Dynamical Collapse in Quantum Theory

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Outline

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2. Markovian theory of dynamical collapse
3. Hypotheses of Universal Dynamical Collapse

1. Statistical Interpretation: One-Shot vs Dynamical
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    - Dynamical Non-selective Measurement, Collapse
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    - Revisit Early Motivations 1970-1980's

Early Motivations 1970-1980's

Interpretation of ψ is statistical.
Sudden 'one-shot' collapse $\psi \rightarrow \psi_n$ is central.

- If collapse takes time?
- Hunt for a math model (Pearle, Gisin, Diosi)
- New physics?

1-Shot Non-Selective Measurement, Decoherence

Measurement of $\hat{A}$, pre-measurement state $\hat{\rho}$, post-measurement state, decoherence: $\hat{A}=\sum_n A_n\hat{P}_n;\quad \sum_n \hat{P}_n=\hat{I},\quad \hat{P}_n\hat{P}_m=\delta_{nm}\hat{I}$

$$\hat{\rho}\rightarrow\sum_n \hat{P}_n\hat{\rho}\hat{P}_n$$

Off-diagonal elements become zero: Decoherence.
Example: $\hat{A}=\hat{\sigma}_Z=|\uparrow\rangle\langle\uparrow|-|\downarrow\rangle\langle\downarrow|,\ \hat{P}_\uparrow=|\uparrow\rangle\langle\uparrow|,\ \hat{P}_\downarrow=|\downarrow\rangle\langle\downarrow|,$

$$\hat{\rho}=\rho_{\uparrow\uparrow}|\uparrow\rangle\langle\uparrow|+\rho_{\downarrow\downarrow}|\downarrow\rangle\langle\downarrow|+\rho_{\uparrow\downarrow}|\uparrow\rangle\langle\downarrow|+\rho_{\downarrow\uparrow}|\downarrow\rangle\langle\uparrow|$$

$$\rightarrow\hat{P}_\uparrow\hat{\rho}\hat{P}_\uparrow+\hat{P}_\downarrow\hat{\rho}\hat{P}_\downarrow=\rho_{\uparrow\uparrow}|\uparrow\rangle\langle\uparrow|+\rho_{\downarrow\downarrow}|\downarrow\rangle\langle\downarrow|$$

Replace 1-shot non-selective measurement (decoherence) by dynamics!

Dynamical Non-Sel. Measurement, Decoherence

Time-continuous (dynamical) measurement of $\hat{A} = \sum_k A_k \hat{P}_k$:

$$d\hat{\rho}/dt = -\tfrac{1}{2}[\hat{A}, [\hat{A}, \hat{\rho}]]$$

Solution:

$$[\hat{A}, [\hat{A}, \hat{\rho}]] = \sum_k A_k^2 \hat{P}_k \hat{\rho} + \sum_k A_k^2 \hat{\rho} \hat{P}_k - 2 \sum_{k,l} A_k A_l \hat{P}_k \hat{\rho} \hat{P}_l$$

$$d(\hat{P}_n \hat{\rho} \hat{P}_m)/dt = -\tfrac{1}{2} \hat{P}_n [\hat{A}, [\hat{A}, \hat{\rho}]] \hat{P}_m = -\tfrac{1}{2}(A_m - A_n)^2 (\hat{P}_n \hat{\rho} \hat{P}_m)$$

Off-diagonals$\rightarrow 0$, diagonals=const

Example: $\hat{A} = \hat{\sigma}_z$, $d\hat{\rho}/dt = -\tfrac{1}{2}[\hat{\sigma}_z, [\hat{\sigma}_z, \hat{\rho}]]$

$$\hat{\rho}(t) = \rho_{\uparrow\uparrow}(0) |\uparrow\rangle\langle\uparrow| + \rho_{\downarrow\downarrow}(0) |\downarrow\rangle\langle\downarrow|$$

$$+ \mathrm{e}^{-2t} \rho_{\uparrow\downarrow}(0) |\uparrow\rangle\langle\downarrow| + \mathrm{e}^{-2t} \rho_{\downarrow\uparrow}(0) |\downarrow\rangle\langle\uparrow|$$

$$\rightarrow \rho_{\uparrow\uparrow}(0) |\uparrow\rangle\langle\uparrow| + \rho_{\downarrow\downarrow}(0) |\downarrow\rangle\langle\downarrow|$$

Master Equations

General non-unitary (but linear!) quantum dynamics:

$$d\hat{\rho}/dt = \mathcal{L}\hat{\rho}$$

Lindblad form — necessary and sufficient for consistency:

$$d\hat{\rho}/dt = -i[\hat{H},\hat{\rho}] + \left(\hat{L}\hat{\rho}\hat{L}^\dagger - \tfrac{1}{2}\hat{L}^\dagger\hat{L}\hat{\rho} - \tfrac{1}{2}\hat{\rho}\hat{L}^\dagger\hat{L}\right) + \ldots$$

If $\hat{L} = \hat{L}^\dagger = \hat{A}$:

$$d\hat{\rho}/dt = -i[\hat{H},\hat{\rho}] - \tfrac{1}{2}[\hat{A},[\hat{A},\hat{\rho}]]$$

Decoherence (non-selectiv measurement) of $\hat{A}$ competes with $\hat{H}$.
General case $\hat{H} \neq 0, \hat{L} \neq \hat{L}^\dagger$: untitary, decohering, dissipative, pump mechanisms compete.

1-Shot Selective Measurement, Collapse

Measurement of $\hat{A} = \sum_n A_n \hat{P}_n$; $\quad \sum_n \hat{P}_n = \hat{I}$, $\quad \hat{P}_n \hat{P}_m = \delta_{nm} \hat{I}$

General (mixed state) and the special case (pure state), resp.

mixed state:

$\hat{\rho} \rightarrow \frac{\hat{P}_n \hat{\rho} \hat{P}_n}{p_n} \equiv \hat{\rho}_n$

with-prob. $p_n = \mathrm{tr}(\hat{P}_n \hat{\rho})$

pure state, $\hat{P}_n = |n\rangle\langle n|$:

$|\psi\rangle \rightarrow |n\rangle \equiv |\psi_n\rangle$

with-prob. $p_n = |\langle n|\psi\rangle|^2$

Selective measurement is refinement of non-selective.

Mean of conditional states = Non-selective post-measurement state:

$$\mathbf{M}\hat{\rho}_n = \sum_n p_n \hat{\rho}_n =$$

$$= \sum_n \hat{P}_n \hat{\rho} \hat{P}_n \qquad\qquad = \sum_n \hat{P}_n |\psi\rangle\langle\psi| \hat{P}_n$$

Replace 1-shot selective measurement (collapse) by dynamics!

Dynamical Non-selective Measurement, Collapse

Take pure state 1-shot measurement of $\hat{A} = \sum_n A_n |n\rangle \langle n|$ and expand it for asymptotic long times:

$$|\psi(0)\rangle \text{ evolves into } |\psi(t)\rangle \rightarrow |n\rangle$$

Construct a (stationary) stochastic process $|\psi(t)\rangle$ for $t > 0$ such that for any initial state $|\psi(0)\rangle$ the solution walks randomly into one of the orthogonal states $|n\rangle$ with probability $p_n = |\langle n|\psi(0)\rangle|^2$!
There are ∞ many such stochastic processes $|\psi(t)\rangle$. Luckily, for

$$\hat{\rho}(t) = \mathbf{M}\,|\psi(t)\rangle\langle\psi(t)|$$

we have already constructed a possible non-selective dynamics, recall:

$$d\hat{\rho}/dt = -\tfrac{1}{2}[\hat{A}, [\hat{A}, \hat{\rho}]]$$

This is a major constraint for the process $|\psi(t)\rangle$. Infinite many choices still remain.

Dynamical Collapse: Diffusion or Jump

Consider the dynamical measurement of $\hat{A} = \sum_n A_n |n\rangle\langle n|$, described by dynamical decoherence (master) equation:

$$d\hat{\rho}/dt = -\tfrac{1}{2}[\hat{A}, [\hat{A}, \hat{\rho}]]$$

Construct stochastic process $|\psi(t)\rangle$ of dynamical collapse satisfying the master equation by $\hat{\rho}(t) = \mathbf{M}|\psi(t)\rangle\langle\psi(t)|$.

- Gisin's Diffusion Process (1984):

$$d|\psi\rangle/dt = -i\hat{H}|\psi\rangle - \tfrac{1}{2}(\hat{A} - \langle\hat{A}\rangle)^2|\psi\rangle + (\hat{A} - \langle\hat{A}\rangle)|\psi\rangle w_t$$

$$w_t : \text{standard white-noise;} \quad \mathbf{M}w_t = 0, \quad \mathbf{M}w_t w_s = \delta(t-s)$$

- Diosi's Jump Process (1985/86):

$$d|\psi\rangle/dt = -i\hat{H}|\psi\rangle - \tfrac{1}{2}(\hat{A} - \langle\hat{A}\rangle)^2|\psi\rangle + \tfrac{1}{2}\langle(\hat{A} - \langle\hat{A}\rangle)^2\rangle|\psi\rangle$$

$$\text{jumps } |\psi(t)\rangle \to \text{const.} \times (\hat{A} - \langle\hat{A}\rangle)|\psi(t)\rangle \text{ at rate} \langle(\hat{A} - \langle\hat{A}\rangle)^2\rangle$$

Dynamical Collapse: Diffusion or Jump - Proof

- Gisin's Diffusion Process (1984):

$$d|\psi\rangle/dt = -i\hat{H}|\psi\rangle - \frac{1}{2}(\hat{A}-\langle\hat{A}\rangle)^2|\psi\rangle + (\hat{A}-\langle\hat{A}\rangle)|\psi\rangle w_t$$

w_t : standard white-noise; $\mathbf{M}w_t = 0, \quad \mathbf{M}w_t w_s = \delta(t-s)$

- Diosi's Jump Process (1985/86):

$$d|\psi\rangle/dt = -i\hat{H}|\psi\rangle - \frac{1}{2}(\hat{A}-\langle\hat{A}\rangle)^2|\psi\rangle + \frac{1}{2}\langle(\hat{A}-\langle\hat{A}\rangle)^2\rangle|\psi\rangle$$

jumps $|\psi(t)\rangle \to$ const. $\times\,(\hat{A}-\langle\hat{A}\rangle)|\psi(t)\rangle$ at rate $\langle(\hat{A}-\langle\hat{A}\rangle)^2\rangle$

If $[\hat{H},\hat{A}]=0$, prove:

- $\hat{\rho}(t) = \mathbf{M}|\psi(t)\rangle\langle\psi(t)|$ satisfies $d\hat{\rho}/dt = -\frac{1}{2}[\hat{A},[\hat{A},\hat{\rho}]]$
- $|\psi(t)\rangle \to |n\rangle$
- $|n\rangle$ occurs with $p_n = |\langle n|\psi(0)\rangle|^2$

Revisit Early Motivations 1970-1980's

Interpretation of ψ is statistical.
Sudden 'one-shot' collapse $\psi \to \psi_n$ is central.

- If collapse takes time? — Why not!
- Hunt for a math model (Pearle, Gisin, Diosi) — Too many models!
- New physics?
  - No, it's standard physics of real time-continuous measurement (monitoring).
  - Yes, it's new!
    - to add universal non-unitary modifications to QM
    - to replace von Neumann statistical interpretation

2 Markovian theory of dynamical collapse

- Open System: Reduced Dynamics
- Master Equation
- Bath=Monitor
- Monitoring the free particle
- Monitoring the atomic decay
- Stochastic unravelling
- Three unravellings out of ∞
- Stochastic Schrödinger vs Stochastic Master Equation
- Summary: Dynamical Collapse

Open System: Reduced Dynamics

Our System (of interest) is part of a bigger closed system.
System+Environment (Reservoir, Bath, etc.) is closed, unitary:

$$d\hat{\rho}_{SB}/dt = -i[\hat{H}_S + \hat{H}_B + \hat{H}_{SB}, \hat{\rho}_{SB}]$$

Reduced state of our (open) System: $\hat{\rho}_S(t) = \mathrm{tr}_B\, \hat{\rho}_{SB}(t)$.
If $\hat{\rho}_{SB}(0) = \hat{\rho}_S(0)\hat{\rho}_B(0)$ then reduced dynamics exists:

$$\hat{\rho}_S(t) = \mathcal{M}(t)\hat{\rho}_S(0); \qquad \mathcal{M}(t): \text{ Completely Positive map}$$

Markovian (+semigroup) approximation: $\mathcal{M}(t) = \exp(\mathcal{L}t)$.
Markovian Master equation:

$$d\hat{\rho}_S(t)/dt = \mathcal{L}\hat{\rho}_S(t); \qquad \mathcal{L}: \text{semigroup generator}$$

Lindblad and GoriniKossakowskiSudarshan 1976:

$$d\hat{\rho}_{(S)}/dt = -i[\hat{H}, \hat{\rho}] + \left(\hat{L}\hat{\rho}\hat{L}^\dagger - \tfrac{1}{2}\hat{L}^\dagger\hat{L}\hat{\rho} - \tfrac{1}{2}\hat{\rho}\hat{L}^\dagger\hat{L}\right) + \ldots$$

Lindblad form is never unique, covariance group is trivial.

Master Equation

- Abstract system in random potential $\hat{V}(t) = -\hat{A}w_t$:

$$d\hat{\rho}/dt = -i[\hat{H},\hat{\rho}] - \tfrac{1}{2}[\hat{A},[\hat{A},\hat{\rho}]]$$

- Free particle in high T bath (friction ignored):

$$d\hat{\rho}/dt = -i[\hat{p}^2/2m,\hat{\rho}] - D[\hat{q},[\hat{q},\hat{\rho}]]$$

- Two-state system in vacuum ($T=0$ bath):

$$d\hat{\rho}/dt = -i[\omega\hat{a}^\dagger\hat{a},\hat{\rho}] - \Gamma\left(\hat{a}\hat{\rho}\hat{a}^\dagger - \tfrac{1}{2}\hat{a}^\dagger\hat{a}\hat{\rho} - \tfrac{1}{2}\hat{\rho}\hat{a}^\dagger\hat{a}\right); \quad \hat{a} = |g\rangle\langle e|$$

- Two-state system in heat bath $T>0$:

$$d\hat{\rho}/dt = -i[\omega\hat{a}^\dagger\hat{a},\hat{\rho}] - \Gamma\left(\hat{a}\hat{\rho}\hat{a}^\dagger - \tfrac{1}{2}\hat{a}^\dagger\hat{a}\hat{\rho} - \tfrac{1}{2}\hat{\rho}\hat{a}^\dagger\hat{a}\right) - \mathrm{e}^{-\omega/T}\Gamma\left(\hat{a}\leftrightarrow\hat{a}^\dagger\right)$$

Primary interpretation: reduced dynamics of various open systems. They all show dynamical decoherence (maybe competing with other mechanisms).

But how does dynamical collapse come in?

Bath=Monitor

$$d\hat{\rho}/dt = \mathcal{L}\hat{\rho}$$

How does dynamical collapse come in? Answer: Bath=Detector!
Footprints of System→Bath.
Bath does time-continuous (dynamical) measurement of the System.
Non-selective so far! To make it selective: monitor the bath!
As a result: you selectively monitor the System.
In ideal case you monitor $|\psi(t)\rangle$ of the system!
Math features: $|\psi(t)\rangle$ is a stochastic process, satisfying the Master Equation (top) by

$$\hat{\rho}(t) = \mathbf{M} |\psi(t)\rangle \langle\psi(t)|$$

Quantum ambiguity: the stochastic process $|\psi(t)\rangle$ depends on what quantum thing you decide to monitor on the bath.

Monitoring the free particle

Free particle (System) in photon beam (bath):

$$d\hat{\rho}/dt = -i[\hat{p}_\perp^2/2m, \hat{\rho}] - D[\hat{q}_\perp, [\hat{q}_\perp, \hat{\rho}]]$$

Monitor: photon scattering angles or $\perp$-locations.
In lab: detect scattering angles without or with lense inserted.
Two different diffusive processes (quantum trajectories) $|\psi(t)\rangle$:

- random Brownian (diffusive) motion, no spatial localization:

$$d\,|\psi\rangle\,/dt = -i(\hat{p}_\perp^2/2m)\,|\psi\rangle + i\sqrt{2D}\hat{q}_\perp w_\perp\,|\psi\rangle$$

- spatial localization competes with unitary spread:

$$d|\psi\rangle/dt = -i(\hat{p}_\perp^2/2m)|\psi\rangle - D(\hat{q}_\perp - \langle\hat{q}_\perp\rangle)^2|\psi\rangle + \sqrt{2D}(\hat{q}_\perp - \langle\hat{q}_\perp\rangle)w_\perp|\psi\rangle$$

  Balance $\hat{p}^2/m \sim D\hat{q}^2$ yields stationary localiation $\Delta q \sim (Dm)^{-1/4}$. (Exact solution: Diosi 1988).

Monitoring the atomic decay

Two-level atom (System) decaying into vacuum (bath):

$$d\hat{\rho}/dt = -\Gamma\left(\hat{a}\hat{\rho}\hat{a}^\dagger - \tfrac{1}{2}\hat{a}^\dagger\hat{a}\hat{\rho} - \tfrac{1}{2}\hat{\rho}\hat{a}^\dagger\hat{a}\right); \qquad \hat{a} = |g\rangle\langle e|$$

Monitor the photon by counter or by heterodyne detector.
Two different stochastic processes (quantum trajectories) $|\psi(t)\rangle$:

- Counter (jump process):

$$d|\psi\rangle/dt = -\tfrac{1}{2}\Gamma(\hat{a}^\dagger\hat{a} - \langle\hat{a}^\dagger\hat{a}\rangle)|\psi\rangle$$

$$\text{jump } |\psi\rangle \rightarrow |g\rangle \text{ at rate } \Gamma\langle\hat{a}^\dagger\hat{a}\rangle$$

  Deterministic gradual decay completed by a random jump (DalibardCastinMolmer 1992: MC Wave Function method)

- Heterodyne (diffusive process):
  $d|\psi\rangle/dt = -\tfrac{1}{2}\Gamma(\hat{a}^\dagger - \langle\hat{a}^\dagger\rangle)(\hat{a} - \langle\hat{a}\rangle)^2|\psi\rangle + \sqrt{\Gamma}(\hat{a} - \langle\hat{a}\rangle)|\psi\rangle\, w_t^\star$
  Diffusive gradual decay (GisinPercival 1992: Quantum State Diffusion, WisemanMilburn1996)

Stochastic unravelling

Open System reduced dynamics' Master Equation:

$$d\hat{\rho}/dt = \mathcal{L}\hat{\rho} = -i[\hat{H},\hat{\rho}] + \left(\hat{L}\hat{\rho}\hat{L}^\dagger - \tfrac{1}{2}\hat{L}^\dagger\hat{L}\hat{\rho} - \tfrac{1}{2}\hat{\rho}\hat{L}^\dagger\hat{L}\right) + \dots$$

Different choices of Bath can lead to same Master Equation.
The stochastic process $|\psi(t)\rangle$ (i.e: quantum trajectory) is called unravelling of the reduced dynamics if $\hat{\rho}(t) = \mathbf{M}\,|\psi(t)\rangle\langle\psi(t)|$ satisfies the master equation.
In general: ∞ many different unravellings. Some describe time-continuous collapse (e.g.: localization), some don't.
Any time-continuous measurement (monitoring) of S via monitoring the bath corresponds to a unique unravelling. Vice versa: Any unravelling corresponds to a unique time-continuous measurement of S via monitoring a suitably chosen bath.
Classification of all diffusive unravellings vs quantum optics monitorig: DiosiWiseman2001,GambettaWiseman2011.

Three unravellings out of ∞

$$d\hat{\rho}/dt = \mathcal{L}\hat{\rho} = -i[\hat{H},\hat{\rho}] + \left(\hat{L}\hat{\rho}\hat{L}^\dagger - \tfrac{1}{2}\hat{L}^\dagger\hat{L}\hat{\rho} - \tfrac{1}{2}\hat{\rho}\hat{L}^\dagger\hat{L}\right) + \dots$$

$$\hat{\rho}(t) = \mathbf{M}\,|\psi(t)\rangle\,\langle\psi(t)|$$

- Quantum State Diffusion ($\Leftrightarrow$ heterodyne measurement on B):

  $$d|\psi\rangle/dt = -i\hat{H}|\psi\rangle - \tfrac{1}{2}(\hat{L}-\langle\hat{L}\rangle)^\dagger(\hat{L}-\langle\hat{L}\rangle)|\psi\rangle + (\hat{L}-\langle\hat{L}\rangle)|\psi\rangle\, w_t^\star + \dots$$

  w_t : standard Hermitian white-noise; $\mathbf{M}w_t=0$, $\mathbf{M}w_t^\star w_s=\delta(t-s)$

- Ortho-Jump Process ($\Leftrightarrow$ tricky counter measurement on bath):

  $$d|\psi\rangle/dt = -i\hat{H}|\psi\rangle - \tfrac{1}{2}(\hat{L}-\langle\hat{L}\rangle)^\dagger(\hat{L}-\langle\hat{L}\rangle)|\psi\rangle + \tfrac{1}{2}\langle(\hat{L}-\langle\hat{L}\rangle)^\dagger(\hat{L}-\langle\hat{L}\rangle)\rangle|\psi\rangle$$

  jumps $|\psi(t)\rangle \rightarrow \text{const.}\times(\hat{L}-\langle\hat{L}\rangle\,|\psi\rangle$ at rate $\langle(\hat{L}-\langle\hat{L}\rangle)^\dagger(\hat{L}-\langle\hat{L}\rangle)\rangle$

- Jump Process ($\Leftrightarrow$ counter measurement on bath):

  $$d|\psi\rangle/dt = -i\hat{H}|\psi\rangle - \tfrac{1}{2}\hat{L}^\dagger\hat{L}|\psi\rangle + \tfrac{1}{2}\langle\hat{L}^\dagger\hat{L}\rangle|\psi\rangle$$

  jumps $|\psi(t)\rangle \rightarrow \text{const.}\times\hat{L}\,|\psi\rangle$ at rate $\langle\hat{L}^\dagger\hat{L}\rangle)$

Stochastic Schrödinger vs Stochastic Master Eq.

$$d\hat{\rho}/dt = \mathcal{L}\hat{\rho} = -i[\hat{H},\hat{\rho}] + \left(\hat{L}\hat{\rho}\hat{L}^\dagger - \tfrac{1}{2}\hat{L}^\dagger\hat{L}\hat{\rho} - \tfrac{1}{2}\hat{\rho}\hat{L}^\dagger\hat{L}\right) + \dots$$
$$\hat{\rho}(t) = \mathbf{M}\left|\psi(t)\right\rangle\left\langle\psi(t)\right|$$

Unravelling⇔Monitoring⇔Stochastic Schrödinger Equation:

$$d\left|\psi\right\rangle/dt = -i\hat{H}\left|\psi\right\rangle + \text{non-linear term} + \text{stochastic term}$$

Equivalent Stochastic Master Equation for $\hat{\rho}^\psi = \left|\psi\right\rangle\left\langle\psi\right|$:

$$d\hat{\rho}^\psi/dt = \mathcal{L}\hat{\rho}^\psi + \text{non-linear stochastic term}$$

$$\mathbf{M}[\text{non-linear stochastic term}] = 0.$$

Example: Monitoring the 1D free particle position
ME: $d\hat{\rho}/dt = \mathcal{L}\hat{\rho} = -i[\hat{p}^2/2m,\hat{\rho}] - D[\hat{q},[\hat{q},\hat{\rho}]]$
SSE: $d|\psi\rangle/dt = -i(\hat{p}^2/2m)|\psi\rangle - D(\hat{q}-\langle\hat{q}\rangle)^2|\psi\rangle + \sqrt{2D}(\hat{q}-\langle\hat{q}\rangle)w_t|\psi\rangle$
SME: $d\hat{\rho}^\psi/dt = \mathcal{L}\hat{\rho}^\psi + \sqrt{2D}(\hat{q}-\langle\hat{q}\rangle)w_t\hat{\rho}^\psi + \text{h.c.}$
Recall: balance $\hat{p}^2/m \sim D\hat{q}^2 \Rightarrow$ stationary localization $\Delta q \sim (Dm)^{-1/4}$.

Summary: Dynamical Collapse

System Reduced Dynamics (v.r.t. S+B) in Markovian Approximation:

$$d\hat{\rho}/dt = \mathcal{L}\hat{\rho} = -i[\hat{H},\hat{\rho}] + \left(\hat{L}\hat{\rho}\hat{L}^\dagger - \tfrac{1}{2}\hat{L}^\dagger\hat{L}\hat{\rho} - \tfrac{1}{2}\hat{\rho}\hat{L}^\dagger\hat{L}\right) + \dots$$

Pure state unravelling, constrained by $\hat{\rho}(t) = \mathbf{M}\,|\psi(t)\rangle\langle\psi(t)|$

Stochastic Schrödinger or Stochastic Master Eq. for each unravelling:

SSE: $d\,|\psi\rangle/dt = -i\hat{H}\,|\psi\rangle$ + non-linear term + stochastic term

SME: $d\hat{\rho}^\psi/dt = \mathcal{L}\hat{\rho}^\psi$ + non-linear stochastic term

Unravelling⇔Monitoring

- Standard physics
  - Theory of monitorig individual atomic systems
  - MC solution of the master equation
- New physics
  - Hypotheses of universal dynamic collapse
  - Make von Neumann measurement theory superfluous

3 Hypotheses of Universal Dynamical Collapse

- Universal decoherence
- 1 G-related and 2 G-unrelated examples
- ... and their Master Equations
- ... and their physical predictions
- Closing remarks

Universal decoherence

- macroscopic superpositions are apparently missing from Nature
- consistent quantum-gravity is apparently missing from Science
- would be better to replace von Neumann by Nature

Suppose mass density $f(\mathbf{r})$ matters. E.g. in Schrödinger Cat State:

$$|Cat\rangle = |f\rangle + |f'\rangle$$

where f and f' are "macroscopically" different.
Macroscopicity (catness) is measured by distance $\ell(f, f')$.
For concreteness: $[\ell^2]$=energy.
Suppose Nature makes $|Cat\rangle$ decay (decohere, or decohere and collapse) at mean life time

$$\tau = \frac{\hbar}{\ell^2(f, f')}$$

Careful choice of ℓ: no decay (extreme large τ) for atomic cats, immediate decay (small τ) for "macroscopic" cats.

1 G-related and 2 G-unrelated exammples

Coarse-grain f at $\sigma \sim 10^{-5}cm$, otherwise ℓ diverges.

- Gravity related, Diosi (1987), Penrose (1996)

$$\ell_G^2(f,f') = G\int\int[f(\mathbf{r})-f'(\mathbf{r})][f(\mathbf{s})-f'(\mathbf{s})]\frac{d\mathbf{r}d\mathbf{s}}{|\mathbf{r}-\mathbf{s}|}$$
$$= 2U(f,f') - U(f,f) - U(f',f')$$

- Gravity-unrelated, Ghirardi et al. (1990-...)

$$\ell_{GRW}^2(f,f') = \sum_k \frac{\hbar\lambda}{m_k}\int\left[\sqrt{f_k(\mathbf{r})}-\sqrt{f'_k(\mathbf{r})}\right]^2 d\mathbf{r}$$
$$\ell_{CSL}^2(f,f') = \frac{\hbar\lambda\sigma^3}{m_p^2}\int[f(\mathbf{r})-f'(\mathbf{r})]^2 d\mathbf{r}$$

$\lambda \sim 10^{-17}s^{-1}$, f_k=f of the k'th constituent of mass m_k, m_p=proton mass.

... and their Master Equations

Coarse-grain f at $\sigma \sim 10^{-5} cm$, otherwise r.h.s.'s diverge.

- Gravity related, Diosi (1987), Penrose (199?)

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] - \frac{G}{2\hbar}\int\int[\hat{f}(\mathbf{r}),[\hat{f}(\mathbf{s}),\hat{\rho}]]\frac{d\mathbf{r}d\mathbf{s}}{|\mathbf{r}-\mathbf{s}|}$$

- Gravity-unrelated, Ghirardi et al. (1990-...)

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] - \sum_k \frac{\lambda}{2m_k}\int\left[\sqrt{\hat{f}_k(\mathbf{r})},\left[\sqrt{\hat{f}_k(\mathbf{r})},\hat{\rho}\right]\right]d\mathbf{r}$$

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] - \frac{\lambda\sigma^3}{2m_p^2}\int[\hat{f}(\mathbf{r}),[\hat{f}(\mathbf{r}),\hat{\rho}]]d\mathbf{r}$$

$\lambda \sim 10^{-17}s^{-1}$, f_k=f of the k'th constituent of mass m_k, m_p=proton mass.

... and their physical predictions

$$|Cat\rangle = |f\rangle + |f'\rangle$$

Rigid Ball Cat: mass M, size R, "water" density, dispacement $\sim R$

Free motion and decoherence (collapse) are balanced if

$$\hbar^2/MR^2 \sim \ell^2 \qquad \Rightarrow R = R_c$$

$R \ll R_c$: free motion dominates; $R \gg R_c$: decoherence dominates.

$$\ell_G^2(f,f') = G\int\int [f(\mathbf{r}) - f'(\mathbf{r})][f(\mathbf{s}) - f'(\mathbf{s})]\frac{d\mathbf{r}d\mathbf{s}}{|\mathbf{r}-\mathbf{s}|} \sim \frac{GM^2}{R}$$

$$\ell_{GRW}^2(f,f') = \sum \frac{\hbar\lambda}{m_k}\int\left[\sqrt{f_k(\mathbf{r})} - \sqrt{f_k'(\mathbf{r})}\right]^2 d\mathbf{r} \sim \frac{\hbar\lambda M}{m_p}$$

$$\ell_{CSL}^2(f,f') = \frac{\hbar\lambda\sigma^3}{m_p^2}\int [f(\mathbf{r}) - f'(\mathbf{r})]^2 d\mathbf{r} \sim \frac{\hbar\lambda M^2\sigma^3}{m_p^2 R^3}$$

For G,GRW,CSL all: $R_c \sim 10^{-5} cm \Rightarrow$ ignorable decoherence on atomic scales, immediate decoherence for bodies $\gg 10^{-5} cm$.

Closing remarks

- Master Equations predict everything that is testable
- G,GRW,CSL have their standard stochastic unravelling each
- Several experiments aim at testing G,GRW,CSL
- Environmental noise outmasks G,GRW,CSL decoherence
- G,GRW,CSL predict but additional universal noise
- More characteristic predictions? $\Leftarrow$ More radical models!