**On canonical coupling of classical geometry to quantum matter**

1. Introduction

$$R_{ab} - \frac{1}{2} g_{ab} R = \frac{8\pi G}{c^4} T_{ab}$$

1.1 classical $T$, classical $R$ — fine.
1.2 quantum $T$, quantum $R$ — nobody knows.
1.3 quantum $T$, classical $R$ — try it canonically!

2. Poisson, Dirac, and Alexandrov brackets

3. Coarse-grained Alexandrov equation

4. Application to gravity

5. Fundamental uncertainty of geometry, decoherence

6. Conclusion
6.1 Semiclassical evolution must be coarse-grained
6.2 Classical part, too, gets noisy.
6.3 Reasonable decoherence obtained.
6.4 Covariant coarse-graining?

2.1 Classical: Poisson bracket, Liouville equation of motion.

$$\{A,B\}_P \equiv \sum_n \Big( \frac{\partial A}{\partial q_n}\frac{\partial B}{\partial q_n} - \frac{\partial A}{\partial p_n}\frac{\partial B}{\partial p_n} \Big)$$

$$\dot\rho = \{H,\rho\}_P$$

where $\rho(q,p) \geq 0$, $\int \rho(q,p) dq dp = 1$.

2.2 Quantum: Dirac bracket, Schrodinger (von Neumann) equation.

$$\{A,B\}_D \equiv -\frac{i}{\hbar}[A,B].$$

$$\dot\rho = \{H,\rho\}_D$$

where $\rho \geq 0$, $tr\rho = 1$.

2.3 Semiclassical: Alexandrov (1981) bracket and equation.

$$\{A,B\}_A = \{A,B\}_P + \frac{1}{2}\{A,B\}_D - \frac{1}{2}\{B,A\}_D$$

$$\dot\rho = \{H,\rho\}_A$$

where $\rho(q,p) \geq 0$, $tr \int \rho(q,p) dq dp = 1$.

3. Coarse-grained Alexandrov equation.

$$H = H(q_1, q_2, p_1, p_2) = H_1(q_1, p_1) + H_2(q_2, p_2) + H_I(q_1, q_2, p_1, p_2)$$

where $(q_1, p_1)$ are quantum, $(q_2, p_2)$ are classical.

$$H_I(q_1, p_1, q_2, p_2) = \sum_\alpha J_1^\alpha(q_1, p_1) J_2^\alpha(q_2, p_2), \qquad (J_1^\alpha, J_2^\alpha \neq 1).$$

*Noisy* interaction:

$$H_I^{noise}(q_1, p_1, q_2, p_2) = \Big(J_1^\alpha(q_1, p_1) + \delta J_1^\alpha(t)\Big)\Big(J_2^\alpha(q_2, p_2) + \delta J_2^\alpha(t)\Big)$$

where $\delta J_1, \delta J_2$ are *classical* noises.

$$H^{noise} = H_1 + H_2 + H_I^{noise}$$

$$\dot\rho = \Big\langle \{H^{noise}, \rho\}_A \Big\rangle_{noise}.$$

Choose white-noices:

$$\Big\langle \delta J_n^\alpha(t') \delta J_n^\beta(t) \Big\rangle_{noise} = \frac{1}{2} \Lambda_n^{\alpha\beta} \delta(t' - t), \qquad n = 1, 2.$$

$$\dot\rho = -\frac{i}{\hbar}[H, \rho] + \frac{1}{2}\{H, \rho\}_P - \frac{1}{2}\{\rho, H\}_P$$
$$-\frac{1}{4}\sum_{\alpha,\beta} \Lambda_2^{\alpha\beta}[J_1^\alpha, [J_1^\beta, \rho]] + \frac{1}{4}\sum_{\alpha,\beta} \Lambda_1^{\alpha\beta}\{J_2^\alpha, \{J_2^\beta, \rho\}_P\}_P$$

Lindblad's condition (1976) for positivity: $\Lambda_1 \Lambda_2 = I$.

4. Application to gravity.

$$(q_1,p_1)\equiv(q,p)\quad \mathit{quantized\ non-relativistic\ matter}$$
$$(q_2,p_2)\equiv(\phi,\pi)\quad \mathit{weak\ classical\ gravitational\ field}$$
$$\phi\equiv\frac{1}{2}c^2(g_{00}-1)\quad \mathit{classical\ Newtonian\ potential}$$
$$f=T_{00}/c^2\quad \mathit{operator\ of\ mass\ distribution}$$
$$H(q,p,\phi,\pi)=H_m(q,p)+\frac{1}{8\pi G}\int_r\left(\frac{1}{c^2}\pi^2+|\nabla\phi|^2\right)+\int_r\phi(r)f(r)$$

Coarse-grained Alexandrov equation:

$$\dot\rho=-\frac{i}{\hbar}[H_m,\rho]-\frac{i}{\hbar}\int_r\phi(r)[f(r),\rho]$$
$$-\frac{1}{4\pi G}\int_r\Big(\frac{1}{c^2}\pi(r)\frac{\delta\rho}{\delta\phi(r)}+\Delta\phi(r)\frac{\delta\rho}{\delta\pi(r)}\Big)+\frac{1}{2}\int_r\Big[f(r),\frac{\delta\rho}{\delta\pi(r)}\Big]_+$$
$$-\frac{1}{4}\int_r\int_{r'}\lambda(r,r')[f(r),[f(r'),\rho]\ ]+\frac{1}{4}\int_r\int_{r'}\lambda^{-1}(r,r')\frac{\delta^2\rho}{\delta\pi(r)\delta\pi(r')}.$$

Reduced dynamics of $\rho_m=\int\int\rho(\phi,\pi)\mathcal{D}\phi\mathcal{D}\pi$ in Newtonian approximation.
Drop term $1/c^2$ and assume $\rho_m$ determines Newton potential $\phi$:

$$\int\rho(\phi,\pi)\mathcal{D}\pi=\prod_r\delta\Big(\phi(r)+\int_{r'}\frac{G/2}{|r-r'|}[f_+(r')+f_-(r')]\Big)\rho_m.$$

Subscripts $+$ and $-$ assure $\rho_m$'s hermiticity.
Integrate both sides over $\phi,\pi$!

$$\dot\rho_m=-\frac{i}{\hbar}[H_m+H_g,\rho_m]-\frac{1}{4}\int_r\int_{r'}\lambda(r,r')[f(r),[f(r'),\rho_m]$$
$$H_g=-\frac{G}{2}\int_r\int_{r'}\frac{f(r)f(r')}{|r-r'|}.$$

5. Fundamental uncertainty of geometry, decoherence

$$\left\langle \delta f(r',t')\delta f(r,t)\right\rangle_{noise} = \frac{1}{2}\lambda^{-1}(r',r)\delta(t'-t),$$
$$\left\langle \delta\phi(r',t')\delta\phi(r,t)\right\rangle_{noise} = \frac{\hbar^2}{2}\lambda(r',r)\delta(t'-t).$$

In Newtonian approximation: $\Delta\langle\phi(r)\rangle = 4\pi G\langle f(r)\rangle$.

$$\Delta\Delta'\lambda(r,r') = (4\pi G)^2\lambda^{-1}(r,r')$$

Diósi and Lukács (1987):

$$\lambda(r,r') = (G/\hbar)|r-r'|^{-1}$$

Diósi (1987):

$$\dot\rho_m = -\frac{i}{\hbar}[H_m + H_g, \rho_m] - \frac{1}{4}\int_r\int_{r'}\frac{G/\hbar}{|r-r'|}[f(r),[f(r'),\rho_m]].$$

Ghirardi et al. (1990): cutoff $10^{-5}cm$.
Decoherence at characteristic time $\tau = \hbar/\Delta U_G$,
where $\Delta U_G$: quantum spread of the Newtonian self-energy.
Pearle and Squires (1995): Verification in proton-decay experiments(!?)

# References


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