Equating quantum and thermodynamic entropy productions (Information erasure in closed system: Nature may operate twirling)

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A new entropy theorem

Product state $\rho = \underset{1}{\sigma'} \otimes \underset{2}{\sigma} \otimes \underset{3}{\sigma} \otimes \underset{\dots}{\dots} \otimes \underset{N}{\sigma}$ entropy:

$$S[\sigma' \otimes \sigma^{\otimes(N-1)}] = S[\sigma'] + (N-1)S[\sigma]$$

Irreversible operation *twirling* $\mathcal{T}$:

$$\mathcal{T}\left(\sigma' \otimes \sigma^{\otimes(N-1)}\right) = \frac{\sigma' \otimes \sigma^{\otimes(N-1)} + \sigma \otimes \sigma' \otimes \sigma^{\otimes(N-2)} + \dots + \sigma^{\otimes(N-1)} \otimes \sigma'}{N}$$

Limit theorem for entropy production:

$$\lim_{N=\infty}\left(S[\mathcal{T}(\sigma' \otimes \sigma^{\otimes(N-1)})] - S[\sigma' \otimes \sigma^{\otimes(N-1)}]\right) = S[\sigma'|\sigma].$$

Csiszár-Hiai-Petz: We don't see how you got the conjecture.
D.-Feldmann-Kosloff: We don't see how you prove it.

Microscopic reversibility

Theory: reversibility in closed systems

$$\rho \to U\rho U^\dagger, \qquad S[U\rho U^\dagger] = S[\rho]$$

Experience: entropy production in large closed systems
Some irreversible mechanism superseds unitary evolution.

$$\rho \to U\rho U^\dagger \to \mathcal{M}^? \rho, \qquad S[\mathcal{M}^? \rho] > S[\rho]$$

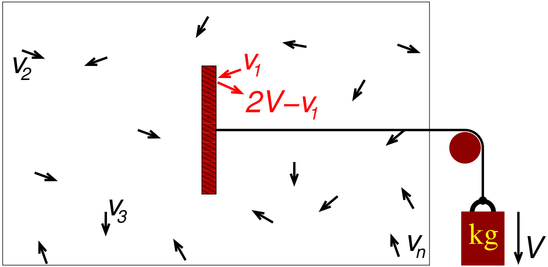
What can $\mathcal{M}^?$ be?
Find a system such that:

- microscopic dynamics U is tractable
- macroscopic friction force is calculabe from U
- $\Rightarrow$ thermodynamic entropy production $\Delta S^{\mathrm{thermo}}$ is calculable
- $\quad \Rightarrow \Delta S^{\mathrm{micro}} = S[\mathcal{M}^? \rho] - S[\rho]$ is strictly given by $\Delta S^{\mathrm{thermo}}$

Then you construct $\mathcal{M}^?$!

Mechanical friction in Maxwell gas

Constant force is dragging a disk at velocity V through the gas.



Friction force: $2\nu mV$
(Epstein 1910)

ν collision rate
m molecular mass
V disc velocity

Thermodynamic entropy-
-production rate: $2\beta\nu mV^2$

$$\rho(v)=\prod_k \exp\left(-\frac{\beta m}{2}v_k^2\right) \Rightarrow \rho_1(v)=\exp\left(-\frac{\beta m}{2}(2V-v_1)^2\right)\prod_{k\neq 1}\exp\left(-\frac{\beta m}{2}v_k^2\right)$$

$$\Delta S^{\mathrm{micro}}/\mathrm{collision} = S[\rho_1]-S[\rho]=0$$
$$\Delta S^{\mathrm{thermo}}/\mathrm{collision} = 2\beta mV^2$$

Impose $\mathcal{M}^? : \rho_1 \to \mathcal{M}^?\rho_1$ such that is dynamically 'innocent' and

$$S[\mathcal{M}^?\rho_1]-S[\rho]=2\beta mV^2$$

Nature may forget...

which one of the N molecules has just collided: $\mathcal{M}^? = \mathcal{T}$

$$\mathcal{T}\rho_1 = (\rho_1 + \rho_2 + ... + \rho_N)/N$$

where

$$\rho_n(v) = \exp\left(-\frac{\beta m}{2}(2V - v_n)^2\right)\prod_{k\neq n}\exp\left(-\frac{\beta m}{2}v_k^2\right)$$

Indeed, in thermodynamic limit $N \to \infty$:

$$\Delta S^{\mathrm{micro}} = S[\mathcal{T}\rho_1] - S[\rho_1] \longrightarrow 2\beta m V^2 + \mathcal{O}(V^4) = \Delta S^{\mathrm{thermo}} + \mathcal{O}(V^4)$$

Twirling Maxwell gas:

- dynamically 'innocent': $\mathcal{T}[H,\rho] = [H, \mathcal{T}\rho]$
- erases information $\Delta S^{\mathrm{micro}}$ coinciding with $\Delta S^{\mathrm{thermo}}/k_B$

D. 2002

'Friction' in abstract quantum gas

Initial Gibbs state:

$$\rho = \left(\frac{e^{-\beta H}}{Z(\beta)}\right)^{\otimes N} \equiv \sigma^{\otimes N}$$

Collision on outside field/object (cf.: 'disk'):

$$\rho \Rightarrow \rho_1 = \sigma' \otimes \sigma^{\otimes(N-1)}$$

where $\sigma' = U\sigma U^\dagger$. Identity for energy change:

$$\Delta E = \mathrm{tr}(H\sigma') - \mathrm{tr}(H\sigma) = S[\sigma'|\sigma]/\beta$$

Suppose ΔE is dissipated, then

$$\Delta S^{\mathrm{thermo}} = S[\sigma'|\sigma].$$

Twirl $\mathcal{T}$ generates exactly this amount: $S[\mathcal{T}\rho_1] - S[\rho_1] = \Delta S^{\mathrm{thermo}}$.

$$\lim_{N=\infty}\left(S[\mathcal{T}(\sigma'\otimes\sigma^{\otimes(N-1)})] - S[\sigma'\otimes\sigma^{\otimes(N-1)}]\right) = S[\sigma'|\sigma].$$

Conjecture D.-Feldmann-Kosloff 2006. Proof Csiszár-Hiai-Petz 2007.

Summary

- Notorious tension: reversible micro vs. irrev. macro
- Case study: mechanical friction in Maxwell gas
- Quantitative entropic constraint on microscopic mechanism
- Nature may use *twirl* to erase information
- Bye-product: new quantum informatic theorem
- Reality: twirling local perturbation of Gibbs state (D. 2012)

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