Schrödinger–Newton Equation: Four Fundamental Catches and Attempts to Relax Them

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Abstract

In 1984, Schrödinger-Newton equation opened the door to a non-relativistic regime of nano/micro-quantum-mechanics, instead of quantum cosmology, where quantum and gravity are equally important. The fundamental difficulties, well-known and less known ones, of this non-linear equation are summarized. Some concepts to relax or merely cope with them are interpreted.

Peaceful coexistence ...

of quantum mechanics and special relativity (Shimony)

- apparent action-at-a-distance in EPR situation
- non-locality in Bell formulation

Still:
action-at-a-distance (**AAD**) & faster-then-light (**FTL**) communication remain **impossible**.

Reason: linear structure of quantum mechanics
Non-linear modifications open door to **FTL** communication! (Gisin)

$$i\hbar\frac{d\psi}{dt} = \hat{H}\psi + \hat{V}_{\psi}\psi$$

allows for **FTL** communication for whatever small (non-trivial) $\hat{V}_{\Psi}$.

Semiclassical Gravity in Cosmology

Effective theory in cosmology/astrophyics: coupling quantized matter and classical gravity, **Semiclassical Einstein Equation**:

$$G_{ab} = \frac{8\pi G}{c^4} \left\langle \hat{T}_{ab} \right\rangle_{\psi}$$

Four fundamental catches:

- Fake action-at-a-distance **AAD**
- Faster-then-light **FTL** communication
- No local autonomy **NoLA**
- No Born statistical interpretation **NoBorn**

Unrelated to relativity, gravity. Related to quantum nonlinearity induced by $\langle \hat{T}_{ab} \rangle_{\psi}$. Transparent in the nonrelativistic limit.

Semiclassical Gravity in the Lab

Newtonian limit of **Semiclassical Einstein Equation**

$$G_{00} = 8\pi G c^{-4} \left\langle \hat{T}_{00} \right\rangle_\psi$$

where $G_{00} = 2c^{-2}\Delta\Phi$, $\hat{T}_{00} = \hat{\varrho}c^2$, hence c cancels:

$$\Delta\Phi = 4\pi G \langle\hat{\varrho}\rangle_\psi .$$

$$\Phi_\psi(x) = -G \int \frac{d^3 r}{|x - r|} \langle\hat{\varrho}(r)\rangle_\psi .$$

The **Schrödinger-Newton Equation (SNE)**:

$$i\hbar\frac{d\psi}{dt} = \hat{H}\psi + \int \Phi_\psi(r)\hat{\varrho}(r) d^3 r \psi$$

Effective theory? Might be fundamental! (D. 1984, Penrose 1996)
All four catches survive: **AAD, FTL, NoLA, NoBorn**

Single body SNE

... for c.o.m. free motion of "pointlike" big mass M:

$$\Phi_\psi(x) = -GM \int \frac{|\psi(r)|^2}{|x-r|} d^3 r$$

$$i\hbar \frac{d\psi(x)}{dt} = -\frac{1}{2M\hbar^2} \nabla^2 \psi(x) + M\Phi_\psi(x)\psi(x)$$

- Self-field Φ_ψ means self-attraction
- Self-attraction yields solitons, size $\sim (\hbar^2/GM^3)$
- Testable soon in nano-Quantum-Mechanics:

"Quantum Gravity in the Lab"

Schrödinger–Newton Cats

- One-soliton: stable ground state. Might be the natural localized state of macroobjects.
  Soliton size: astronomic for elementary particles, subsubmicroscopic for large objects, crossover at 10^{-5}cm.
- Two-soliton: total mass M is shared between the parts. They **attract each other** gravitationally. Can penetrate each other. Any classical two-body motion is stable approximately, e.g. Keplerian motion.
  Example: two solitons, mass $M/2$ each, can oscillate along a line, penetrating each other, with the classical period $T = \pi\sqrt{\ell^3/2GM}$ where ℓ is the amplitude of their distance.
- . . .

The Four Catches: AAD, FTL, NoLA, NoBorn

Alice and Bob, far away from each other, own a qubit and a SN-Kepler Cat, respectively, in entangled state:

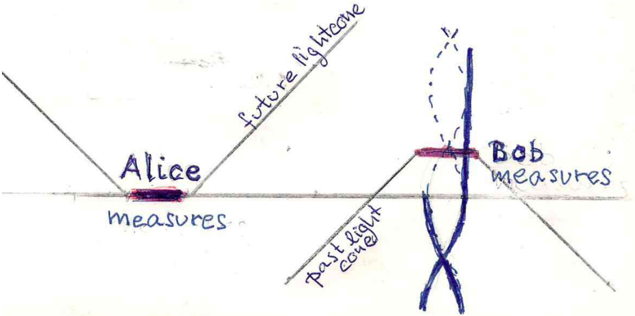
$$\frac{1}{\sqrt{2}}\Big(|\uparrow\rangle \otimes |\text{“half-cat”-1}\rangle + |\downarrow\rangle \otimes |\text{“half-cat”-2}\rangle\Big)$$

At $t = 0$ Alice i) does nothing or ii) measures her qubit. At some time $t > 0$ Bob measures position of his Cat. Bob finds that his Cat is on the orbit [case i)] or that the Cat left the orbit [case ii)].

- **AAD**: Alice local action changed dynamics at Bob's location.
- **FTL**: She did it superluminally if their distance was large.
- **NoLA**: Bob can in general not predict the dynamics of his system.
- **NoBorn**: ...

Nonlinear SNE without FTL & NoBorn ?

Delay collapse (caused by Alice) causally at Bob's location!
(Kent 2005, Bedingham 2016, Helou & Chen 2017)
Standard collapse is instantaneous:



Proposal: Bob's local state be not collapsed by Alice because her measurement falls outside Bob's past-lightcone.

Stochastic Semiclassical Gravity: AAD, FTL, NoLA, NoBorn Have Gone!

Persecute Schrödinger–Newton Cats! (Tilloy & D. 2016-17)
Assume universal (spontaneous) measurement of mass distribution $\hat{\varrho}(x)$, yielding the measured outcome $\varrho(x,t)$ of the form

$$\varrho = \langle\hat{\varrho}\rangle_\psi + \text{noise}$$

On r.h.s. of semi-classical Newton-Poisson eq., replace $\langle\hat{\varrho}\rangle_\psi$ by ϱ:

$$\Delta\Phi_\psi = 4\pi G\Big(\langle\hat{\varrho}\rangle_\psi + \text{noise}\Big)$$

Optimize trade-off between noise and precision of $\hat{\varrho}$-measurement.
Result: the **Gravity-Related Spontaneous Collapse** theory, known from alternative considerations (Penrose & D.)

Statistics Require Linearity

- Suppose any **dynamics** $\hat{\rho}^f = \mathcal{M}[\hat{\rho}^i]$, not necessarily linear.
- Consider statistical **mixing** of $\hat{\rho}_1, \hat{\rho}_2$ with weights $\lambda_1 + \lambda_2 = 1$:

$$\hat{\rho} = \lambda_1 \hat{\rho}_1 + \lambda_2 \hat{\rho}_2$$

In von Neumann standard theory

mixing and dynamics are interchangeable:

$$\mathcal{M}[\lambda_1 \hat{\rho}_1 + \lambda_2 \hat{\rho}_2] = \lambda_1 \mathcal{M}[\hat{\rho}_1] + \lambda_2 \mathcal{M}[\hat{\rho}_2]$$

Recognize the condition of $\mathcal{M}$'s linearity!

- Interchangeability excludes deterministic non-linear Schrödinger equations
- Without interchangeability statistical interpretation collapses

Catch NoBorn is non-quantum, it's classical statistical! (D.: *A Short Course in Quantum Information Theory*, Springer, 2007, 2011)

Summary: Catches and Loopholes

However hard the **conflict between non-linearity and statistics** is, Schrödinger–Newton equation deserves attention in foundations.
Just we should keep in mind catches:
The SNE

- allows for
  - fake action-at-a-distance (maybe extreme weak)
  - faster-than-light communication (maybe too hard to realize)
- does not allow for
  - local autonomous dynamics (unless you have a pure state)
  - Born statistical interpretation (maybe a substitute works?)