hbar}M\omega_G^2|\mathrm{x}-\langle\mathrm{x}\rangle|^2\psi(\mathrm{x}) + \frac{1}{2\hbar}M\omega_G^2\langle(\Delta\mathrm{x})^2\rangle\psi(\mathrm{x})$$

where $\langle(\Delta\mathrm{x})^2\rangle = \langle\mathrm{x}^2\rangle - \langle\mathrm{x}\rangle^2$. **Exact ground state solution is easy (if there is no external potential):**

$$\psi(\mathrm{x}) = \mathcal{N} exp\left(-\sqrt{-i}\frac{\mathrm{x}^2}{4\Delta x_G^2}\right), \qquad \sqrt{-i} = \frac{1-i}{\sqrt{2}}$$

- **Irreversible nonlinear eq., no divergence problem for** $R=0$**!**

- **But: no interpretation for the rest of the solutions which are not simple solitons.**

MATCHING DECOHERENCE WITH POINTER STATES

- Master Eq. that realizes decoherence at scale t_G:

$$\frac{d\rho(\mathrm{x},\mathrm{x}')}{dt} = \text{standard q.m. terms} - \frac{1}{\hbar}[U(\mathrm{x}-\mathrm{x}')-U(0)]\rho(\mathrm{x},\mathrm{x}')$$

- Distinguished Stochastic ME, that realizes collapse to pointer states:

$$\frac{d\rho(\mathrm{x},\mathrm{x}')}{dt} = \text{standard q.m. terms} - \frac{1}{\hbar}[U(\mathrm{x}-\mathrm{x}')-U(0)]\rho(\mathrm{x},\mathrm{x}') + \frac{1}{\hbar}[W_t(\mathrm{x})+W_t(\mathrm{x}')-2\langle W_t\rangle]\rho(\mathrm{x},\mathrm{x}')$$

where W is random field: $\mathrm{M}[W_t(\mathrm{x})W_t(\mathrm{x}')] = -\hbar U(\mathrm{x}-\mathrm{x}')\delta(t-t')$.

For long time, this SME drives any initial state $\rho(\mathrm{x},\mathrm{x}')$ into localized pure state (pointer state) while the SME reduces to:

$$\frac{d\psi(\mathrm{x})}{dt} = \text{standard q.m. terms} - \frac{1}{\hbar}\int U(\mathrm{x}-\mathrm{x}')|\psi(\mathrm{x}')|^2 d\mathrm{x}'\ \psi(\mathrm{x}) + \frac{1}{\hbar}U_G\psi(\mathrm{x}) + \frac{1}{\hbar}[W_t(\mathrm{x})-\langle W_t\rangle]\psi(\mathrm{x})$$

Conjecture: the pointer state (in its co-moving system) is the ground state solution of the frSN equation. Proof exists in the $\Delta_G\mathrm{x} \ll R$ limit:

$$\frac{d\psi(\mathrm{x})}{dt} = \text{standard q.m. terms} - \frac{1}{2\hbar}M\omega_G^2|\mathrm{x}-\langle\mathrm{x}\rangle|^2\psi(\mathrm{x}) + w_t\sqrt{\frac{M}{\hbar}}\omega_G(\mathrm{x}-\langle\mathrm{x}\rangle)\psi(\mathrm{x})$$

where w_t is standard white-noise.

- The SME predicts the pointer states correctly even for $R = 0$.
- But: The process of collapse necessitates a cutoff.