Centre of mass decoherence due to time dilation: paradoxical frame-dependence

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Two stories for one model

Effect: Positional decoherence of composite objects, $\propto g/c^2$.
Pikovski-Zych-Costa-Brukner, *Nature Phys.* **11**, 668 (2015).

- Method: $1/c^2$ GR correction to composite object QM.
- Arguments: relativistic, semiclassical
- Claim: universal decoherence due to gravitational time dilation

Same Hamiltonian, pedestrian story [L.D. arXiv:1507.05828]:

- Method: $1/c^2$ SR correction to composite object QM.
- Arguments: non-relativistic, exact dynamics
- Claim: frame-dependent decoherence due to $1/c^2$ coupling between c.o.m. and i.d.o.f.
  SR/GR arguments for frame-dependence:
  Bonder-Okun-Sudarski PRD92, 124050, (2015)
  Pang-Chen-Khalili PRL117, 090401 (2016)

Newtonian Equivalence Principle

http://wigner.mta.hu/∼diosi/tutorial/freefalltutor.pdf

Free-Falling observer: $g = 0$.
Laboratory observer: $g = 9.81\mathrm{cm}/\mathrm{s}^2$.
Example: center-of-mass (c.o.m.) motion of free mass m.

Free-Falling: $\hat{x}, \hat{p}$; $\quad \hat{H}_0 = \dfrac{\hat{p}^2}{2m}$

Laboratory: $\hat{X}, \hat{P}$; $\quad \hat{H}_g = \dfrac{\hat{P}^2}{2m} + mg\hat{X} \qquad (X : \text{vertical})$

Canonical transformation:

$$\hat{U} = \exp\left(-igt^2\hat{p}/2\right)\exp\left(imgt\hat{x}\right)\exp\left(img^2t^3/6\right)$$

$$\hat{X} = \hat{U}\hat{x}\hat{U}^\dagger = \hat{x} - gt^2/2$$

$$\hat{P} = \hat{U}\hat{p}\hat{U}^\dagger = \hat{p} - mgt$$

$$\hat{H}_g = \hat{U}\hat{H}_0\hat{U}^\dagger - i\dot{\hat{U}}\hat{U}^\dagger$$

Relativistically: c.o.m. couples to internal d.o.f.

Internal Hamiltonian $\widehat{H}_i$ is additive: $\widehat{H}^{\rm tot}_{0/g} = \widehat{H}_{0/g} + \widehat{H}_i$.

Special relativistic correction, try $m \to m + \widehat{H}_i/c^2$.

Free-Falling: $\widehat{x}, \widehat{p}, \widehat{o}_i$; $\quad \widehat{H}^{\rm tot}_0 = \dfrac{\widehat{p}^2}{2(m + \widehat{H}_i/c^2)} + \widehat{H}_i$

Laboratory: $\widehat{X}, \widehat{P}, \widehat{O}_i$; $\quad \widehat{H}^{\rm tot}_g = \dfrac{\widehat{P}^2}{2(m + \widehat{H}_i/c^2)} + (m + \widehat{H}_i/c^2) g \widehat{X} + \widehat{H}_i$

Canonical transformation $\widehat{U}$ (as before, just $m \to m + \widehat{H}_i/c^2$):

$\widehat{X} = \widehat{U}\widehat{x}\widehat{U}^\dagger = \widehat{x} - gt^2/2$ pure kinematics, as before

$\widehat{P} = \widehat{U}\widehat{P}\widehat{U}^\dagger = \widehat{p} - (m + \widehat{H}_i/c^2) gt$ mixing i.d.o.f. to $\widehat{p}$

$\widehat{O}_i = \widehat{U}\widehat{o}_i\widehat{U}^\dagger = \exp(ic^{-2} gt \widehat{H}_i \widehat{x}) \widehat{o}_i \exp(-ic^{-2} gt \widehat{H}_i \widehat{x})$ mixing $\widehat{x}$ to i.d.o.f.

Note: $\widehat{U}\widehat{H}_i\widehat{U}^\dagger = \widehat{H}_i$.

C.o.m. positional decoherence due to g

$$\widehat{H}_g^{\mathrm{tot}} = \frac{\widehat{P}^2}{2m} + \frac{g}{c^2}\widehat{X}\widehat{H}_i + \widehat{H}_i$$

A wonderful coupling betwen Laboratory c.o.m. $\widehat{X}$ and $\widehat{H}_i$.
If initial state $\widehat{\rho}^{\mathrm{tot}} = \widehat{\rho}_{\mathrm{cm}} \otimes \widehat{\rho}_i$ where $\widehat{\rho}_i = Z^{-1}\exp(-\beta\widehat{H}_i)$, that's typical system-bath situation, yields c.o.m. positional decoherence:

$$\langle x_1|\,\widehat{\rho}_{\mathrm{cm}}(t)\,|x_2\rangle \approx e^{-\frac{1}{2}t^2/\tau_{\mathrm{dec}}^2} \times \langle x_1 - \tfrac{1}{2}gt^2|\widehat{\rho}_{\mathrm{cm}}(0)|x_2 - \tfrac{1}{2}gt^2\rangle$$

$$\text{decoherence rate: } \frac{1}{\tau_{\mathrm{dec}}} = \frac{g}{\hbar c^2}\sqrt{k_B C}\,T|x_1 - x_2|.$$

$m{=}1\mu$g, $C{=}10^{-5}$cal/K, $T{=}300$K, $x_1{-}x_2{=}1\mu$m:

$\Rightarrow \tau_{\mathrm{dec}} \sim 1$ms.

- Positional decoherence $\propto g$ in Laboratory frame
- No positional decoherence in Free-Fall frame

Frame-dependence of positional decoherence?

Hm ..., that's counterintuitive.

If $|x_1\rangle + |x_2\rangle$ *decays in the Laboratory and* $|X\rangle = |x - \frac{1}{2}gt^2\rangle$ *then in the Free-Fall frame* $|X_1\rangle + |X_2\rangle$ *should, too, decay.*

This argument is just false: $|X\rangle \neq |x - \frac{1}{2}gt^2\rangle$.

No closed map exists between Laboratory eigenstates $|x\rangle$ and Free-Fall eigenstates $|X\rangle$! Why:

$$\widehat{X} = \widehat{U}\widehat{x}\widehat{U}^\dagger = \widehat{x} - gt^2/2 \quad \text{pure kinematics}$$

$$\widehat{P} = \widehat{U}\widehat{P}\widehat{U}^\dagger = \widehat{p} - (m + \widehat{H}_i/c^2)gt \quad \text{mixing i.d.o.f. to } \widehat{p}$$

C.o.m. generic observables are frame-dependent.

Split $\mathcal{H}_{\rm cm} \otimes \mathcal{H}_i$ is frame-dependent.

Hilbert space $\mathcal{H}_{\rm cm}$ is frame-dependent.

You don't expect this. It is just so if you start with

$$\widehat{H}^{\rm tot}_{\rm FF} = \frac{\widehat{p}^2}{2(m + \widehat{H}_i/c^2)} + \widehat{H}_i$$

and change for Laboratory frame, or vice versa.

Frame-dependence of positional decoherence!

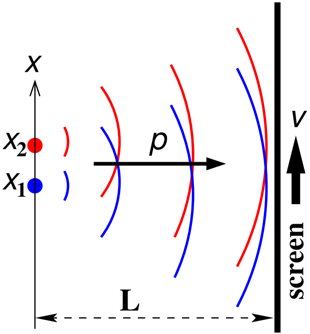
Yes! In Earth gravity g:

- Free-Falling screen detects no decoherence
- Laboratory (fixed) screen detects positional decoherence

In gravity-free ($g=0$) frame:

- Static screen detects no decoherence
- Accelerated screen detects positional decoherence

Lucid proof: Pang-Chen-Khalili [PRL 117, 090401 (2016)]:



Fringes shifted $\propto$ arrival time:

$$\cos\left[\frac{p(x_1-x_2)/L}{\hbar}\left(x_{\rm screen}-v_{\rm screen}\frac{Lm}{p}\right)\right]$$

m is random since $m\to m+H_i/c^2$.

Visibility supressed $\propto v_{\rm screen}$.

Choice $v_{\rm screen}=gt$ recovers $\tau_{\rm dec}$ just like in Earth's Laboratory frame.

Summary: Pikovski et al. theory for pedestrians

Pedestrian=non-relativistic thinker, sees different depths.

i) SR (not GR) correction to standard Hamiltonian:

$$\widehat{H} = \frac{\widehat{p}^2}{2(m + \widehat{H}_i/c^2)} + \widehat{H}_i$$

A piece of SR, but no Lorentz inv., no general cov.

ii) Exact Galilean inv. and Newtonian Equivalence Principle.

iii) We can interpret everything in non-relativistic terms - plus the fact that m contains the correction $\widehat{H}_i/c^2$.

iv) Positional decoherence is missing in inertial frames. It emerges in accelerating frames only.

v) Moving ($v \ll c$) detector sees different interference fringes, accelerating detector sees same fringe as static one in gravity.

With these pedestrian lessons can we put the theory back to SR/GR context (and re-attribute positional decoherence to time dilation).