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2010 J. Phys.: Conf. Ser. 218 012007

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Linear waves on fixed Kerr background and their relevance in jet formation *

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Abstract. In the conventional models of astrophysical jets magnetohydrodynamics is believed to play the most role. Recently there were several qualitative arguments emphasizing that the role of pure gravitational effects might be more important than expected before. Here we present some preliminary but quantitative results clarifying the role of gravity in the formation of astrophysical jets.

1. Introduction

A great number of astrophysical observations, see, e.g. [Mirabel&Rodriguez1994], [Tingay1995], justified the existence of high energy highly collimated matter streams. The sources are usually compact celestial objects while the yielded matter currents are referred to as astrophysical jets. The conventional explanation of their existence is based on the use of magnetohydrodynamics. The emitting compact object is assumed to possess strong magnetic field which interacts with the surrounding ionized matter. The corresponding investigations justified the formation of matter currents confined to the axis of the magnetic field (see, e.g., Marti 2003).

Recently, there were several authors proposing alternative explanations for the formation of jets where gravitational effects were assumed to play central role. For instance, in Gariel (2007) the following possibility, making use of the Penrose process, was considered. A steady fall of particles of an accretion disc towards the central Kerr black hole was considered and the Penrose process (see below) was used to explain the formation of strongly collimated streams of outgoing particles along the axis of rotation. However, in Gariel (2007) beside introducing a possible mechanism no quantitative investigations were made whence no reliable estimation on the relevance of the proposed effect could be provided. The main motivation for the present authors was to derive these type of estimates by studying the evolution of a linear field on a fixed Kerr background.

2. Penrose process and super-radiance in Kerr black hole spacetime

It is well-known (see, e.g., Wald 1984) that each rotating Kerr black hole possesses a subregion – called the ergoregion of the black hole – in the domain of outer communication, where the

* This article is dedicated to the memory of our friend and colleague Péter Csizmadia, the original author of the GridRipper. Péter Csizmadia disappeared in October 2009 during a first ascent attempt of the unclimbed peak Ren Zhong Feng in the Chinese Himalayas.

asymptotically timelike Killing vector field t^a is spacelike. In this region, the energy $g_{ab}t^ap^b$ of a point particle with timelike momentum vector p^a may become negative. The Penrose process can be used to extract energy from a rotating Kerr black hole as follows. Shoot an unstable point particle into the ergoregion which is prepared such that it decays into two particles there with one of them having negative energy. This negative energy particle cannot exit the ergoregion as the energy of point particle in the asymptotic region has to be positive. Thereby, it is captured by the black hole. Then, in virtue of the energy conservation, the other yielded particle, has to have positive energy – in fact larger than that of the original particle sent into the ergoregion – whence it escapes to infinity. What was shown in Gariel (2007) is that in a rotating Kerr black hole spacetime there exists a class of causal geodesics, the elements of which start at the ergoregion, and leaves the ergoregion asymptotically parallel to the rotation axis. If these trajectories were used by the escaping particles of the Penrose process one could get a viable explanation of jets.

Recall now that the super-radiance provides a much less ambiguous process of energy extraction from black hole spacetimes. As shown in Carter (1968), the d'Alembert operator over Kerr spacetime may be separated in Boyer-Lindquist coordinates t, r, ϑ, φ . The timelike Fourier transform of a solution may be written in the form

$$\sum_{l=0}^{\infty} \sum_{m=-l}^l R_{l,\omega}^m(r) S_{l,\omega}^m(\vartheta, \varphi),$$

where ω is the frequency in the time translation direction, $S_{l,\omega}^m$ is the oblate spheroidal harmonic function with oblateness parameter ω and with angular momentum quantum numbers l and m . The $R_{l,\omega}^m$ radial functions may be determined by the radial field equations. If one takes a plane wave solution of the form $\exp(-i\omega t)\phi(r, \vartheta)\exp(im\varphi)$, then the energy for a massless Klein-Gordon field absorbed through the event horizon can be written as

$$\frac{1}{2}\omega(\omega - m\Omega_H)|\phi|^2,$$

where Ω_H is the angular velocity of the black hole (Wald 1984). Thus the energy inflow is negative, whenever $0 < \omega < m\Omega_H$. If this happens the backscattered wave carries more energy than the wave sent towards the black hole. This field theoretical analogue of the Penrose process is called super-radiance which is known to occur for all matter fields satisfying linear wave equations.

3. The anisotropy of linear waves on Kerr background

This paper is to clear up the significance of collimation effect proposed by Gariel (Gariel 2007). In doing so we shall consider the time evolution of massless Klein-Gordon field on fixed rotating black hole spacetime where super-radiance may occur. The applied numerical method provides the opportunity of making quantitative investigations.

3.1. The numerical method

To carry out our investigations, we developed the 3+1 dimensional version of the PDE solver, GridRipper (Csizmadia 2009, Csizmadia 2006, Csizmadia 2007). The former version of GridRipper was an 1+1 dimensional fourth order finite difference PDE solver with adaptive mesh refinement (AMR) for solving hyperbolic equations. It is based on the Berger-Oliger algorithm: refinement is performed whenever the Richardson error of a predefined field quantity exceeds a threshold. In the 3+1 dimensional version the polar directions are taken into account by expanding the fields with respect to the L^2 basis of spherical harmonics, and the spectral

components up to $l = 24$ are evolved. Since on a rotating Kerr spacetime the wave equation immediately involves multilinear expressions of the multipole components the applicability of involved spectral method is not immediately obvious. The applicability of spectral methods may be justified by applying Sobolev embedding arguments which can be found in Csizmadia (2010).

3.2. Results and discussion

As indicated above we considered the evolution of the massless Klein-Gordon equation on a fixed Kerr spacetime. The black hole is parameterized by its mass M and its specific angular momentum a . The initial data was assumed to be radially non-expanding possessing the form

$$\begin{aligned}\Phi|_{t=0} &= \exp(-i\omega t) \phi(r, \vartheta) \exp(im\varphi)|_{t=0}, \\ \partial_t \Phi|_{t=0} &= \partial_t (\exp(-i\omega t) \phi(r, \vartheta) \exp(im\varphi))|_{t=0},\end{aligned}$$

where the profile of ϕ was chosen such that the initial condition represented a rotating toroidal ring. The parameter ω was non-negative real, while m was chosen to be an integer. The energy density, angular momentum density and energy current density distributions are shown on Fig. 1, relevant for a configuration with parameters $M = 1$, $a = 0.5$, $\omega = 1$, $m = 1$. On the right panel an apparent preference of the axis of rotation is visible.

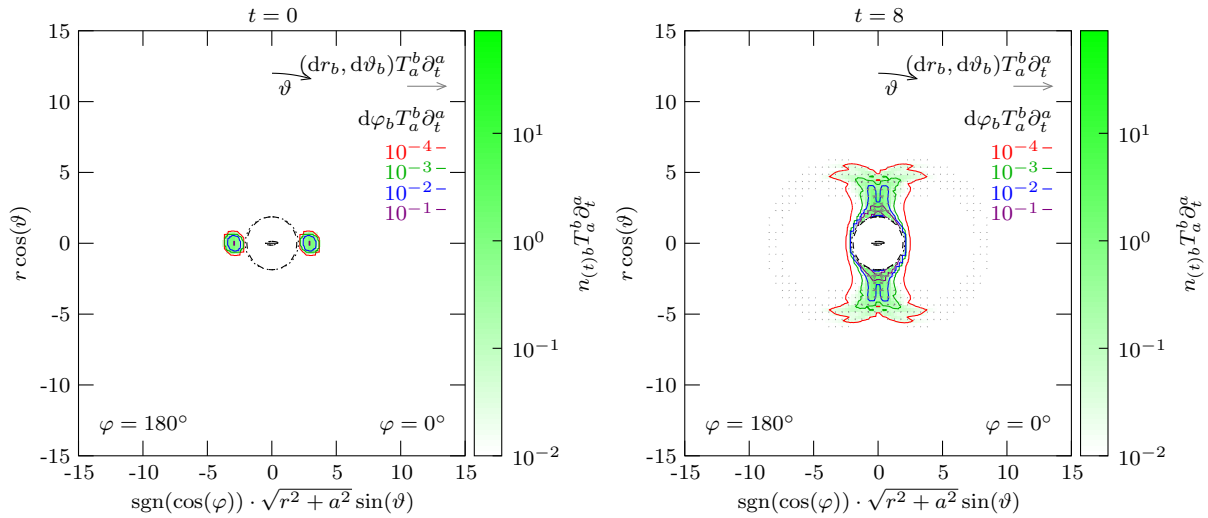


Figure 1. Initial condition at $t = 0$ (left panel) and intermediate state at $t = 8$ (right panel) of a massless Klein-Gordon field over Kerr background spacetime, with parameters $M = 1$, $a = 0.5$, $\omega = 1$ and $m = 1$.

To characterize the radiation anisotropy, the following measure of anisotropy was used. Denote by $E_{\text{out}}|_{\bar{\vartheta}}(t)$ the outwards radiated energy within the cone with half-angle $\bar{\vartheta}$, $0 \leq \bar{\vartheta} \leq \pi$, measured at an outer boundary as the function of coordinate time t . Then the quantity

$$\mathcal{E}_{\bar{\vartheta}}(t) = \frac{4\pi}{2\pi(1 - \cos(\bar{\vartheta}))} \cdot \frac{E_{\text{out}}|_{\bar{\vartheta}}(t)}{E_{\text{out}}|_{\pi}(\infty)}$$

provides a measure of angular preference of the energy of the radiation. $\mathcal{E}_{\bar{\vartheta}}(t)$ tends to 0 whenever the radiation prefers the equatorial directions, whereas it tends to 1 in the case of spherically symmetric radiation, while $\mathcal{E}_{\bar{\vartheta}}(t)$ tends to a value larger than 1 when the radiation prefers the axis of rotation. Obviously, an analogous quantity $\mathcal{L}_{\bar{\vartheta}}(t)$ can also be constructed to measure the angular momentum anisotropy of the radiation.

The preliminary results of our numerical investigations are shown on Fig. 2 for the specific choice $\bar{\vartheta} = 30^\circ$ and for spacetimes $M = 0, a = 0$ (Minkowski limit), $M = 1, a = 0$ (Schwarzschild limit) and $M = 1, a = 0.5$ (a rotating Kerr spacetime). The parameters of the initial field configurations are $\omega = 1, m = \pm 1$ (for co-rotating or counter rotating field) and $\omega = 1, m = 0$ (for non-rotating field). As it is shown by these panels the energy radiation anisotropy is a little bit higher for the case of black hole spacetimes. However, as the energy anisotropy measured in case of Schwarzschild or Kerr black holes are almost the same it cannot be attributed to the super-radiance. The slight difference indicated by the right bottom panel justifies that the rotation of the background yields an anticipated asymmetry in the angular momentum anisotropy. These results do not seem to support the method proposed in Gariel (2007).

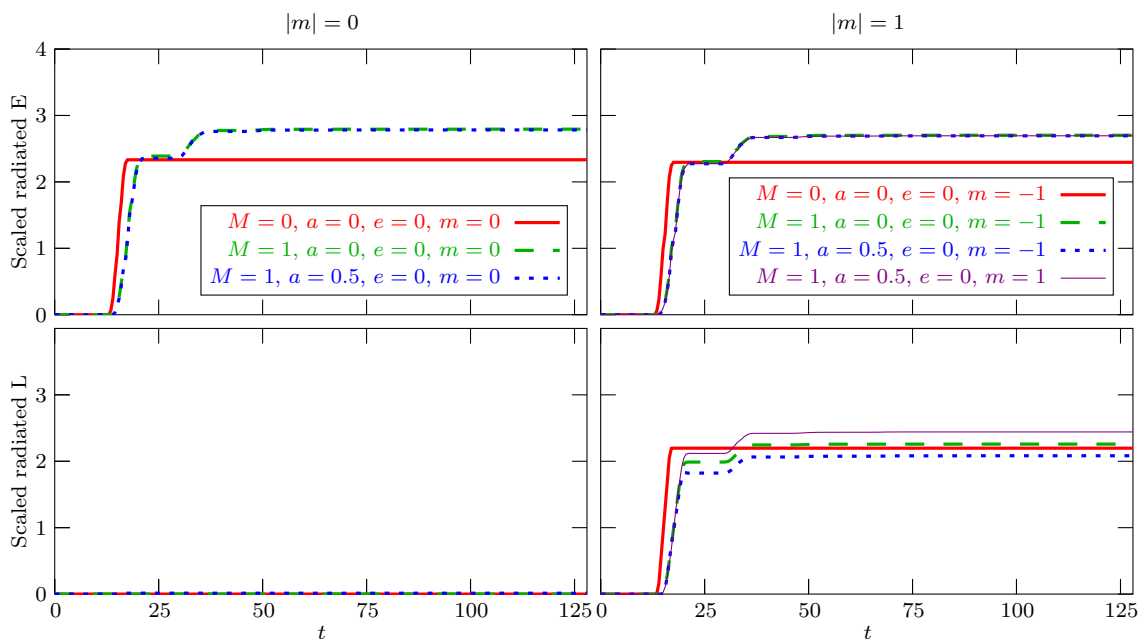


Figure 2. Energy (top panels) and angular momentum (bottom panels) radiation anisotropy in the $\bar{\vartheta} = 30^\circ$ cone, for initially non-rotating (left panels) and initially rotating (right panels) fields are shown.

Acknowledgments

This research was supported in part by OTKA grant K67942.

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