Does Planck length challenge non-relativistic quantum mechanics of large masses?

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Does Planck scale require relativistic motion?

- Space-time continuum is likely to break down at $\ell_{\rm Pl}$.
- Hence Planck scale puts a limit to standard physics.
- Early Big Bang high energies hit the Planck scale.
- At lower energies than that, we remain on the safe side.
- Which is not quite true.

When quantum mechanics enters the Planck scale:

$$\underbrace{\lambda_{\rm deBroglie}}_{\frac{2\pi\hbar}{mv}\sqrt{1-v^2/c^2}} \sim \underbrace{\ell_{\rm Planck}}_{\sqrt{\frac{\hbar G}{c^3}}}.$$

$\lambda_{\rm dB}$ sinks to $\ell_{\rm Pl}$ in two ways:

- Relativistic way: when elementary particles velocity v closes c.
- Non-relativistic: *when mass grows macroscopic*.

Nonrelativistic plane wave versus Planckian scale

Non-relativistic plane wave, $v \ll c$:

$$\begin{aligned}\Psi(x,t) &= \exp\left(-iEt/\hbar + ipx/\hbar\right)\\ &= \exp\left(-i\frac{mv^2}{2\hbar}t + i\frac{mv}{\hbar}x\right)\\ &= \exp\left(-2\pi i\frac{t}{\tau} + 2\pi i\frac{x}{\lambda}\right)\end{aligned}$$

$$\text{Periodicity in } t \text{ and } x: \begin{cases}\tau &= (4\pi\hbar/mv^2)\\ \lambda &= (2\pi\hbar/mv)\end{cases}$$

$\Psi(x,t)$ is legitimate as long as $\tau \gg \tau_{\mathrm{Pl}}$ and $\lambda \gg \ell_{\mathrm{Pl}}$.

$$\text{Planck time and length: } \begin{cases}\tau_{\mathrm{Pl}} &= \sqrt{\hbar G/c^5} \sim 10^{-43}s\\ \ell_{\mathrm{Pl}} &= \sqrt{\hbar G/c^3} \sim 10^{-33}cm\end{cases}$$

For atomic m that's the case: $\tau/\tau_{\mathrm{Pl}} \gg 10^{18}$ and $\lambda/\ell_{\mathrm{Pl}} \gg 10^{18}$.
But larger m will push $\Psi(x,t)$ towards the Planckian scales.

Large mass non-relativistic wave function

- $\tau = \frac{2\pi\hbar}{mv^2/2}$ sinks to $\tau_{\rm Pl} \sim 10^{-43}$s if $mv^2 \sim m_{\rm Pl}c^2$, i.e:
$$m \sim \frac{c^2}{v^2} m_{\rm Pl} \sim \frac{c^2}{v^2} 10^{-5} g.$$
- $\lambda = \frac{2\pi\hbar}{mv}$ sinks to $\ell_{\rm Pl} \sim 10^{-33}$cm if $mv \sim m_{\rm Pl}c$, i.e.:
$$m \sim \frac{c}{v} m_{Pl} \sim \frac{c}{v} 10^{-5} g.$$

With growing m, non-relativistic $\Psi(x,t)$ becomes illegitimate.
The bell rings for spatial periodicity first.
Example I, free motion:
$$m = 10g, v = 10km/s, \Rightarrow \lambda = (2\pi\hbar/mv) \sim \ell_{\rm Pl}$$
Example II, rigid body elastic vibration mode:
$$m = 10kg, \omega = 100kHz, a(\text{mplitude}) = 10^{-2}cm \Rightarrow \lambda = (2\pi\hbar/ma\omega) \sim \ell_{\rm Pl}$$

Ignorable effects per atoms accumulate

Suppose a global constant "uncertainty":

$$x \Rightarrow x + u.$$

$|u| \sim \ell_{\mathrm{Pl}}$ is space's error/bluriness/foaminess/fluctuation.
Many-particle state:

$$\Psi(x_1, x_2, \dots, x_{10^{23}}) \Rightarrow \Psi(x_1 + u, x_2 + u, \dots, x_{10^{23}} + u).$$

u is irrelevant for non-relativistic individual particles, but its *effect accumulates* for the many-particle c.o.m. x.
Best seen in momentum representation:

$$\widetilde{\Psi}(p_1,p_2,\dots,p_{10^{23}}) \Rightarrow \exp\left(\frac{i}{\hbar}u(\underbrace{p_1+p_2+\dots+p_{10^{23}}}_{P})\right)\widetilde{\Psi}(p_1,p_2,\dots,p_{10^{23}})$$

C.o.m. reduced state decoheres in momentum if u is stochastic:

$$\rho(P,P') \Rightarrow \exp\left(\frac{i}{\hbar}u(P-P'\right)\rho(P,P') \Rightarrow \exp\left(-\frac{\ell_{\mathrm{Pl}}^2}{2\hbar^2}(P-P')^2\right)\rho(P,P')$$

Is c.o.m. operator x of 10^{23} atoms observable?

Sure, it is!

In quantum optomechanics, magnetomechanics:
Spatial motion of suspended, flexibly located, levitated or trapped macroobjects is controlled in their quantum regime.

E.g.: Each and every photon in mirror-optomechanics interacts with the mirror as a whole.

Masses are still much less then those requested for $\lambda \sim \ell_{\mathrm{Pl}}$.
But much larger than before 20 years we beleived in.

Facts and questions

- We don't need extreme high energies to explore Planck scale.
- Nonrelativistic QM *of massive d.o.f.* does explore it.
- And breaks down there. What way, we don't yet know.
- Plausible: space-time "uncertainty" yields noise/decoherece.
  - Holographic noise? (Hogan [Genovese's talk]
  - G-related decoherence of massive d.o.f. ? (D-Penrose)
- Breakdown depends on spectrum of "uncertainty" u.
- Can come much earlier than for global static u.

D-Penrose: nonrelativistic

- Can it be the non-relativistic footprint of Planck scale "uncertainties"?
- If it decoheres massive d.o.f. before their λ_{dB} sinks to ℓ_{Pl}?

Closing remarks

The knowledge that
i) *Planckian "uncertainty" of space-time accumulates for large non-relativistic objects*,
ii) *we might therefore study Planckian footprints in the lab non-relativistically*,
has been *implicit* in various works (Károlyházy, D., Penrose, Hogan, Bekenstein ...), all using *sophisticated arguments*.
What I'm adding is
explicit and elementary evidence.

Remember:

$$\lambda_{\mathrm{dB}} = \frac{2\pi\hbar}{10g \times 10km/s} = 4.2\times 10^{-33}cm \sim \ell_{\mathrm{Pl}}.$$