# Can the Thermodynamic and Quantum Entropies be made equal in friction



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Why should $S$ and $S_Q$ be equal?

In equilibrium: $S = S_Q$

Out of -"- : $\dot{S} \stackrel{?}{=} \dot{S}_Q$

Famous conflict: $\dot{S} \geq 0$ (2nd Law)

(pity) $\dot{S}_Q \equiv 0$ (microscopic reversibility)

My example: friction

in fluid
in Maxwell-Boltzmann gas
in abstract quantum "gas"
in ideal Bose/Fermi gas

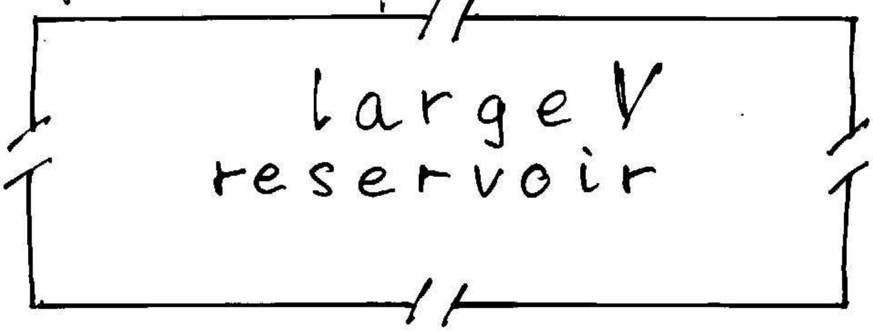
Aug 27, 2007
Safed, Israel

$$\begin{cases} R, V, E : \text{Reservoir, Volume, Energy} \\ S(E) : \text{thermodynamic entropy} \\ S'(E) \equiv \beta = 1/T : \text{inverse temperature} \\ \qquad\qquad (k_B = 1) \end{cases}$$

$$\begin{cases} \hat{\rho}, \hat{\sigma}, \ldots : \text{density matrices} \\ S_Q(\hat{\rho}) = -\mathrm{tr}(\hat{\rho} \log \hat{\rho}) : \text{quantum entropy} \\ S_Q(\hat{\sigma} | \hat{\rho}) = \mathrm{tr}\left[\hat{\sigma}(\log \hat{\sigma} - \log \hat{\rho})\right] : \text{q. relative e.} \end{cases}$$

$$\begin{cases} \rho_R(\vec{p}) \equiv \rho_R(p_1, p_2, \ldots, p_N) : \text{Maxwell-distribution} \\ S_{MB}(\rho) = -\int \left(\rho(\vec{p}) \log \rho(\vec{p})\right) d^N p \end{cases}$$

## Entropies in equilibrium

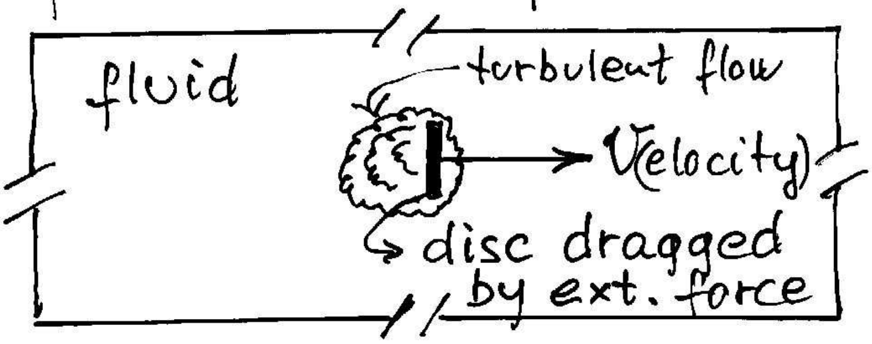



$E$ $\qquad\qquad\qquad$ $\hat{\rho}_R = \frac{1}{Z(\beta)} e^{-\beta \hat{H}}$

$S(E)$ $\qquad\qquad$ $S_Q(\hat{\rho}_R) = -\mathrm{tr}(\hat{\rho}_R \log \hat{\rho}_R)$

Choose $S(0)=0$, $k_B = 1$ $\Rightarrow$ $S = S_Q$

## Entropies in non-equilibrium




$\dot{S} = \eta \beta V^2$ $\qquad\qquad$ $\dot{S}_Q \equiv 0$ (pity)

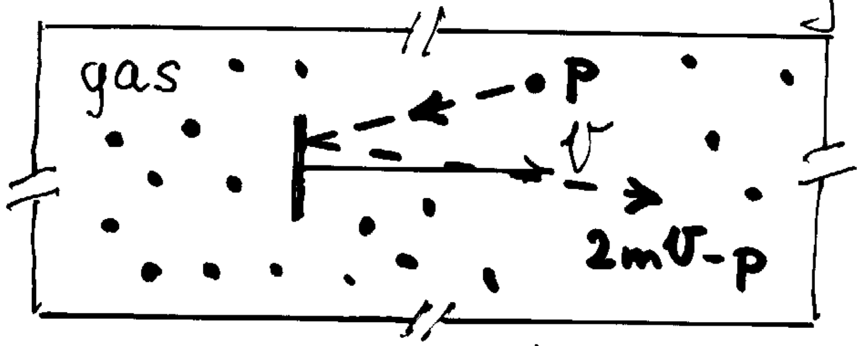
$\uparrow$ constant of friction

$\downarrow$ coarsening $\hat{\rho}_R$

$\dot{S}_Q > 0$

What coarsening?

# Friction in Maxwell-Boltzmann gas




initially $\rho_R \sim e^{-\frac{\beta}{2m}\sum_{n=1}^{N} p_n^2}$

1 collision with the disc

$$p_r^{\parallel} \xrightarrow{\text{coll.}} 2mv - p_r^{\parallel}$$

Cunningham 1910
Epstein 1924

$$\Rightarrow \eta = 2\nu m \qquad \nu = \text{frequency of collisions}$$

$$\Rightarrow \dot{S} = 2\nu m \beta v^2$$

Let's discuss $\dot{S}_{MB}$!

$$\rho_R(\vec{p}) \xrightarrow{\text{coll.}} \rho_R^{(r)}(\vec{p}) \equiv \rho_R(\vec{p})\, e^{-2\beta v p_r^{\parallel} + O(v^2)}$$

symmetric non-symmetric

$$\dot{S}_{MB} =: \nu\left[S_{MB}(\rho_R^{(r)}) - S_{MB}(\rho_R)\right] = 0 \quad \textbf{(pity)}$$

coarse

$$\rho_R^{(r)} \longrightarrow \mathcal{M}\rho_R^{(r)} = \frac{1}{N}\sum_{r=1}^{N} \rho_R^{(r)}$$

$$\Rightarrow \dot{S}_{MB} =: \nu\left[S_{MB}(\mathcal{M}\rho_R^{(r)}) - S_{MB}(\rho_R)\right] = 2\nu m\beta v^2$$

D. 2002

# Friction in abstract quantum gas

initially $\hat{\rho}_R = \hat{\rho}^{\otimes N}$ ; $\hat{\rho} = \frac{1}{Z(\beta)} e^{-\beta \hat{H}}$

def.: unitary map $\hat{U}\hat{\rho}\hat{U}^{+} = \hat{\sigma}$

1 "collision"

$$\hat{\rho}_R \equiv \hat{\rho}^{\otimes N} \xrightarrow{\text{coll.}} \hat{\rho}_R^{(r)} \equiv \hat{\rho}^{\otimes(r-1)} \otimes \hat{\sigma} \otimes \hat{\rho}^{\otimes(N-r)}$$

$$\dot{S} =: \nu\beta\Delta E = \nu\beta \,\mathrm{tr}\big[\hat{H}(\hat{\sigma}-\hat{\rho})\big] = \nu S(\hat{\sigma}|\hat{\rho})$$

$$\dot{S}_Q =: \nu\big[S_Q(\hat{\rho}_R^{(r)}) - S_Q(\hat{\rho}_R)\big] = 0 \quad (\text{pitty})$$

$\downarrow$ coarse

$$\hat{\rho}_R^{(r)} \longrightarrow \mathcal{M}\hat{\rho}_R^{(r)} \equiv \frac{1}{N}\sum_{r=1}^{N} \hat{\rho}_R^{(r)}$$

$$\dot{S}_Q =: \nu\big[S_Q(\mathcal{M}\hat{\rho}_R^{(r)}) - S_Q(\hat{\rho}_R)\big]$$

$$\Rightarrow \dot{S} = \dot{S}_Q \quad \text{if } [\ldots] = S(\hat{\sigma}|\hat{\rho})$$

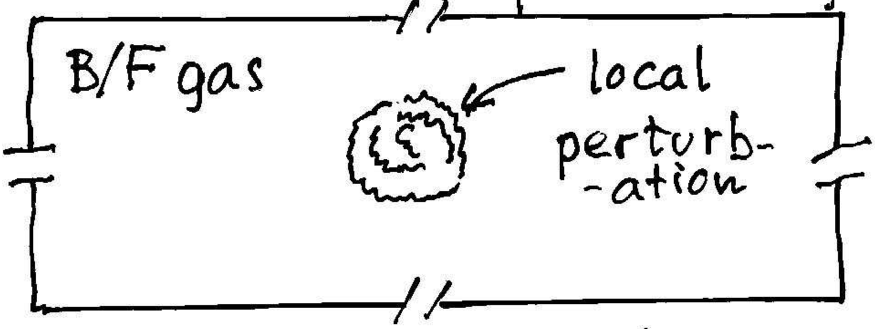
New math. conjecture (DFK 2006)

$$S_Q\big(\mathcal{M}(\hat{\sigma}\otimes\hat{\rho}^{\otimes N})\big) - S_Q(\hat{\sigma}\otimes\hat{\rho}^{\otimes N}) \longrightarrow$$

$$\xrightarrow[N\to\infty]{} S(\hat{\sigma}|\hat{\rho})$$

Proof: Csiszár, Hiai, Petz 2007

# Friction in ideal quantum gas




initially $\hat{\rho}_R \sim e^{-\sum_k \beta E(k)\hat{n}(k)}$

local unitary perturbation: $\hat{U}$

$$\hat{\rho}_R \longrightarrow \hat{U}\hat{\rho}_R\hat{U}^{+} \equiv \hat{\rho}_R^U$$

$\Rightarrow \Delta E = \operatorname{tr} \sum_k E(k)\,\hat{n}(k)\left(\hat{\rho}_R^U - \hat{\rho}_R\right)$

If $\Delta E$ "got" dissipated

$\Delta S =: \beta \Delta E = S(\hat{\rho}_R^U | \hat{\rho}_R)$ (great)

$\Delta S_Q =: S(\hat{\rho}_R^U) - S(\hat{\rho}_R) = 0$ (pity)

$\downarrow$ coarse

$$\mathcal{M}\hat{\rho}_R^U = \frac{1}{V}\int e^{ix\hat{P}}\hat{\rho}_R^U e^{-ix\hat{P}} d^3x$$

$$\hat{P} \equiv \sum_k k\,\hat{n}(k)$$

Math. conjecture:

$$S(\mathcal{M}\hat{\rho}_R^U) - S(\hat{\rho}_R^U) \xrightarrow[V\to\infty]{} S(\hat{\rho}_R^U | \hat{\rho}_R)$$

If so, then

$$\Delta S = \Delta S_Q$$